Introduction to Fluid Mechanics
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Lecture – 19
Stability of solid bodies in fluid-Part-II

We were discussing earlier about the stability of floating bodies and we will continue with that. So, if you recall this sketch which we were discussing in the previous class, we identified a point known as metacentre and the motivation behind identifying this point is that when it is a floating body and it gets displaced and its submerged portion within the water changes its configuration the centre of buoyancy does not remain fixed at its own position. So, referring to a fixed centre of buoyancy might not work as stability criteria.

Now, we will try to find out as what we have done for the case of totally submerged bodies that how we can find out stability criterion for floating bodies. As you can see the presence of or the existence of this metacentre is something which is equivalent to the presence or existence of the centre of buoyancy for a case when a centre of buoyancy for a case when a centre of buoyancy remains at its own position. So, the metacentre somehow reflects the location of the centre of buoyancy with respect to the axis of symmetry of the body. So, it has some relationship with the centre of buoyancy definitely, but by specifying the metacentre or by specifying the centre of buoyancy just as their absolute locations it is not just possible to talk about the stability criteria.

So, we have look into more details to look in to more details we will set up some coordinate axis say we have x axis like this, say we have z axis like this and the y axis is perpendicular to the plane of the figure.

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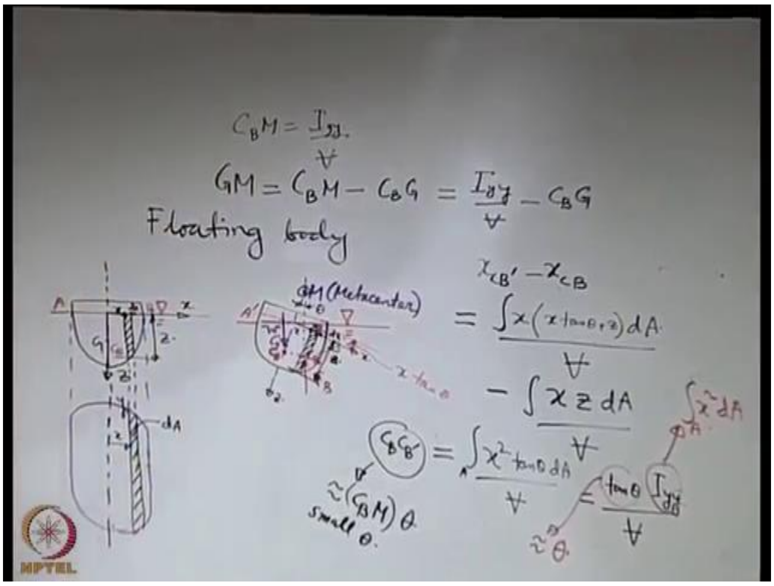

So, the y axis is this dotted line which represents an axis in the other view now if you consider that how we calculate the submerged volume the volume of the solid which is there within the fluid let us say that at a distance of x we take a small element of width dx and this element has a depth of z. So, this element if you look in to the other view will be like this which is basically located at a distance x from the y axis. Let us say that this shaded area is dA.

So, you can say that z into dA is a representation of elemental volume of this shaded portion. Now when it is even when it is tilted we fix the x and z axis with respect to the body and we keep them as same. So, we still have this as the x axis and may be this as the z axis these are fixed relative to the body. So, all though the body has tilted we are using a body fitted set of axis so to say. So, now, let us say the angle of tilt is theta, if the angle of tilt is theta then what happens? We can find out that; what is now the displaced volume. So, if you consider again say at a distance x from here some steep of width dx.

Now, if you consider the displaced volume the displaced volume is of course, corresponding to this part this part like z same as this one because it is the same axis that has got tilted. So, if this is z this displaced part is also z, but you have the additional displaced part to which may let us mark with the different colour. So, this is an additional displaced volume. So, this additional displaced part will correspond to a particular length along the z axis, and what is this? Remember that if this angle is theta then the angle of tilt of the x axis with respect to the horizontal is also theta.

So, what is this dimension? You know that this is x. So, that is x tan theta. So, if we calculate what is the displacement in centroid of the displaced volume that is what is the difference between the x coordinate of say the original centre of buoyancy let us give it a name that there was a original centre of buoyancy say C B and now the new centre of buoyancy is C B prime. So, the difference in x coordinate of C B prime and C B that is what we are interested to find out. What is that? Centre of buoyancy you can calculate by utilizing the formula for centroid of a volume which is here the displaced volume. So, this will be integral of x d v. So, x is this x; what is d v? So, you have this total height as x tan theta plus z dA, dA is the other area in the other view. So, the depth times the d A is the volume. So, these divided by the volume of the immersed part because of this symmetry the volume of the immersed total immersed volume does not change, whatever comes the same volume goes up above the water. So, the total displaced volume does not change; minus what was the original one integral of x z dA divided by V.

It may be easier if we mark the original C B and the displaced C B here. So, there has been a displacement from C B to C B prime the location of the centre of buoyancy has got shifted. So, this is nothing, but equal to C B, C B prime that is the length that we are talking about and this is equal to integral of x square tan theta d A divided by V, theta being like a constant parameter tan theta you can take out of the integral and this area integral is carried out over the shaded area dA. So, what does it represent if you take tan theta out of the integral? Second moment of area with respect to which axis;

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With respect to;

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You see it is, you refer to this plan view integral of x square dA is the second moment of area with respect to the y axis. So, it is tan theta in to I yy by V where what is I yy? It is integral of x square d A.

Now, referring to this figure you can say that for a small angle theta C B, C B prime this particular small distance is a small arc for may be a circle. So, you can write this as approximately C B M into theta for small theta. It is just like s equal to r theta for a small part of an arc of a circle because this is very small we are assuming a small displacement we test stability not with a large displacement, but with a small displacement.

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Meta centre M is the metacentre. Now for the same smallness tan theta will be approximately equal to theta of course, you are expressing theta in radiant that is understood implicitly. So, then if you equate the 2 parts what you will get C B M is equal to I yy by V. Now look in to this figure. In this figure you see that M is above G when m is above G the couple movement created by the forces is trying to restore the body to its original configuration; however, if G was above M it would have been just the opposite case that you can clearly visualize. So, what is important here is the location of M relative to G just like for a totally submerged body it was location of B relative to or the centre of buoyancy relative to G. So, what we are interested to find out is what is the height GM.

So, GM you can write it as C B M minus C B G, so I yy by V minus C B G. See why we have written the formula in this way? In the right hand side you have terms which are independent of the deformed configuration or deflected configuration. So, if you see here I yy V these are, this can calculated based on what is the original configuration and C B is the location of the original centre of buoyancy relative to the body, G is the location of centre of gravity relative to the body and the distance between those 2 that does not change with deflection. So, we are able to express something which should be a function of deflection in terms of certain things which are not functions of deflection therefore, in is also not a function of deflection and M remains sort of fixed, but you have to keep in mind that there are certain assumptions that go behind this what are the important assumptions?

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Theta is small. So, this analysis is valid only when you having a small theta not only that we have implicitly assumed that the z axis is the axis of symmetry for calculating or for coming up this expression. So, we are essentially dealing with a simplified case with symmetric bodies that we also have to keep in mind. Let us look an look into an example where we illustrate the use of this expression for finding out the stability criteria, so what is the stability criteria here let us summarize. If M is above G it is stable, if M is below G it is unstable and obviously if M coincides with G it is neutral. So, you can clearly understand that M plays the role of equivalent role of centre of buoyancy in this case, but it is not exactly the centre of buoyancy.

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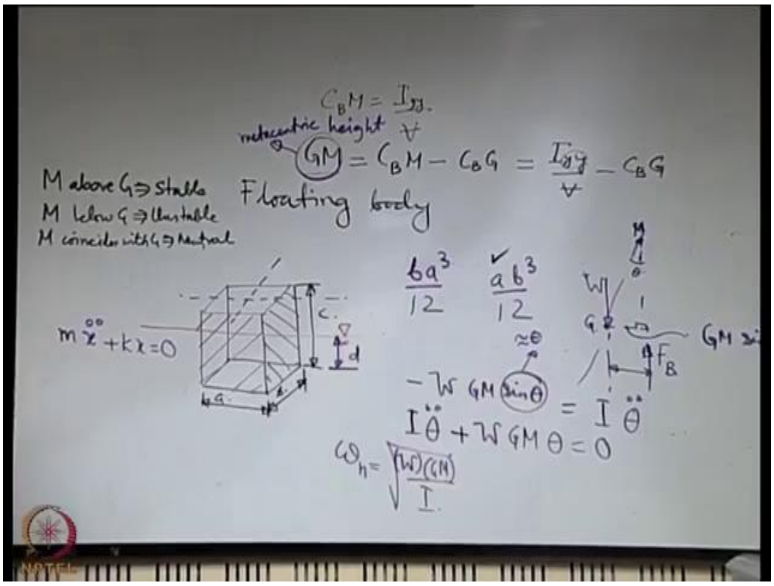

We consider a very simple example and we will illustrate the use of this through that example. Let us say that you have body of this shape rectangular parallelepiped shape and you can give dimensions say let us say a maybe this is b and let us say this is c, say this is partially immersed in a fluid, so it is like a floating body.

Now let us try to see that how we can apply this stability criterion here and what are the important issues. First important issue is that when you apply these criteria what should be the y axis with respect to which you are considering the second moment of area. If you look in to this problem very critically you will see that there are certain not trivial issues like, you can consider maybe this as one of the frontal surfaces you might consider this as one of the frontal surfaces and even the third one.

So, accordingly it is not very straight forward to say. So, when we drew the picture of the boat see it could be this front part it could be this side part both are exposed to the water I mean both are within the water. So, if you could say that what should be the corresponding axis that you need to consider or does the choice of the axis change if you shift your attention from the frontal area to the side area at the end what you are bothered with you are bothered with the plain view. So, when you have the plain view which is like the top view of this one, what is the plane view it is the intersection of the body with the free water surface or free fluid surface.

So, that particular view it boils down to the same when you consider this surface or this surface. But again when you look from the side and when you look from this one maybe in one case you are considering this as the y axis, in another case you are considering this as the y axis both axis are relative to the top view. So, the question is with respect to which axis you should evaluate this. See this is what this is like evaluation of safety criteria that it will be stable. So, when you want that it should be stable what should be the guideline? The guideline should be that like you want this to be positive right; that means, this height is greater than this height then like M is above G. So, this, the way in which this is written the right hand side has to be positive. So, more positive means so to say, it is like it is more stable so to say.

So, if we take the least of this one least possible value of this one and still find that the right hand side is positive then for all other cases it should be positive, it is like a safety design. So, you look in to the most adverse condition make sure that it is positive for even for that. So, for better conditions it would always be better. So, out of these 2 axis for one axis it will be what will be the I, it will be b a cube by 12, another will be a b cube by 12. So, you take the smaller one; that means, if b is smaller than a as an example you take this one. So, then when you substitute that in this expression and still you calculate this as positive you are assured that it is safe because with respect to other axis when you calculate the metacentric height it will definitely be greater this is known as metacentric height.

And really when you calculate different metacentric heights based on different views these represent different types of angular motions like rolling, pitching, these are different technical names based on with respect to what type of axis it is tilting. But just for design simple design you can consider the smaller one and you divide it by the volume, the volume not the total volume, but the volume of the submerged part so that you can easily calculate based on what part is submerged. Let us say that d is the submerged depth it is easy to calculate this because you can use the equilibrium that for equilibrium the buoyancy force must be equal to the weight.

So, from that if you know the density of the fluid and the density of the solid body you can come up with what should be the equilibrium d, just by very simple equating of the 2 forces. So, you can calculate what is the; what is the volume which is immersed in the fluid. Centre of buoyancy location you can get from the centroid of the displaced volume centre of gravity also has the fixed position with respect to the body. So, you can clearly substitute this dimensions and find out what is the metacentric height. So, this effectively requires only the calculation of the immersed volume, the calculation of the second moment of area with respect to this axis and specification of the locations of the centre of buoyancy and centre of gravity with respect to the axis fixed on the body. So, that is sufficient information to calculate the metacentric height.

So, what we can clearly see is that if M is above G that makes it stable; that means, load the location of the centre of gravity it is having a greater chance that it would be stable because then it is a greater chance that M is above G lower the location of G, so location of G if it goes higher and higher it might make a previously stable system or convert it into an unstable one. Let us look in to an animated example to consider this case. So, in this animated example what we will see, we will see that how the stability maybe disturbed because of the shifting of the distribution of the weight of the body.

So, let us just look in to it carefully. So, there is a body which is given a slight displacement and you will see that for a small displacement it oscillates like a pendulum we will, we may easily derive what is the time period of that. Now you see that the distribution of the weight is being altered. So, the centre of gravity is being shifted higher and higher by putting the load more and more towards the top.

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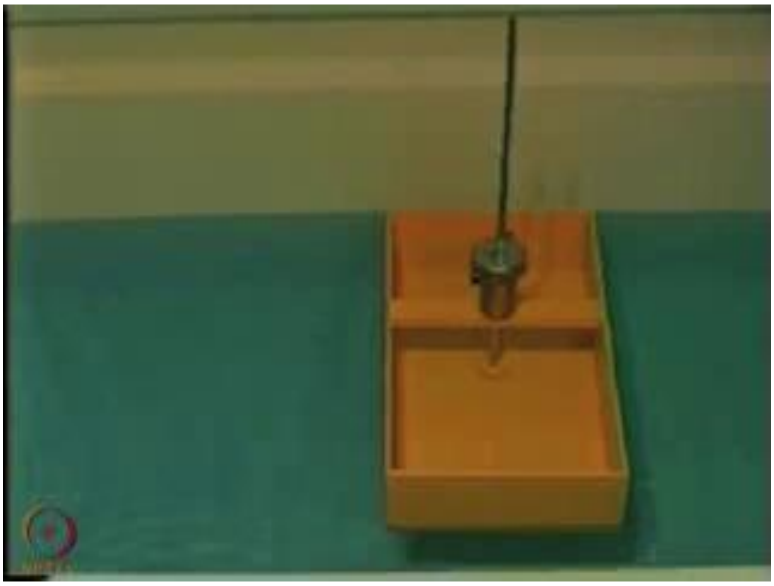

And you see that it topples so; obviously, this is clear illustration of this concept that we have learnt in this example that if you have meta centre above the centre of gravity it makes it more stable and otherwise it is not so. So, now, for small oscillations you could see that or for small deflection you could see that it oscillates like a pendulum.

So, when it oscillates like a pendulum it has a time period and that may be calculated by calculating the moment of the resultant force with respect to the axis. So, if you have a tilted axis like this and if you have a force F B which is the buoyancy force, this force has a moment with respect to the axis of the body. So, what is the moment of this force with respect to the axis of the body? You can just find out the perpendicular distance of this force from the axis of the body. So, it will be just what will be the perpendicular distance of this buoyancy force line of action of the buoyancy force, I mean you can see that it eventually passes through the axis of the body right. So, it is not this force individually which is important it is the couple moment that you are having, the so called the restoring couple. So, you have also the W and it is basically the couple moment of this forces that you need to consider. So, the perpendicular distance between these 2. In terms of the metacentric height, could you express this so if have this as the metacentric M.

So, we are interested about this distance. So, it is possible to express it in terms of the metacentric height. So, this angle being theta it is what is this perpendicular distance? G M sine theta, so the moment of this force is W GM sine theta and the resultant moment of all the forces is nothing, but if it is like rotation of rigid body with respect to a fixed axis it is a restoring moment. So, you should have a minus sine associated with one; that means, whatever is the chosen positive direction of theta, theta dot, theta double dot this has a direction opposite to that to bring it to its original configuration.

So, you can write this as; so I is what is now? I is the mass moment of inertia, it is not the same I that we were talking about it is the real mass moment of inertia with respect to the axis of the body relative to which it is tilting. So, this plus for small theta again this is approximately equal to theta that is equal to 0. So, it is just like equation of a spring mass system m x double dot plus k x equal to 0. So, what is the natural frequency of oscillation of this system?

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Root over equivalent stiffness by equivalent mass. So, W into GM divided by I that is the natural frequency of oscillation of the system, it is an angular oscillation not a linear one. So, you can clearly see that greater the metacentric height greater will be the frequency of oscillation. So, if it is the ship it will be more uncomfortable to the passenger. So, these are 2 conflicting designs. See greater the metacentric height you expect it to be a safe in terms of stability, but it will have more oscillation within that stability regime; that means, for a passenger it may be quite uncomfortable. At the same time if it is for a used for a particular say critical purpose like warfare and so on. So, there stability ship is more important than the comfort of the crew and they are; obviously, it is the stability that should be the driving factor for design. So, when a ship is designed you have 2 conflicting things one is the comfort another is the stability and comfort factor comes from this high frequency of oscillation and the stability comes from the metacentric height or location of the metacentre relating to the centre of gravity.