Introduction to Fluid Mechanics
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Lecture – 23
Acceleration of fluid flow

We were discussing about different flow visualization lines in our last class and we will discuss one more flow visualization line which is called as timeline.

So, what is the time line? If you have a snapshot at a particular time in the flow field where you mark nearby particles, so nearby fluid particles which are located in the flow field at a given instant of time, if you somehow mark those particles by somewhere then if you now get the snapshot at different times, it will give a picture of evolution of the flow field as a function of time and that is known as a timeline.

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So, it is nothing but like snapshot of nearby fluid particles at a given instant of time that is called as a timeline. So, let us look into a small movie to see that what we mean by a timeline. So, if you see now this gives a snapshot at different instance of time of nearby fluid particles and in a way, it gives a sense of the velocity profiles at different instance of time you can see that in this example the flow passage is narrowing another flow passage is narrowing the fluid is moving faster to make sure that the mass flow rate is conserved we will see later on that formally this is described by the continuity equation and in may be a differential form or an integral form.

But at least this gives us a visual idea of what the timeline is all about. Now with this background on the flow visualization lines we have now understood that how we can visualize the fluid flow in terms of some imaginary description like through the stream lines, strip line, path line or maybe the timeline. Next we will go into the description of acceleration of fluid flow. So, we have discussed about the velocity. The next target is the acceleration. Let us say that you have fluid particle located at a position P at specifically the location P 1 at time equal to t and how the velocity is described here? The velocity described is described here through a velocity vector v which is a function of r 1 that is the position vector of the point P 1 and the time t.

This is nothing, but the Eulerian description, if you write it in terms of components; you can write an equivalent scalar component description that you have u as a function of x, y z and t; v as a function of x, y, z, t and w as another function of x, y, z t. So, we are trying to describe it in terms of Cartesian coordinates, it is not always necessary to do that, but it may be a simple way to demonstrate one we use other coordinate systems as well. So, if you are using a Cartesian coordinate system 3 independent coordinates space coordinates plus time coordinate that together give the velocity at a particular point.

So, if the fluid particle is located at P 1, the velocity at that point is basically the velocity of a fluid particle located at that point and that is given by these components. Now let u say that at a time of t plus delta t, this thing get changed now at a time plus delta t what happens? This fluid particle is no more located at this point the fluid particle is located at a different point. So, let us say that the fluid particle is located at a point P 2. So, at the point P 2, now let us say that the velocity is whatever some arbitrary velocity. So, initially it may be velocity at the point 1 say v 1.

Now, it is v 2, which is again a function of its local position and time. So, you have this v 2, this one a function of what? So, let us say that it is given by its components u plus delta u, v plus delta v, w plus delta w, these are functions of what these are functions of the new position vector, the new position vector say is r 1 plus delta r 1. So, in terms of scalar components it may be x plus delta x, y plus delta y, z plus delta z and the time has also now changed, it has become t plus delta t. So, we are thinking about a small interval of time delta t over which the fluid particle has undergone some displacement which is a change in position vector having components delta x delta y and delta z that is what we are trying to understand.

So, we can clearly see that there is an original velocity in terms of its 3 components, there is a change velocity in terms of its 3 components and if you want to find out the acceleration see the basic definition of acceleration is based on a Lagrangian reference frame that is the rate of time rate of change of velocity in a Lagrangian frame not in an Eulerian frame the all the basic definitions in newtonian mechanics that we have learnt earlier are based on Lagrangian mechanics. So, when you say that its rate of time rate of change of velocity then that has to deal with the time rate of change of velocity of maybe an identified fluid particle which earlier was at P 1.

Now, is at P 2. So, if you want to find out the chain. So, you can write of course, you can write it in terms of the 3 different components, but just for simplicity let us just write for the x component similar things will be there for y and z component. So, how can you write u plus delta u as a function of u? So, u plus delta u is now dependent on the local position of the particle and the time that has elapsed. So, it is a function of it depends on what it depends on the original u plus the change. So, what was the original you that was u plus see it is a function of 4 variables. So, you again it is a same mathematical problem that there is a function of 4 variables it is known at a given condition.

Now, you make a small change in each of these variables and you want to find out the new function again, you can express it through a Taylor series expansion. Now it is a function of multiple variables instead of a single variable. So, we will use the Taylor series expansion into keep in mind that now you are having 4 variables. So, let us first consider the time variable maybe because it is bit different in characteristic then the earlier one. So, this is with regard to the time then with regard to the space plus higher order terms this we have just written the first order term in the Taylor series.

Since it is a function of 4 variables, you have 4 first order derivative terms, similarly you will be getting second order derivative terms and so on but we will neglect the higher order terms by considering that these delta x, delta y, delta z and delta t are very small. So, we have to keep in mind that all these are tending to 0 and because all they are tending to 0, we are neglecting their higher orders. So, you can first thing what you can do you can cancel u from both sides and what is the definition of acceleration along x for from a particle mechanics view point, well Lagrangian view point. So, you have to find out the change in velocity x component of velocity because we are writing acceleration along x divided by the time delta t in the limit as delta t tends to 0.

Very simple straightforward Lagrangian description, when you do that basically what we are doing? We are dividing the left hand side by delta t. So, right hand side is also divided by delta t and the limit is taken as delta t tends to 0. So, the first term is straight forward, let us look into the next terms. So, first we will evaluate the limits, limit as delta t tends to 0, delta x divided by delta t that multiplied by the derivative with respect to x, similarly the other terms let us just complete it. So, what we are doing is we are trying to find out that because of the changes in velocity component along different directions, what is the net effect in acceleration and these terms are basically representatives of that.

We will formally see that how they represent such a situation. So, now, let us concentrate on this limiting terms say the first limiting term what it is representing it is representing the time rate of change of displacement along x of the fluid particle over the period delta t. Now we have to keep in mind that we are thinking about a limit as delta t tends to 0. This is a very important thing, what is the significance of this limit as delta t tends to 0 when delta t tends to 0, P 1 and P 2 are almost coincident; that means, let us say that P 1 P 2 all those converts to some point P and that point is a point at which say we are focusing our attention to find out what is the change of velocity that is taking place.

So, when in the limit delta t tends to 0 we are considering the Eulerian and Lagrangian descriptions merge, this is very very important. So, we are trying to see, what is our motivation? We know something and we are trying to express something in terms of what we know. What we know? We know the straight forward Lagrangian description of acceleration, we are trying to extrapolate that with respect to an Eulerian frame, to do that we must have an Eulerian Lagrangian transformation and essentially we are trying to achieve the transformation in a very simple way that as the delta t tends to 0, Eulerian and Lagrangian descriptions should go inside.

And then what does it represent it represents the instantaneous velocity x component of the instantaneous velocity of the fluid particle located at p; that means, it represents the x exponent of the fluid particle located at P, since you are focusing on attention on pay itself and the velocity of the fluid particle if it is neutrally buoyant is same as the velocity of flow, we can write that this is same as what this is same as u at the point P see writing, this as u is these very straight forward understanding its it conceptually is not that trivial and straight forward is the Eulerian and Lagrangian descriptions did not merge we could not have been able to write this because this is on the basis of a Lagrangian descriptions and this is the Eulerian and velocity field.

How these 2 can be same, they can be same only when we are considering a particular case when in the limit as delta t tends to 0. So, wherever we are focusing on attention at that particular point this represents the velocity of the fluid particle if the fluid particle is neutrally buoyant with the flow then it is like can inner stress or particle moving with the flow and then it would have the same velocity as that of the flow at that point at that point at that instant; however, if the fluid particle has a different density than that of the flow then this would be u of the fluid particle. So, fundamentally this is u of the fluid particle.

Not u of the flow field if it neutrally buoyant then it becomes same as u of the flow field if it is an inner stress are article in the flow which is the definition of the fluid particle then it is definitely same as u at that point, but if it is a fluid if it is a particle of a different characteristic different density characteristic than that of the flow, it may be different from that of the velocity field at that point so that you have to keep in mind. So, if you complete this description of this term what you will get you will get a x is equal to that is the straight forward follow up of this expression because the other limits you can express in terms of v and w again with the same understanding as we expressed as we used for expressing the first term.

Now, if you clearly look into this acceleration expression there are 2 different types of terms, one is this type of term which gives the time derivative and other gives the special derivative, you will see that this expression; we will give you a first demarcating look of how the expression is different in terms of what we express in a Lagrangian mechanics in a Lagrangian mechanics it is just the time derivative that comes into the picture here you also have a positional derivative and what do these terms represent we will give a formal name to this terms, but before that first let us understand that what these 2 terms represent say you are located at a point 1, now you go to a point 2 in the field.

So, when you go there; there are 2 ways by which your velocity get changed how one is may be from 1 to 2 when you go you have a change in time and because of a change and you also have a change in position you have a change in velocity and that is. So, only time dependent phenomena how can you understand what is the component of the time dependent phenomena if you did not move to 2, but say you confined yourself to one say you are not moving with the flow field you are confining yourself with one then you are freezing your position, but still at the point one may be a change in velocity because of change in time it if it is an unsteady flow field.

So, because of that it might be having acceleration. So, the acceleration that acceleration component is because of what the time rate of change of velocity at a given point at a given location. So, that is reflected by this one, but by the time when you are making the analysis the fluid particle might have gone to a different point even if its local velocity that is velocity at a point is not changing with time it has gone to a different point, there it encounters a different velocity field. So, here it was encountering a particular velocity field because of its change in position. So, what it has done? It has got advected with the flow it has moved with the flow and it has come to a new location where it is encountering a different u, v, w.

So, because of the change in u, v, w, with the change in position it might be having acceleration. So, acceleration is not directly because of the time rate of change of velocity at a given point, but because of the special changed since the particle the fluid particle by the time has travels to a different location where it finds a different flow field and be and since we are considering that it is an inner stress of particle it has to have the same velocity locally as that is there in the new position. So, because the next combination of terms it represents the change in velocity, only due to change in position.

So, the total the net change is because of two things, one is if you keep position fixed and you just change time because of unsteadiness in the flow field and maybe an acceleration the other parties even in the flow field is steady, but you go to a different point because of non uniformity because of change in velocity due to change in position the fluid particle might have a change in velocity. So, the change in velocity in the fluid particle maybe because of tourisms one is because of the change in velocity due to change in time even if it were located at the same position as that of the original one and the other one is not because of change in time, but because of change in position as it has gone to a different position because of non uniformity in the flow field it could encounter a define velocity.

And the result and acceleration is a combination of this two. So, let us let us take a very simple example to understand it say you are you are travelling by flight from Calcutta to Bombay. So, when you are taking the flight before taking the flight you see that it is raining very very heavily and then say you take two to and half hours to reach Bombay and you find that it is a very sunny weather. So, the question is now if you if you want to ask yourself a question does it mean that when you when you departed from Calcutta it was raining in Bombay or when you departed from Calcutta it was funny at Bombay or when you have reached Bombay is it still raining at Calcutta or is it still is it sunny at Calcutta.

It is not possible to give an answer to anyone of this because the net effect that you have seen is a combination of 2 things you have traversed with respect to time. So, you have ill of certain time by which maybe it was raining at Calcutta, but right now it is not raining at Calcutta, maybe it was sunny at Calcutta and right now it has started raining. So, it is like at a particular location the weather has changed because of change in time, but the other effect is that you have migrate to a different location and because of the change in location maybe it was before two years raining at Bombay.

Now, it is sunny or it might. So, happened that, it was sunny 2 hours back in Bombay, still it is sunny. So, you can see that individual effects you can; may be try to isolate, but what is the net combination of changing with respect to position in time that is the net effect of this and it might not be possible to isolate these effects. So, when you think about the total acceleration. So, it is just like a total change. So, when you have the total change it is the change because of position and because of time and that is why this a x or may be a y or a z this is called as the total derivative of velocity.

So, it is given a special symbol capital D D t. So, capital D D t has the special meaning it is called as total derivative it is to emphasis that it is result in change because of change in position and change in time. So, with respect to change in time if you have a change then it is called as a temporal component of acceleration. Temporal stands for time temporal or transient or local these a certain names which are given again by the name localities clear local means confined to a particular position only with respect to change in time and this is known as the convective component.

So, convective component is because of the change in position from one point to the other and this therefore, is the total or sometimes known as substantial. So, the total derivative is the very important concept mathematically here we are trying to understand this concept physically, but it is not just restricted to the concept of acceleration of fluid flow it is applicable in any context in any context where you are having an Eulerian type of description.

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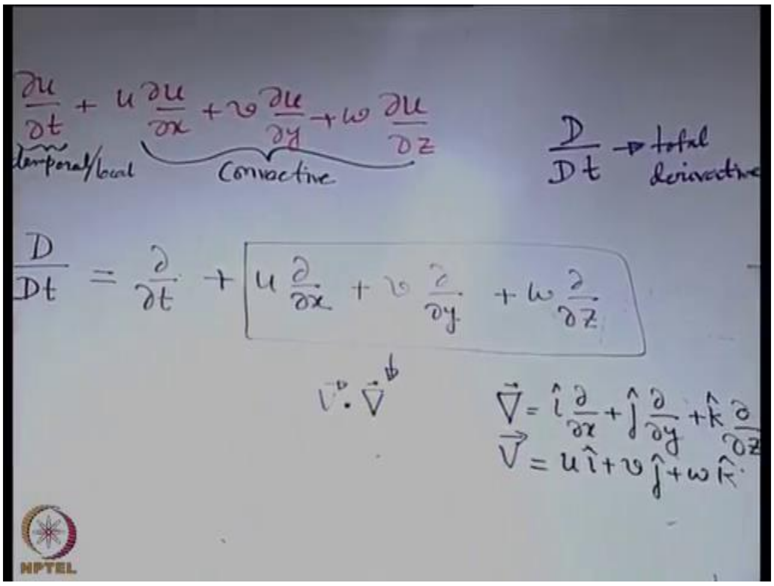

And it is therefore, possible to write the general form of the total derivative as this way where it has local component and a convective component. So, we can try to answer some interesting and simple questions and see and get a feel of the difference of these with again the Lagrangian mechanics.

So, if you ask the question, is it possible that there is an acceleration of flow in a steady flow field that the flow field is steady, but there is acceleration. It is very much possible because if it is steady only the first term will be 0, but there, but if the velocity components change with position then the remaining terms may not be 0. So, this is the like; these are certain contradictions that you will first face when you compare it with Lagrangian mechanics. In Lagrangian mechanics if there is something which does not change with time its time derivative is obviously, 0.

But here even if it does not change with time the total derivative is it may not be 0. On the other hand, it may be possible that it is changing with time at a given location, but acceleration is 0 because I mean in a very hypothetical case it may so happen that the local component of acceleration say it is 10 meter per second square convective component is minus 10 meter per second square. So, the some of that to be 0, but individually each or not 0; that means, it is possible to have a time dependent velocity field, but 0 acceleration.

And it is possible to have a non 0 acceleration even if you have time independent velocity field. So, these are certain contrasting observations from the straight forward Lagrangian description. So, you can write the x component of acceleration in these way and I believe it will be possible for you to write the y and z components which are very straight forward and you have to keep in mind that when you write y component this D D t operator will act on v and when you write the z component it will have act on w. So, you can write the individual components of acceleration vector and the vector sum will give the result and acceleration.

Now, you can write these terms in a somewhere in a somewhat compact form. So, this you can also write as v dot del where del is the operator given by and v is the velocity vector you know that is u i plus v j plus w k. So, if you clearly make a dot product of these 2, we will see that this expression will follow. So, it is the compact vector calculus notation of writing the convective component of the derivative. So, we have got a picture of what is the acceleration of flow how we describe acceleration of flow in terms of a expressions through simple Cartesian notations and maybe also through vector notations.

Thank you very much.