Introduction to Fluid Mechanics

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Lecture – 22
Concept of different flow lines

Now, what we will do? We will try to see that; what are the conceptual lines which are important for quantifying these visualizations and for that we will learn certain concepts, so, the first concept that we will learn is something which you have heard of earlier and that is the concept of a streamline.

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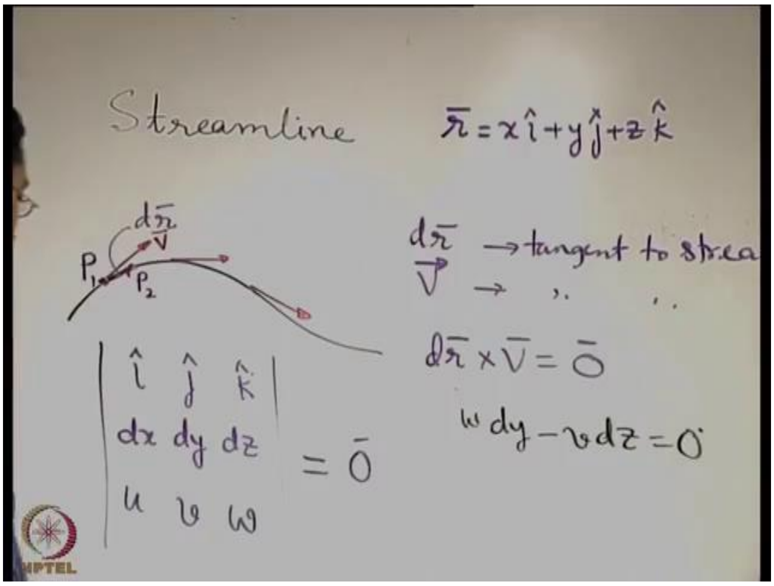

So, we are discussing about some conceptual paradigms which help us in visualizing fluid flow. So, when we say streamlines, how do you define streamlines? Streamlines are imaginary lines in the flow field; these are not existing in reality.

So, imaginary lines, what type of imaginary lines? These are such lines that at an instant of time; tangent to the streamline at any point represents the velocity vector at that point. So, if you have a streamline like this say. So, when you have a steam line like this you may have tangent to it at different points and these tangents are representatives of the velocity vector at these points one important concept that we miss many times is that it is defined at a particular instant of time; that means, at different times you may get different streamlines. Only when the flow field is not changing with time that is a steady flow you get same stream lines at all instance of time.

Otherwise you may get different stream lines. Now to express the streamline in terms of some equation, so this is a line, this is the locus. So, it should be expressible in terms of certain equations. Let us try to see that how we can express that towards that we will first recognize that let us say that there is a point P or P 1 located on the streamline, the fluid particle at a particular time, the fluid particle is located also here. It is coincident with this point when the fluid particle is coincident with this point then after sometime; the fluid particle has come to a different point and so on. We are not bothering about the motion of the fluid particle we are just bothered about say 2 points which are located on the streamline which are quite close to each other.

Say P 1 and P 2 not that P 2 represents the location of the fluid particle at a different time not like that just it is another point on the steam line which is very close to the point P 1. The vector P 1, P 2 let us say we denote it by change in position vector d r and let us say that v is the velocity at that particular point. When we give it a name d r we have to keep in mind that it is very small and it is it is differentially small it is as good as writing delta r as delta tends to 0. So, when delta tends to 0 then what is the status of the points P 1 and P 2? They are almost coincident, when they are almost coincident; that means, P 1 P 2 then represents tangent to the streamline at the point P 1.

So, what is the tangent? Tangent is the limit of a cord in the limit as the gap the distance between the 2 points becomes infinite decimal. So, in the limit P 1, P 2 becomes tangent to the streamline at the point P 1. So, d r in that differential limit is tangent to the streamline at which point at P 1 by definition v is also tangent to the streamline at P 1 that is the definition of the streamline; that means, these 2 are parallel vectors. If these 2 are parallel vectors their cross product should be a null vector. So, you have d r cross v is a null vector.

So, we can write d r and v in terms of components you have r as; how do you write r? X i, y j and z k where i j k are the unit vectors along x y z. So, you can write d r as d x i plus d y j plus d z k. So, when you write these cross products, it is possible to write it in a determinate form. So, let us write that i j k then components of d r d x d y d z and components of the velocity vector u v w; that is equal to null vector. We can easily see that it boils down to 3 scalar equations for each for the x y and z components.

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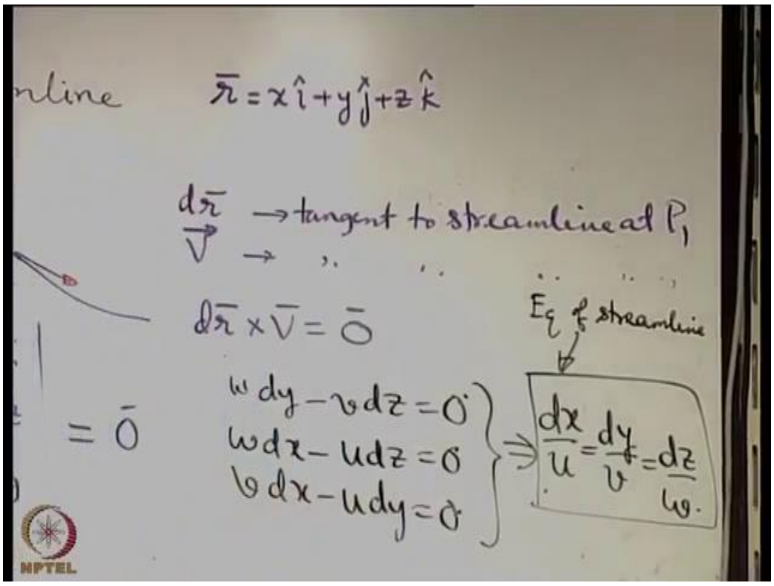

So, what are these scalar equations? W d y minus v d z equal to 0 then w d x minus u d z equal to 0 and v d x minus u d y equal to 0, so if we combine these 3 together, you can get a compact expression d x by u equal to d y by v equal to d z by w which is nothing, but the equation of the streamline this the locus that we are looking for you can easily obtained the locus by keeping in mind that u v w are functions of position like x y z and also time, but when you are considering a streamline you are freezing the time. So, at a given instant of time. So, that does not become a variable in this case. So, x y z are the variable. So, you if these are substituted as functions of x y z you may integrate these to find out the locus that is very straight forward later on we will work out some examples to illustrate that how we can do that. So, this is the concept of a streamline.

Now, related to these concept there are certain other terminologies again sometimes they are confusing because streamline is more commonly used. Those are not very commonly used, but those are sometimes more fundamental and more relevant and the streamline.

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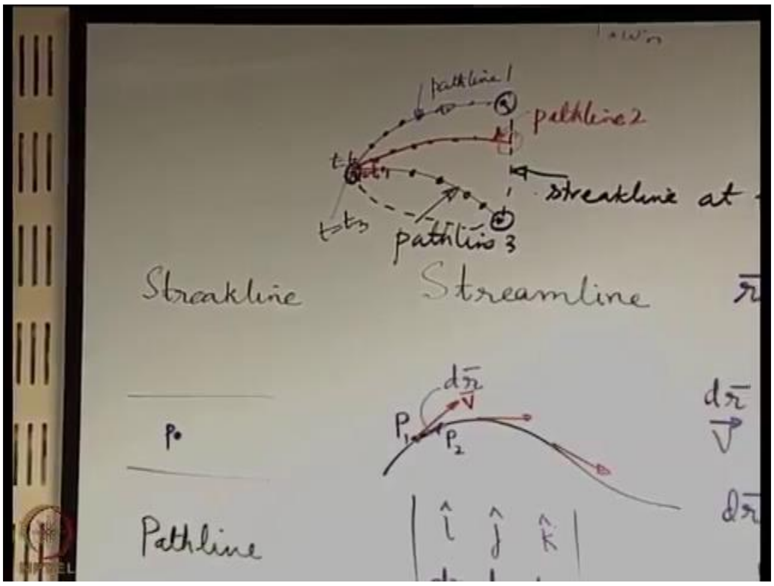

So, we will see the next example that is called as the streak line. So, what is a streak line? Let us say you want to visualize the flow we will try to identify the concept from where it has come. So, when you want to visualize the flow. So, this is the flow field you want to visualize a flow a very common technique is what? You take at injection syringe in that injection syringe you take some colored dye; say a blue colored dye a common name of a blue colored dyes called a thymol blue say you have taken a thymol blue.

Looks like the ink, so when you have taken that blue colored dye and say that you are trying to put that blue colored dye, inject that blue colored dye through this point P. So, the blue colored dye is coming here through an injection syringe. So, now, you are going on injecting the dye here. So, what is happening? Whatever fluid molecules or fluid particles which are passing through this point at different instants of time they are illuminated by the dye and so, wherever they go that tag of illumination remains. So, when you get an illuminated line it is at a particular time then what does it represent it essentially represent locus of all fluid particles which at some earlier instant of time pass through this point of dye injection.

That is how they were colored by the dye. So, when you see a colorful line in the flow field it represents that locus and that locus is called as a streak line. So, what is a straight line streak line is the locus of all fluid particles which at some instant of time at some prior instant of time all of which had passed through a common point mathematically we stop here, but physically we understand that that common point is the point of dye injection. So, that is called as a streak line. Let us try to conceptually draw a streak line let us say that this is a point at which dye is injected. So, when a fluid particle passes through, this point say at time equal to t 0.

So, when at time equal to t 0, the fluid particle passes through this point, the fluid particle then under goes a locus. So, what is this locus? We introduce third line which is called as a path line path line is the simplest and the most trivial concept to understand it is the description of a flow from a Lagrangian view point. So, it is the locus of an identified fluid particle. So, if you identify a fluid particle how it moves the path stressed by that that is called as the path line that is very simple and trivial requires no explanation. So, when we want to draw different path line see at say time t equal t 0, you have one fluid particle which pass through this point of dye injection that fluid particle at subsequent instance of time, it is passing through different points. So, this is the path line 1 say what is that path line one path line 1 is the path line of a fluid particle which pass through the point of dye injection at time equal to t 0.

Let us say there is another fluid particle which has passed through these at time equal to t 1 and let us say that this red line represents it locus. So, this is something which was injected at t equal to t 1, the path may be different because it may be an unsteady flow. So, it is possible that the velocity field has change with time. So, when just change with time the particle may be forced to move along a different locus. So, this is path line 2. Let us consider a third path line may be for completeness let us say that we have a third path line like this which corresponds to that injection here at time equal to t 3.

And again the path line is different because the velocity might have changed at different points with time. So, this is path line 3. Now say we are bothered about at time equal to t say now. So, at time equal to t that particle which passed earlier through this point at t 0 say now it is here that particle which passes through this point at t 1 is now at here and the particle which passes through this point at say t 3 is here right now at time equal to t. So, there is one more fluid particle that is injected just here if it is a continuous process. So, the locus of all these which at some earlier instance of time pass through the point of dye injection that locus is now the streak line at time t.

So, you can clearly demarcate between streak line and path line let us take the example of a very special case, but a very interesting case what is that is what is that special case the special case is for a steady flow.

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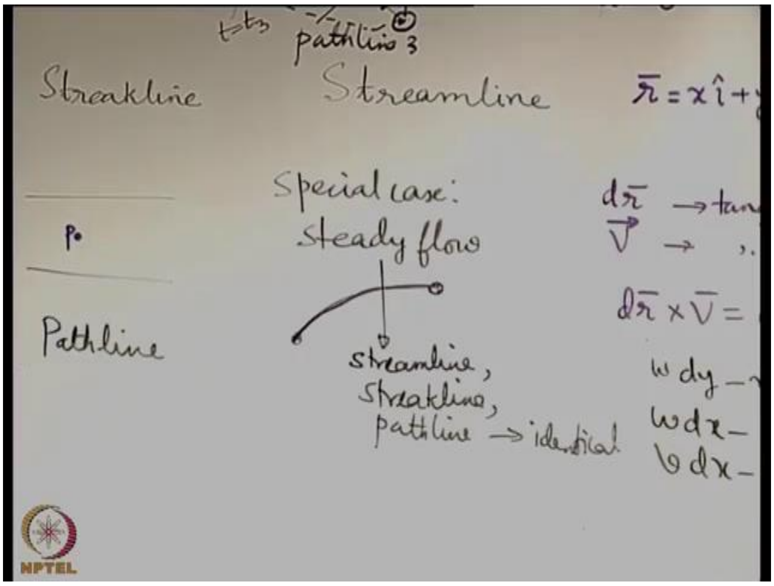

So, when you have steady flows then let us try to draw these path lines. So, first let us draw the path line of that particle which passes through this point of injection at t equal to t 0. So, let us say that that path line is this one now another particle which passes through this at t equal to t 1 that will also follows this line because with time the velocity field has not changed.

So, it will be constrained to follow the same line. So, it will follow this line similarly the third one that is in the one injected at t equal to t 3 that will also be constrained to follow this line and what is the specialty of this line this line is the locus of the fluid particle so; that means, at some time tangent to this line represents the direction of the velocity vector. So, we can understand that this is also a streamline this also the path line and again this is also the streak line because whatever are the locations of fluid particles those are always constrained within this line. So, we can say that for steady flow streamline streak line and path line are identical that is one very important concept that we should remember.

That in this case, you have streamline streak line and path lines are identical they are identical. So, whenever we visualize a flow let us look into some example maybe some images through streamlines or streak lines.

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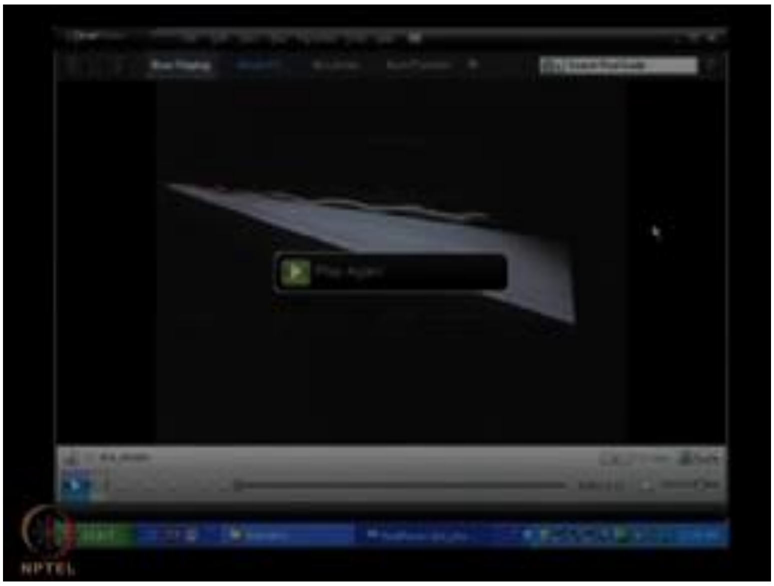

So, let us say that we see first this example; you can see these are streak lines. So, dye streaks which are which are injected at the team you can see that now at different instance of time they are forming different colored images and if you track individual one then it is like a path line.

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Now, if you see, let us look into a path line example maybe see this is like a long sprinkler. So, many times it is used to sprinkle water in a garden we can see that if you track the water droplets you can clearly see that the path what they are following. So, it is it is something like a visual representation of a path line.

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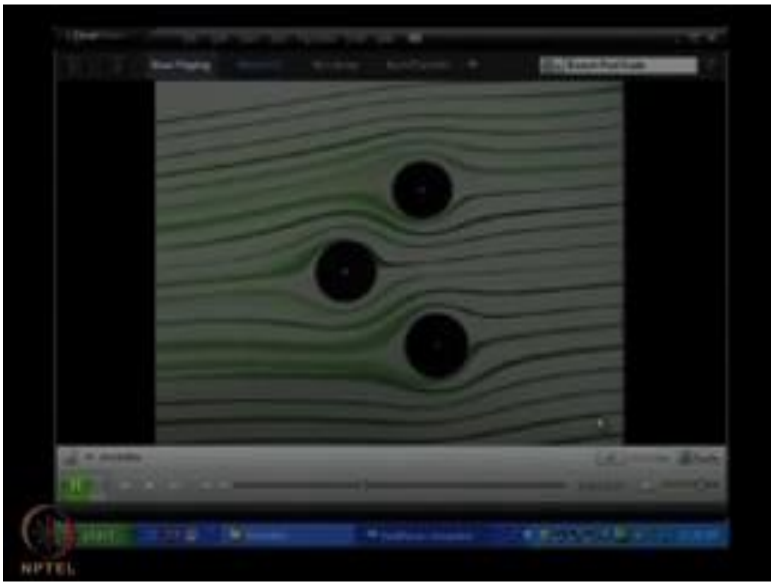

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Let us look into a streamline example, these are 3 cylinders in a steady flow and you can see that this green colored dye is giving an appearance of a streamline fundamentally this is actually a streak line, but because it is a steady flow streamline streak line path line these are all identical.

So, these represent different streak lines or different streamlines or different path lines whatever you say if it is steady flow; if it is a; if it is an unsteady flow these will represent streak lines rather than streamlines. So, we can clearly see that these visualizations of fluid flow that we have seen as concepts these may also.

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We visualized in experiments many times we are interested to construct the velocity vectors. So, see the example of the boat that we saw earlier and you may constructs such velocity vector. So, it is not a direct visualization, but you may do it in 2 ways by post processing the visualization of the particles which are injected into the fluid or by doing a computer simulation and sometimes this may be equivalently compared computer simulation of course, is an idealization because you are using certain boundary conditions certain properties which might not exactly prevail in reality.

But sometimes it gives a very important idea of how the fluid flow is taking place and it is used for advanced designs also. So, this is known as computational fluid dynamics or c f d. So, that is that is a separate area of research all together where the whole idea is to computationally solve the equations of fluid flow to get a picture of the velocity field with this understanding we will try to quickly work out an example to illustrate the concept of these lines streamline streak line and path line.

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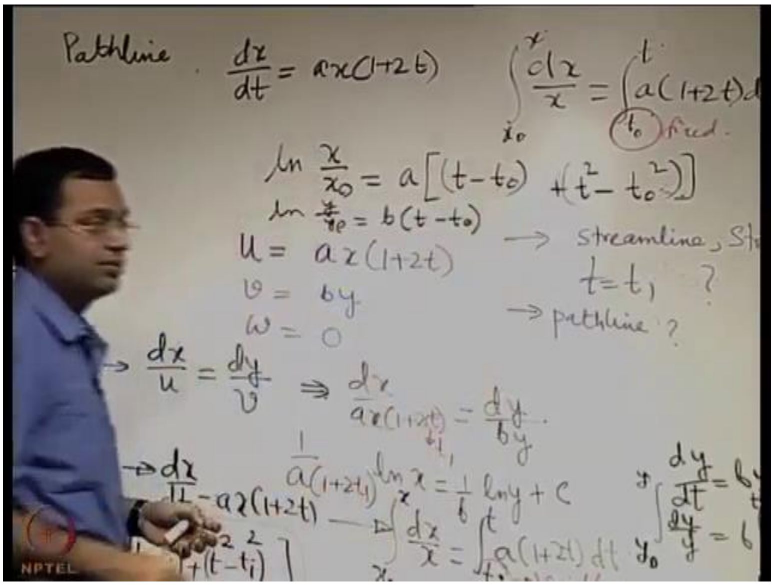

So, an example we will consider 2 dimensional velocity fields as an example. So, let us say that velocity field has these types of components.

U is given by this a x into 1 plus 2 t v is given by b y and w is 0. So, it is a 2 dimensional field usually whenever you have a velocity field we call it 1 dimensional, 2 dimensional or 3 dimensional depending on the number of independent velocity components that you are having. So, you are having 2 independent velocity components, it is a 2 dimensional velocity field a and b are some parameters and x y are the coordinates t is the time and. So, a and b have certain dimensions with adjust this. So, that you get the dimensions of velocities at the end. So, these are dimensional parameters, but constants.

Let us say we are interested to find out the equation of streamline at a time say t equal to t 1, we are interested to find out streamline and streak line at time equal to t 1 that is one objective the other objective is to find out equation of path line. So, to find out equation of streamline that is the easiest part let us do it first. So, you have d x by u equal to d y by v d c by w is not really when because it is a 2 dimensional flow; so d x. So, we are talking about first streamline. So, d x by a x into one plus 2 t is equal to d y by b y at what time we are focusing our attention at time equal to t 1.

So, you replace this t with t 1. So, when you are considering a stream line you are freezing the time at the instant that you are considering it is clearly an unsteady flow. So, at different times you will different stream lines. So, we can integrate this and what we will get it 1 by a into 1 plus 2 t 1 l n of x is equal to 1 by b l n of y plus say some constant of integration c. How do we evaluate the constant of integration you must be given a point on the streamline say the streamline passes through some point. So, when you are given that the steam line passes through that point from that you can find out c that is if you know that at time equal to t 1 whatever streamlines you arr drawing there is one point on it.

So, that point when substituted x say x a 1 and y equal to y 1, we will give the value of c. So, that will give the equation of the streamline if you arrange it properly and in a compact form let us consider the streak line to consider the streak line may have to remember one thing that this is the velocity; that means, if a fluid particle is injected at a point it will also represent its rate of change of position; that means, you will have d x d t is equal to a x into 1 plus 2 t where x represent the x component of displacement of a fluid particle which is subjected to this velocity field remember fluid particle is inert to the velocity field. So, whatever the velocity field is field is imposing on it do it will do that. So, this is what it is imposing.

Now, it is possible to find out how x changes with time it is straight forward, but conceptually not that straight forward to understand why it is conceptually a bit more involved we will parallelly write the equation of the path line. So, let us write the path line this streak line is not yet complete, but we will draw parallel analogy with the path line and see, where is the difference? So, for the path line again you see path line is what it is the locus of the fluid particle. So, for path line also there is no need to believe that it should be something different, then this one similarly d y d t. Now the difference in approach comes in the concept by which we integrate these 2.

So, when we integrate this one say we integrate the equation of the streak line. So, how we do it? We write d x by x is equal to a into 1 plus 2 t d t. Similarly here also we write the same thing d x by x is equal to a into 1 plus 2 t d t. So, in the left there is in there are going to be limits of x and in the right limits of t, here also same, now what limits we will put? Let us say that you are injecting the dye at some point x naught coma y naught this is the point of dye injection the dye injection starts at t naught and the time the dye injection ends at t f the final time.

So, this is the interval over which the dye injection takes place and the time that we are bothering about t 1 is something in between t naught and t f. So, when we write the integration for the streak line what we will do we will integrate at time at sometime say t i x equal to x naught and say at some time equal t equal to whatever say t 1 x is equal to x when you are considering the streak line you have to keep in mind one important thing that importance will be clear when you write the integration here. So, when you write the integration here at time equal to now here the time equal to t naught because you are finding the locus of the particle.

When we are talking about the path line; so when at time equal to t naught x equal to x naught at time equal to t x equal to x, what is the difference between these 2? See look carefully into the limits of t. So, when you say this is t naught this is the fixed t naught; that means, you are finding the locus of a particle which at time equal to t naught passed through the point of dye injection here you are dealing with a variable t naught because you are dealing with locus of all particles which at different instance of time pass through the point of dye injection. So, t i is a variable which may be anything between t naught and t f.

So, this is the variable limit. So, this actually needs to be eliminated we do not know what is this only what we know is that t i has to be between t 0 and t f, but it is not a specified time this is fixed specified time and that is how that is how you are going to find the path line. So, so similarly when you. So, when you write this in terms of x. So, you have l n x by x naught is equal to a into t minus t i plus t minus t i square where t is a variable. Fortunately we will also get another equation involving y. So, you can write similarly that d y d t is equal to b y. So, d y by y is equal to b d t again we integrate with respect to the same limits x naught to x and t i to t.

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T square minus t i square, very good. So, this is t square minus t i square. So, when you do this, this will be not x but y these limits are y naught to y so that will give you what l n of y by y naught is equal to b into t minus t i. So, you have an expression here which involves t i, you have another expression here which involves t i and you can eliminate t i from these 2 to find out the relationship between x and y here that is not necessary because here you can straight forward write this.

So, you can write a l n x by x naught is equal to a into t minus t naught plus t square minus t naught square and l n y by y naught is equal to b into t minus t naught here what is the variable here actually t is a variable parameter because at different time it will have different position to get that locus it is it may be convenient even you may right it in a parametric form, but conceptually you may eliminate t to write the locus y as a function of x whereas, to write this you have to eliminate t i. So, here is t i is a variable where has here t is a variable. So, conceptually it looks very similar, but there is a subtle difference and that subtle difference needs to be appreciated in the context of streak line and path line streamline it becomes more or less straight forward. So, I hope you get the distinctive concepts between streamline, streak line and path line and how to find out their equations given a flow field we stop here today.

Thank you very much.