**Introduction to Fluid Mechanics**
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**Lecture – 24**
**Deformation of fluid elements Part-I**

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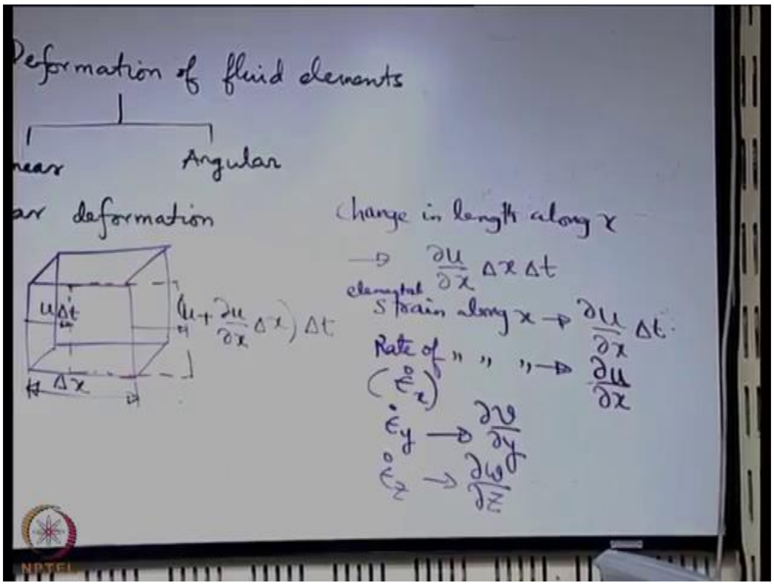


Next what we will do we will start analyzing the deformation of fluid elements. Why this is very important because we have seen that fluids are characterized by deformation they undergo continuous deformation on the under the action of even a very small shear force and the relationship between the sheer force and or the shear stress and the rate of deformation is something which is unique to the constitutive behavior of different fluids.

So, we must first understand that how to characterize deformation of fluid elements in terms of the velocity components. Once we understand that it will be possible for us to mathematically express different types of deformations in terms of the velocity components say u v and w, when we do that we have to keep in mind that we will be essentially bothering about 2 types of deformations, one is the linear and another is the angular deformation. When we talk about the linear deformation, it may eventually give raise to a change in volume of the fluid element also because if you have a length element and the linked element gets changed volume element is comprising of several such length elements.

So, if length linear dimension gets change, the volume is also likely to get changed. Initially we will think of how we can say estimate the linear deformation. So, we will start with the linear deformation. To understand or to get a visual feel we will consider a fluid element like this maybe we may considered even a 3 dimensional fluid element if you want, but that will not make the thing more complicated because at the end we will be dealing with linear deformations in individual directions. See why we use coordinate system for analyzing problem? The reason is like say when you think of x, y, z, is a Cartesian coordinate systems there are independent coordinates a combination of which describe the total effect in the system.

So, when you are thinking of a linear deformation along x, you maybe decoupled from what is the linear deformation along y and z and these individual effects you can super impose because you are dealing with linearly independent components and these vector components actually give you linearly independent basis vectors like components along x, y or z. So, similar concept whenever we are considering change along x may be we are bothered, only with respect to like what is the change in the linear dimension along x, this regarding what happens along y and z.

So, let us keep that target let us say that delta x is the length of the fluid element which is originally there and now what is happening? Now we are having a change in time and because of a change in time, now you see that let us consider the front face of this cuboid. So, this left face over a time interval of delta t will traverse a displacement will undergo a displacement, what is that displacement? If u is the velocity at this location simply u into delta t. We are considering the time interval delta t it is very small. So, it is like just a product of the velocity into delta t. The right phase we will also undergo some displacement, what is that?

So, if this is the x direction, the new u here is same is not the same as the u at the left face, but this is because of change in u due to change in x. So, this is the new u time’s delta t. So, if you consider only the front face and only subjected to this motion, say we freeze all other events just for a clear picture say may be now, it is having a new configuration shown by this dotted line. So, what is the change u needs? Length along x that is the final length minus the original length. So, what is the final length? So, what is the net change? See the right hand face has got displaced by this amount the left hand face has got displaced by this amount.

So, the net displacement is the difference between these 2. So, what is that change in length? So, change in length along x, this is the change in length along x, what is therefore, the strain along x, the change in length per unit length. So, the strain along x this is the elemental strain because we have considered only a small part of the fluid which is having an extent of delta x. So, this is elemental strain along x as we have discussed earlier we are not just interested about the strain for a fluid because if you allow it to grow in time the strain will be more see if this delta t is larger and larger and we integrate it over a large interval of time this we will trivially more and more.

So, measuring strain in a fluid is nothing that is important it is just a function of the time that is elapsed what is more important is the rate of deformation or the rate of strain. So, the rate of strain along x what is that it is basically this divided by delta t as delta t tends to 0. So, when it is say rate; that means, the time rate we always implicitly mean that. So, that will be simply the partial derivative of u with respect to x we may give it shorter notation say epsilon dot x similarly just from your common sense you can say what will be epsilon dot y and what will be epsilon dot z. So, what is epsilon dot y and epsilon dot z is this? So, we have been successful in finding out a very simple thing, what is the rate of linear deformation along x, y and z in terms of the velocity components?

So, if you are given u as a function of position, v as a function of position and w as a function of position, by simple partial differentiation it will be possible to find out the rates of change. Now we are interested not only just in terms of the rate of change in the linear dimension, but may be rate of change in the volume. So, to understand that; what is the rate of change in the volume: let us say that we are having this fluid element which has dimensions along x, y, z as delta x, delta y and delta z.

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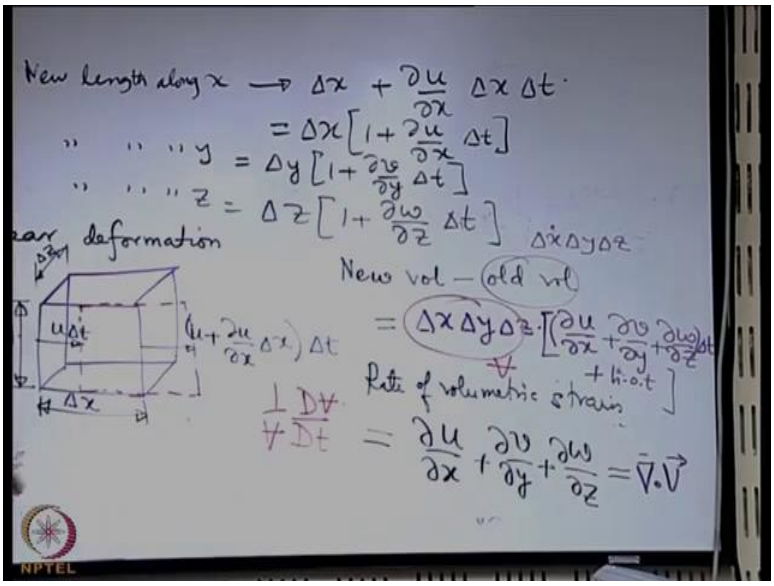


So, we set up coordinate axis as this is x, this is y and this is z, delta x, delta y and delta z. Now what is the new length? So, we are interested to get the new volume. So, what is the new volume? The new volume is new length along x into new length along y into new length along z. So, what is the new length along x? That is the old length plus the change. So, the old length is delta x plus the change is this one. So, we can take delta x common and write this one similarly it is possible to write what is new length along y and new length along z. So, let us complete those expressions. So, the new volume is a product of these 3.

So, what is the new volume minus the old volume, yes? So, what you have to do you have to find out the product of this then subtract the old volume that is delta x into delta y into delta z we will see that the first the delta x into delta y into delta z that term will go away will get cancelled then out of the remaining terms you have to neglect the terms of may be higher order in delta. So, like if you have products like delta t square or delta x into delta t square that type of term you tend to neglect because those are higher order terms. So, retain only the leading order terms because you have to keep in mind that you are dealing with a situation again as delta x delta y delta z delta t all tending to 0 and then what will be the term that is remaining here, yes.

So; obviously, a product of these 3 is there when then what is the remaining term? This plus higher order terms will be there that into delta t right other terms will be of order higher then delta t. So, what is the volumetric strain the rate of volumetric strain? So, the rate of volumetric strain is the change in volume per unit, volume per unit time just like what we found out for the linear strain. So, when you say find out per unit volume you are basically dividing it by this delta x delta y into delta z. So, this is like the original volume, let us give it a symbol v with a strike through. So, that is the original volume. So, what is the rate of volumetric strain? This change in volume divided by volume divided by delta t and take the limit as delta t tends to 0 when the all other higher order terms in the limit will be 0.

So, it is not that we are neglecting the one delta t, here will remain even after division by delta t that will be tending to 0 as in the limit delta t tends to 0. So, then what will be the final expression of this some of these 3. So, we may write it in terms of the total derivative see the volumetric strain it maybe it may be due to many things change in time change in position and the combination. So, we are not bothered about that what is the individual effect? We are bothered about the total effect. What is the net change in the fluid element volume because of this? So, this should be expressible in terms of the total derivative. So, it is capital D D t of the volume with per unit volume. This is the rate of volumetric strain and in terms of the vector calculus notation, you can also write it as the divergence of the velocity vector.

This leads to a very important definition the definition is with regard to incompressible flow. So, when we say that a flow is incompressible. So, incompressible flow by the name it is clear that we are looking for a case when the fluid element does not change in volume.

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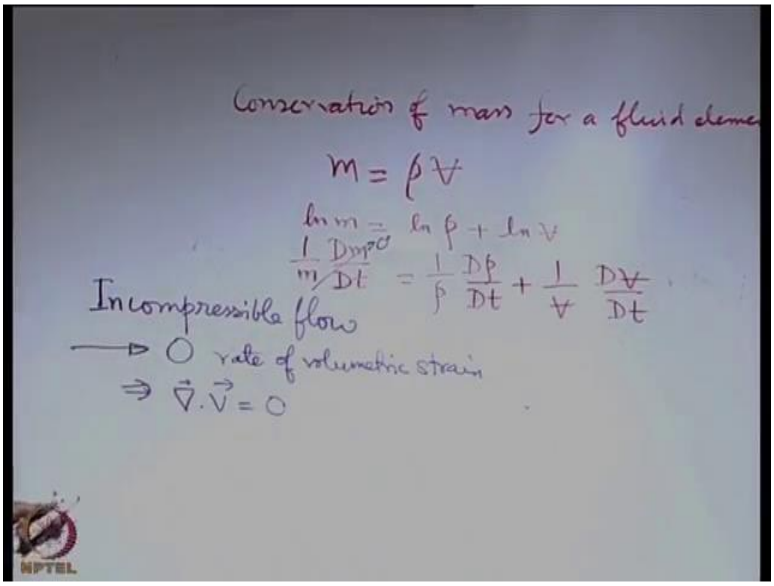


So, incompressible flow, it will have what signature one and only important signature 0 rate of volumetric strain because the fluid element may not be changing its volume that is the meaning of that is even the literal meaning of incompressible that you cannot really compress it. So, 0 volumetric rate of volumetric strain and that boils down to the divergence of the velocity vector is equal to 0.

So, if you are given a velocity field and your ask to check whether it is compressible or incompressible flow then it is possible to check by looking into the fact whether it is satisfying this equation or not if its satisfies this equation we say that it is an incompressible flow keep in mind the distinction between this definition and incompressible fluid definition. So, earlier we also introduced the concept of incompressible fluid and we said that a fluid is incompressible if its density does not significantly change with change in pressure. So, that is incompressible fluid now you are talking about incompressible flow and these 2 are again related, but different concepts that we have to keep in mind. So, when you are having an incompressible flow it is possible to characterize the particular flow in terms of its mechanism by which its satisfies the overall conservation of mass to understand how it does let us try to write an expression for conservation of mass of the fluid element.

So, we will now write conservation of mass for fluid for a fluid element expression. Let us say that a mean the mass of a fluid element, you can express it in terms of the density and the volume. Let us say that rho is the density and v is the volume since the mass is conserved of a fluid element. So, there will be 0 rate of change of mass. So, the since we know already the expression for the volumetric strain and in that volumetric strain one by v appears it may be useful to utilize that expression by taking log of both sides and then differentiating because then one by we will automatically come out.

So, let us take the log of both sides and then differentiate with respect to time when we say we want to differentiate with respect to time it has to be a total derivative. So, because it is a fluid element now it may have change with respect to change in position time whatever we are bother about now the total effect because the conservation of mass is not for individual effects is a combination of total effects that give rise to a mass of a fluid element is conserved. So, when we write say when we differentiate it with respect to time by keeping that in mind we have the left hand side like this which again becomes 0 because the mass of the fluid element is conserved the right hand side 1 by rho.

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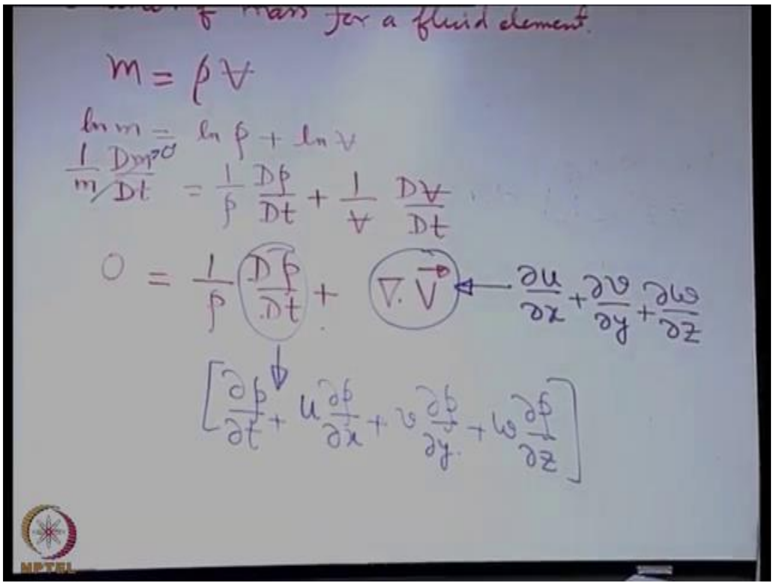


So, you have 0 equal to 1 by rho d rho d t plus 1 by v d, v d t is what is the divergence of the velocity vector or if you write in terms of the Cartesian coordinates. When it till you write it in a vector from it is a coordinate system independent, but when you write its corresponding say components then the components depend on how you take your reference. So, in a Cartesian reference, it is this now let us write this one what will be this use the definition of the total derivative now you can multiply both sides by rho because density of the fluid is not 0. So, you can multiply both sides by rho and then if you multiply both sides by rho.

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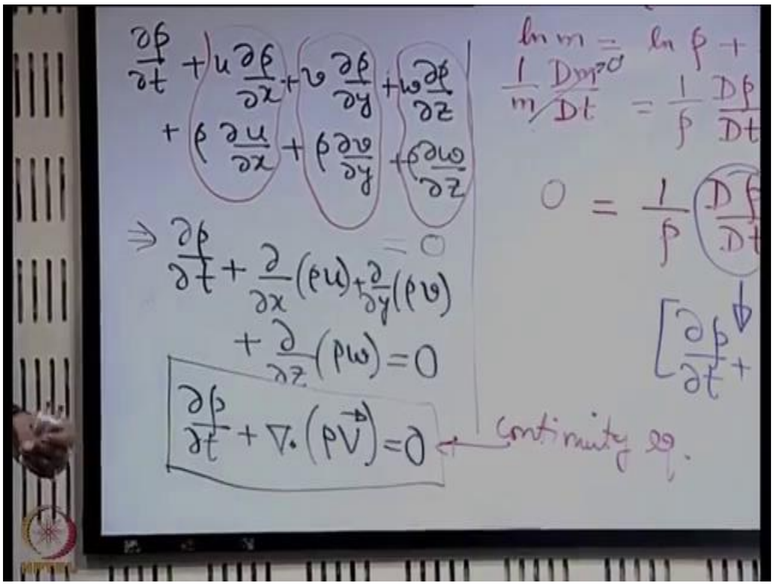


What you get? Now you can combine these types of terms and write then in a compact form by using the product rule of differentiation.

So, this equal to 0 will imply this equal to 0 just by using the product rule of differentiation. It is again possible to write it in a compact vector notation this becomes divergence of rho into velocity vectors that is equal to 0 this equation in a general understanding is supposed to be the most fundamental differential equation in fluid mechanics because no matter how complex or how simple the flow field is it should satisfy the law of conservation of mass.

So, this is a differential equation expressing the law of conservation of mass for a fluid element and this is known as continuity equation. So, if you are given a velocity field you must first check whether it is satisfying the continuity equation if it does not satisfy the continuity equation it is an absurd velocity field it may be mathematically something, but it does not physically make any sense because it has to satisfy the mass conservation now briefly let us look into certain special cases of this.

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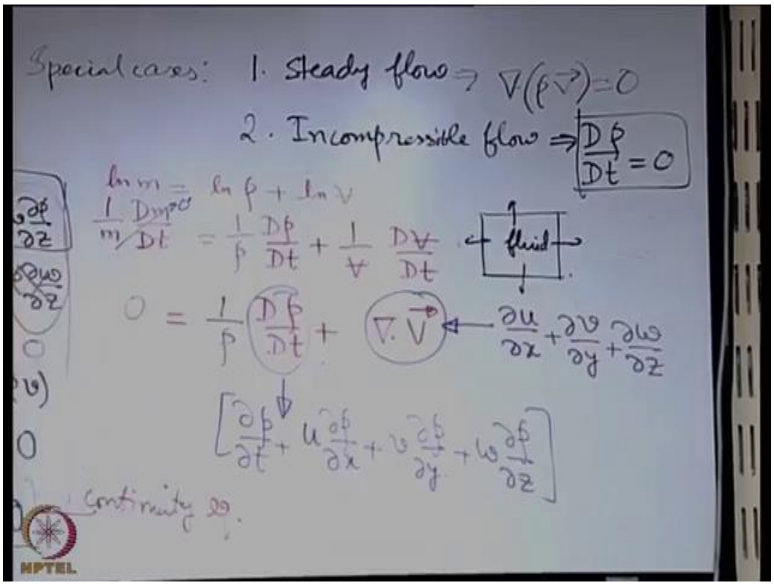


So, what are the special cases? The first special case; we considered as steady flow. So, when you consider a steady flow then how this equation gets simplified to. So, steady flow means the first term at a given remember what is the definition of steady at a given position any fluid property will not change with time. So, at a given position that is why the partial derivative with respect to time; that means, keeping the position frozen you are trying to find out the change in density with respect to time. So, that is 0 if it is a steady flow. So, steady flow will have that term equal to 0. So, that will boil down to, but that does not ensured that rho is a constant rho might not be a function of time, but might be a function of position.

So, still rho remains inside the derivative, it does not get disturbed, let us consider a second case incompressible flow, we have to keep in mind that there is a very big miss concept that we should try to avoid, what is that? When many times we loosely saying incompressible means density is the constant it is the special case of incompressible flow, but it is not a general case of incompressible flow because general case of incompressible flow is what the divergence of velocity vector equal to 0 that is the definition.

Now, where does it ensure that rho is a constant? That basic definition never ensured that rho is a constant. At the same time, it can be shown that if rho is a constant and this will be satisfied. So, the converse is true; that means rho is a constant is a special case of incompressible flow, but it is not a general case. What is the general case? Let us look into that. So, when you are looking about the general case you have to see the continuity equation. So, if you when you look at the continuity equation look into this primitive form that is not the compact form what does form before that. So, here when you have an incompressible flow which terms will go away these terms will go away because divergence of the velocity vector is 0. So, then what you are left with the total derivative with respect to total derivative of density with respect to time is 0.

So; that means, in compressible flow means d rho d t capital d rho d t equal to 0 see a very interesting things it does not mean that rho is a constant because rho might be a function of position and time in such a way that these collection of terms eventually gives rise to 0 if rho is a constant these collection will definitely give it to be 0, but that is a trivial solution; that is the trivial solution to the case that is rho is a constant therefore, any derivative of rho with respect to time or position is 0, but even if any derivative of rho with respect to position and time is not 0 still the net effect may be 0 and then even though rho is a variable we will say that the flow is incompressible.

So, a variable density flow may also be an incompressible flow this is a very important concept. So, incompressible flow need not always be a constant density flow. So, typical example is let us say that you have domain like this within this is there is a fluid now this fluid changed its face say it was in a particular phase say it was in liquid phase now it becomes a vapor phase. So, when it becomes vapor phase it becomes light a. So, the same as now cannot occupy these volumes.

So, there is. So, it wants to occupy an extra volume, but given a particular volume what it will do some extra mass will leave because you are constraining the volume and if you are having a change in density you must have a flow to accommodate at change in density so that whatever fluid is there now is acc accommodated within the volume that was given to you. So, you can see that you might have a change in density at a fixed position with time because may be with time the face change has triggered. So, with time the density has changed. So, this has to be now adjusted with some u, v, w, so that the net effect may still be 0. So, it might so happen that now here the net affect; it may be 0. It may not be 0. So, let us let us take an example, but the net effect is 0, what is that example maybe there was a fluid?

Now, it is getting frozen and because of freezing, its volume gets changed. So, it is possible. So, its density has got changed, but we do not call say a liquids or solids as compressible fluids. So, what has happened because if with freezing there is shrinkage then there will be a deficit in volume here maybe to satisfy the deficit in volume there might be a material supply from all sides. So, it is possible that to make a balance of what is happening locally and what is happening over the volume element you might have to adjust these things with a velocity across the different faces of the element.

So, in summary, we can say that incompressible flow definition is the total derivative of rho with respect to t is 0, but not just rho is a constant. We will stop here today, we have just seen one way of deriving the continuity equation. What we will learn more by having different ways of deriving the continuity equation and that we will do in the next class.

Thank you.