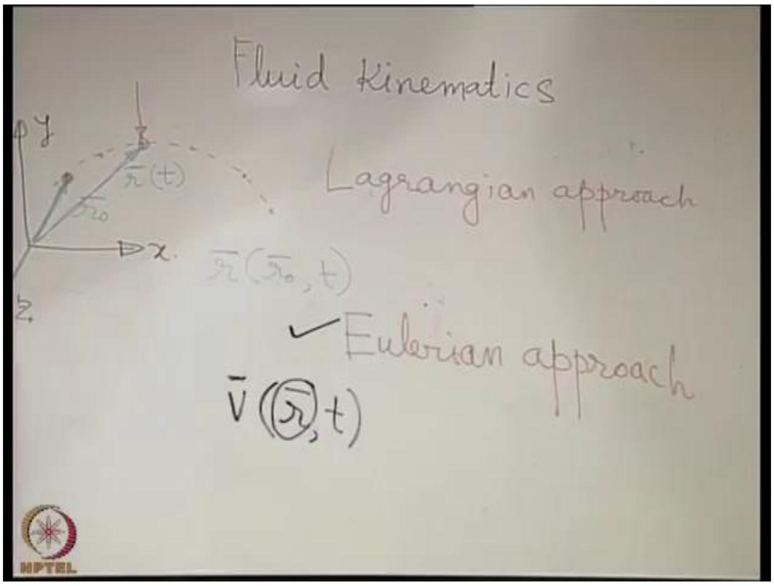
Introduction to Fluid Mechanics
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 21
Lagrangian and Eulerian approaches

(Refer Slide Time: 00:23)



We will start today with the discussions on Fluid Kinematics. As you understand when we will be concentrating on fluid kinematics, we will be somewhat abstracted from whatever are the forces which are giving rise to the motion. But primarily we will be now concentrating on the nature of motion; that means, displacement, velocity, acceleration when we say motion we also mean angular motion, deformation, we have earlier seen that deformation is one of the very important criteria for the de marketing a fluid as compared to that of a solid.

So, we will try to see that how we can characterize the kinematic features of a fluid motion. To do that we have to go through certain formalities, because we need to first appreciate that there is a necessity of describing the fluid motion in a manner different from that of a solid. Usually whenever we are dealing with particle mechanics, what we are trying to do? Most of the times you have like a particle, you are trying to find the evolution of locus of the particle as it moves with time.

So, you have an identified particle which stag like this, this particle at different instance of time comes to different positions, and the locus of the particle can be obtained by joining those position successively; this type of approach is known as Lagrangian approach.

So, Lagrangian approach is nothing, but tracking of individual particles. So, you identify particles and you tag those particles and crack the particle so to say as they evolve. This Lagrangian approach is good for particle mechanics, but when you come to fluid mechanics you see that it has certain limitations; not that it cannot be employed in principle, but when it comes to practice, because fluids have numerous particles well solids also have numerous particles, but the difference is that fluid particles when they are in motion they are continuously deforming. So, it is very difficult to keep track of individual particles and therefore this type of description of motion is not so convenient; if it was convenient how would we have described that motion?

So, let us say that we would have set up a coordinate axis like say x y z and let us say that this was the initial position of the particle which we try to designate buy some position vector say r not. So, the position vector at any other instant of time say the particle is at this location, at an instant of time t and the position vector is r. So, are at time t we can say in this approach is a function of what? It is a function of the initial position and the time that has elapsed. So, when you describe the displacement in this way or the position vector rather than this is a Lagrangian description. As we have discussed that for fluid it is difficult, because you have numerous particles which are continuously evolving over space and time.

So, for fluid there is a more convenient approach, which is known as the Eulerian approach. So, in the Eulerian approach what we are doing; you are not tracking individual particles. Instead of tracking individual particles what you are trying to do you are trying to focus your attention not on a particle, but on a position. So, let us say that you are focusing your attention on this position. Now what is happening there will be many particles which had come to this position and had left, you are not bothered about which particle has come from where and which particle is going where, as if you are sitting with a camera and the camera is focusing on a position, you are bothered about what is the change that is taking place across this position.

So, you are what you are bothered about is that maybe what is the velocity at this point. So, velocity at this point is nothing, but the velocity of a fluid particle at that instant which is passing through that point. So, what is a fluid particle I mean there are involved concepts associated with it, but in a very simple term it is just like an inert trace of particle moving with the flow. So, does not interact with the flow, it does not have any density difference with respect to the flow. So, it is just a passive inert particle moving with the flow. So, if such was a particle then at a particular instant, the velocity of the particle which is seen by an observer who is focusing the attention here is known as the that velocity is known to be the velocity that is obtained by using Eulerian approach.

Again that velocity would be same as the velocity of a tracer particle that at that instant of time was passing through that position right. So, what you can say from here is that we can write the Eulerian description in the following way. So, we can write the just like in this way we could write the Lagrangian approach, the Eulerian approach we can say that the velocity at a particular position, is a function of what? It is a function of the position r and the time t, because it also varies with time. So, the key difference is here you focus your attention on a specified r, across which you are trying to see the changes in a fluid property.

So, here velocity is the fluid property that you are looking for as an example. Regarding the fluid particle one important concept is like this, that all of us know that fluids are comprising of molecules at the end. So, where from the concept of particle comes; if you try to extrapolate the concept of molecules to particles, then it can be thought of in this way. So, there may be a collection of molecules in the fluid which on a statistical average sense have common group behaviour, and that collection may be conceived as a free particle. And the behaviour of that is equivalent to the behaviour of a inert traces particle, put in the fluid and it just moves with the flow.

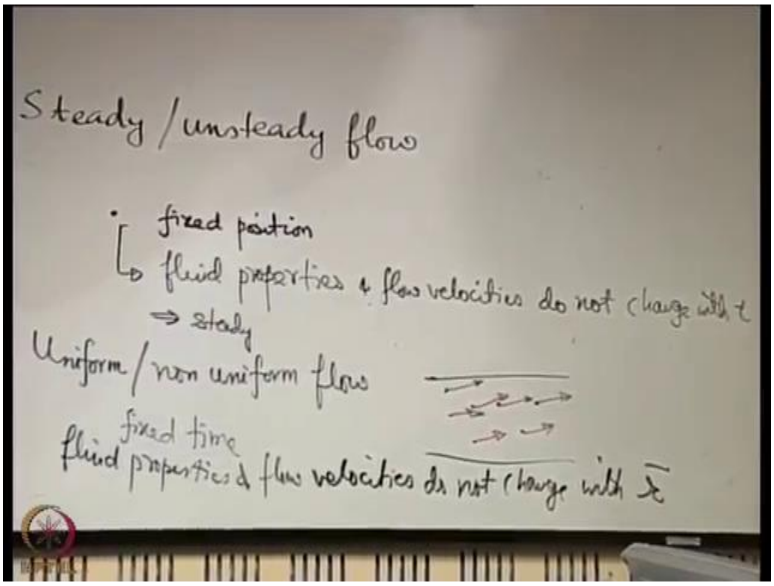
So, it does not do anything, it does not interact with the flow, it is so passive, that wherever flow takes it is just goes there and it does not have any density difference. So, it does not have a net buoyancy force also, because of its existence on the flow. And such a particle which has the density same as that of the fluid conceptually, is known as a neutrally buoyant particle; that means, a particle which does not have any resultant buoyancy effect, because of its interaction with the flow; that means, the densities of the fluid and densities of the particles are considered to be the same.

Clearly the Eulerian approach is easier for implementing for fluids, because when you are implementing that you are extracted of individual particles, you are only concerned on a particular position and you see that some particle is coming and some particle is going, you are not bothered about the identity of the particles which are coming there and leaving that place, and that makes it more convenient for employment for analysis of fluid force.

So, we will mostly be bothering about the Eulerian approach. Keep in mind that the basic laws of Newtonian mechanics were originally derived by using Lagrangian approach. So, a major emphasis on fluid mechanics will be to convert those expressions into equivalent forms, which are implementable using the Eulerian approach. So, the expressions that we will be writing say equivalent to Newton's second law of motion, in terms of Eulerian approach will not exactly write in the we will not exactly look in the same form as we are familiar with for simple particle mechanics for I mean for solid mechanics say, but we will see at the end that these approaches are inter convertible, and it is possible to derive one approach or expressions in perspective of one approach from the expressions of the other and that we will take up in a letter chapter.

But we will keep in mind that Eulerian approach is something that will be trying to follow for most of the problems in fluid mechanics. Now regarding the use of the Eulerian approach we will try to understand or we will try to learn some basic terminologies; the basic terminologies are as follows.

(Refer Slide Time: 11:12)



First concept is steady and unsteady flow. When do we say that a flow is steady? Whenever we are trying to focus our attention at a point say by following the Eulerian approach, we are interested to see that the flow velocity of fluid properties at that particular point are changing with time or not.

So, if at a specified location the fluid properties or the flow velocities are changing with time, then we call it as unsteady flow, but if that is not changing with time then we call it a steady flow. So, what are the keywords here that we are having a fixed position and what we are trying to see is the fluid properties. So, when say fluid properties we have discussed many fluid properties maybe density, may be viscosity whatever fluid properties and flow velocities do not change with time; if that is the case at a fixed position then we say that it is a steady flow.

If that is not satisfied as obvious that is unsteady flow we will learn related, but not the same concept that is uniform and non-uniform flow. What is that? Just like steadiness discusses about change with respect to time at a fixed position, uniformity discusses about change with respect to position at a fixed time; that means, let us say that you have a domain in which flow is taking place, in technical terminology sometimes we call it a flow field, what is a field? Field is a domain over which some influence is felt.

So, when you have a flow field is basically a domain over with the influence of the flow is failed. So, you are having some velocities at different locations. So, in the flow field at different points let us identify different points, let us say that these different points have different velocities. So, let us try to mark this different velocity, say the different velocities are like this. If they are such that at a given instant of time they are same at all points then we say that the flow field is uniform.

So, not only the velocities, but also again the fluid properties; so fluid properties and flow velocities. So, what we are doing now we are fixing the time. So, fixed time that is our concern, at a fixed time we are trying to see that fluid properties and flow velocities are invariant with position do not change with position. So, if that is the case that at a fixed time we are finding that field properties and flow velocities do not change with r, we call it a uniform flow field. So, the distinction between steadiness and uniformity is quite clear.

Let us ask ourselves certain questions related to this, if the flow is steady could it be non-uniform that is very much possible. So, if you have a domain like this with different points. So, you have velocity at different points different, but whatever those are those are not changing with time. So, it is possible to have a steady, but non uniform flow. Is it possible to have a uniform, but unsteady flow. So, 50 percent of you are saying yes and 50 percent no; let us look into this example. So, let us at some time instant these are the velocity vectors, let us say in the next time instant you have the velocity vectors like this assume that they are same.

So, with change in time at a given point if you focus your attention, you can see that the velocity has changed. So, it is unsteady, but at a given instant of time it is same at all locations. So, it is uniform. So, it is possible that you may have unsteady, but uniform flow, just like it is possible to have steady and non-uniform flow. So, the steadiness and uniformity should not be confused as equivalent concepts, they relate to two different things of course, there may be relationships between these two in very special cases.

The other important remark is that whenever we are talking about a steady and unsteady flow, we have to keep in mind that steadiness and unsteadiness is not something which is absolute, it depends on the choice of the reference frame with respect to which you are analysing the flow. And this type of choice of reference frame is very important in mechanics fluid mechanics is of no exception. Let us try to understand this through an example again, let us say that there is a river, on the top of the river there is a bridge, and there is an observer standing on the bridge, the whole idea is to observe just qualitatively observe how the fluid flow is taking place below the bridge on the river. Now suddenly a boat comes the boat approaches the bridge, and it just crosses the bridge below it.

So, what happens to the water the water was say earlier stagnant, now the boat has come. So, it has disturbed the water, and the boat has gone again after sometime the water will come back to its original state. So, to the observer who is standing on the bridge how the flow will appear? It will appear to be changing with time. So, initially he is focusing his attention on a particular location he is finding initially it is stagnant, then velocity is changing because of disturbance created by the boat and then again the velocity is coming back to its original state. So, it is a strongly unsteady flow; let us take the same example in a different perspective, say the boat is moving relative to the river at a constant velocity, and a person sitting on the boat is trying to observe the flow.

So, when the person sitting on the boat is trying to observe the flow of the same river what is the interpretation? Because the velocity of the water relative to the boat is not changing and the person is only observing that relative velocity, for that person the same flow of the river water is appearing to be steady, not changing with time. So, the same flow may appear to be steady or unsteady depending on the choice of the reference frame. So, these two were two special reference frames: one is a fixed reference frame just like the bridge, another is a reference frame that is moving with a boat like moving with something at moving with a reference at constant velocity. And you can see that with respect to the reference frame moving at a constant velocity, it is possible to analyse a flow which is otherwise unsteady in terms of a steady concept; that means, it is possible to convert the notion or transform the notion from an unsteady to a steady flow by such a transformation of reference, and this in mechanics is known as Galilean transformation.

So, we will try to may be have a visual example of what type of flow takes place as a boat is moving, I cannot show you the bridge in this example, but at least I can show the boat.

(Refer Slide Time: 20:54)

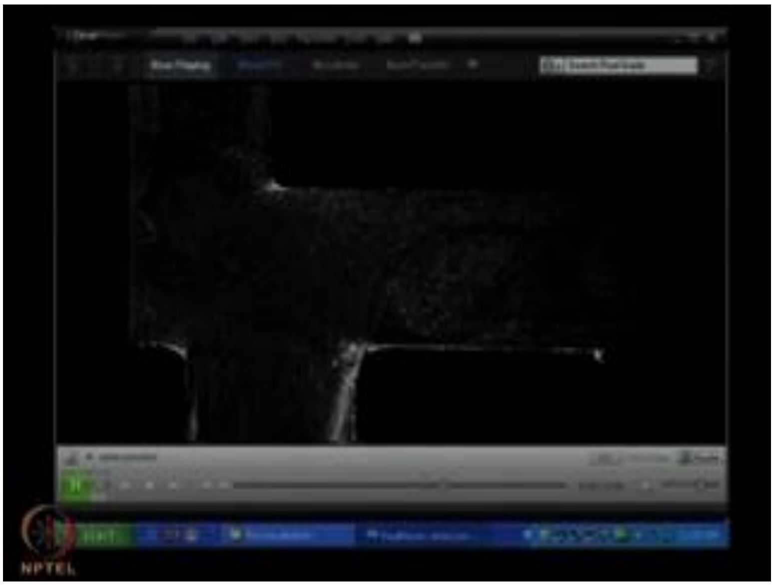
So, we will try to follow very clearly that what is happening across the boat as the boat is moving. It is an artificial tunnel these channels is not very white, and just see that what is happening to the water. Water is not of course, very clean, but it will help you to visualise it better. See the boat was moving relative to the water at a uniform speed, but what you can see is that after the boat has gone, there is a change in the velocity pattern. We are now observers with respect to fix reference frame and we are observing it to be changing with time; see one important thing is we are not talking about inertial and non-inertial frame both are inertial frames, because when something is moving with a uniform linear velocity that is still an inertial frame.

So, do not confuse this with the concept of inertial and non-inertial frame. So, Galilean frame is still an inertial frame, because it is moving with a uniform velocity it is not an accelerating reference frame that we need to understand. So, with this background what we will do next, we will try to see that we have got a basic concept of what should be the description of a flow in terms of change in position, change in time, and the description of the flow velocities. The flow velocity is as you can see that the velocity vector v, this may have their individual components like components along x y and z in fluid mechanics we give them certain common notations the velocity. Velocity component along x we give a name commonly as u, velocity component along y we give a common name v, and velocity component along z as w. So, these are just names I mean just notation. So, to say commonly used in textbooks; in the index notation of course, you can use U i where i equal to one will imply v x, i equal to two will imply v y, and equal to three will imply v z because it is a vector. So, we have seen how to write vectors in index notations velocity is of no exception.

Now, whenever a flow field is having a velocity it is not so easy to map the velocity vectors in a flow. So, there should be some mechanisms by which we are trying to develop a feel of how we visualise a flow, and for that we have to consider certain imaginary lines in the flow; just like you have lines of force in a magnetic field by which you try to visualise how the depth, how the magnetic forces are acting similarly whenever you have a flow field, you must develop some concept of imaginary line, these lines are not exactly existing in the flow, but as if these lines were there to aid you to visualise the flow, both qualitatively as well as quantitatively.

So, we have earlier seen some examples of flow visualisation, may be what we will do is we will try to see some more examples to see different methods of flow visualisation.

(Refer Slide Time: 24:28)



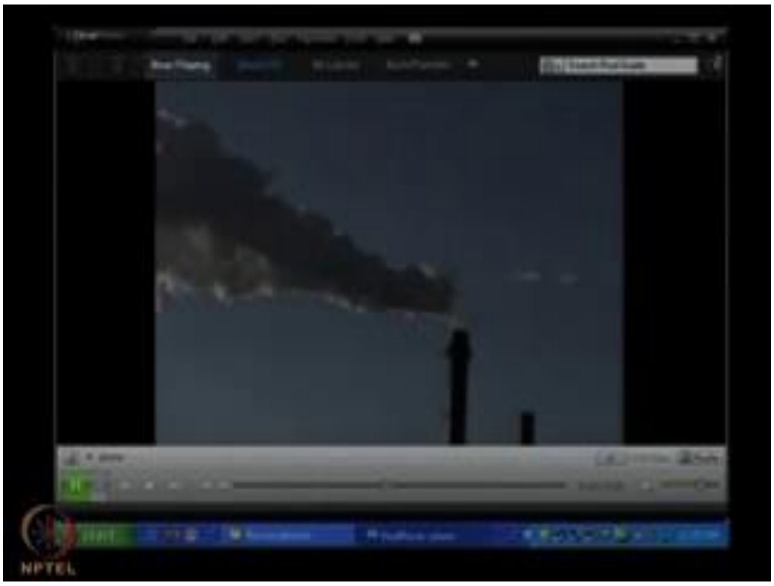
See this first example. So, in this example it is like flow in an artery junction. So, there are certain tracer particles. So, how this flow is visualised; you have tracer particles. So, these particles may be very small beads of may be nanometre size or few micron size, and these beads are put in the flow and the flow is illuminated by say a laser source. So, in that illuminated condition, when the particles are moving it is possible to not track individual particles all particles as such, but it is possible to statistically track them; that means, it is possible to get a statistical picture of the displacement and velocities of these particles, and from that it is possible to have a post processing and map the velocity field also. By a statistical operation on the motion of selected particles, and that principle is followed in one of the devices which we commonly used for flow visualisation in advance research applications known as particle image velocimetry or PIV.

(Refer Slide Time: 25:51)

Now, we will not going to search analysis for this elementary course, but if you just try to look into some other examples, see this is particle visualisation this is also particle based visualisation, but visualisation through smoke emission. So, you can clearly see basically a wing of aircraft has passed and when it has passed it has created a vortex.

See the rotations and by the smoke emission that is being visualised. So, by illuminating with a smoke, smoke is a natural emission and it is possible to visualise the flow at least qualitatively by using that.

(Refer Slide Time: 26:26)

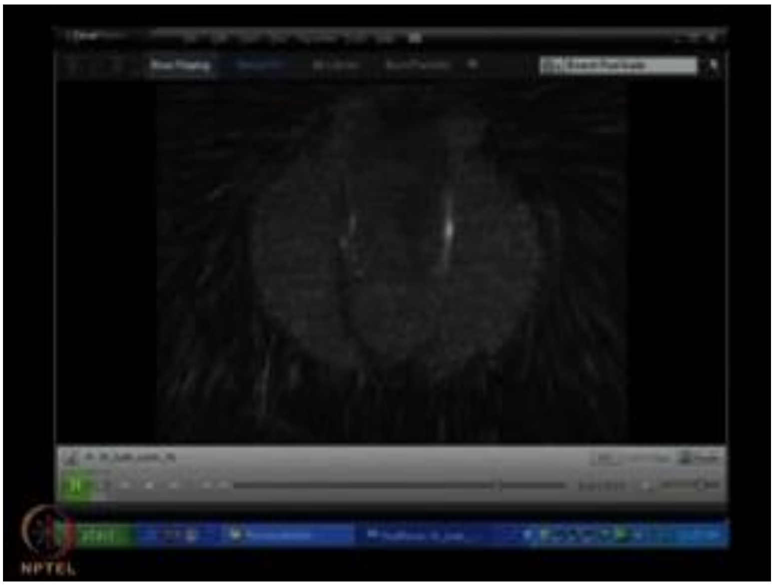


The same type of thing that is possible to be observed in the chimney of a power plant and we have seen this example earlier when we were discussing about the introductory concepts, and you can see that that also give rise to a good visualisation of the flow field.

(Refer Slide Time: 26:47)

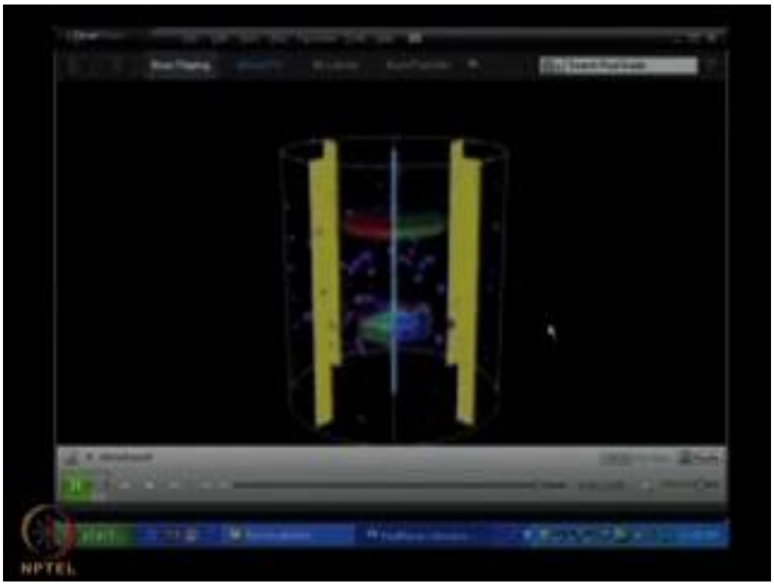
When you have these particles put in the flow and those particles are in forms of beats, so we can see that nice rotating structures or vortices can be observed with these particles. So, these are called as streaks and we will see that why these are called as streaks.

(Refer Slide Time: 27:04)



We will look into one or two more visualisations with the streaks, and then we will try to formulise that how to mathematically write or express these concepts of visualisation.

(Refer Slide Time: 27:22)



This is a numerically or computationally simulated flow with the help of synthetic particles. So, these are not really particles in the physical sense, but this is just the entire flow is simulated in computer to have this kind of visualisation we stop here today.

Thank you very much.