Introduction to Fluid Mechanics
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Lecture – 18
Force on a surface immersed in fluid- Part-III
Stability of solid bodies in fluid- Part-I

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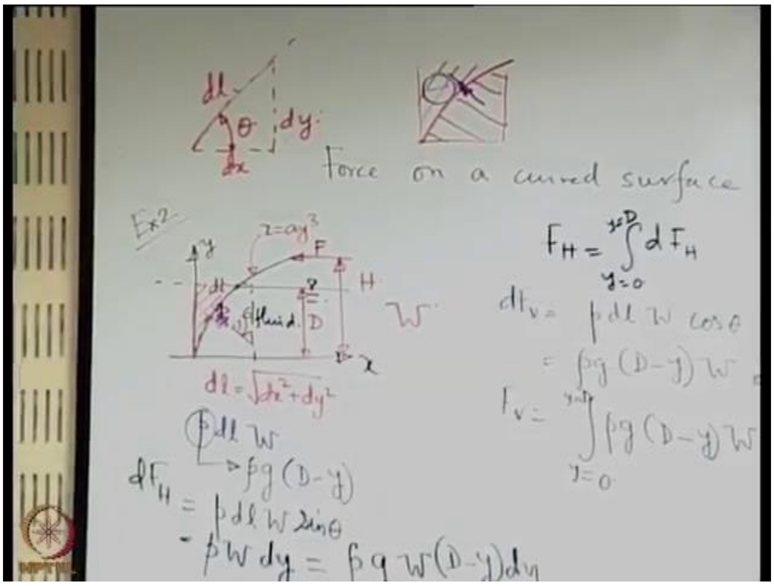

Let us consider a second example, let us say that you have a curved gate, instead of a plane gate you have a curved gate, and there is fluid on one side of it say you have fluid on this side. There is a free surface of the fluid given by this, and this fluid tries to exert some force on this curved surface, and there is some balancer. So, there is some external force which is applied here, to keep it in equilibrium. And the location of this external force is given by this H, and the depth to which this fluid is filled up is capital D.

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So, why such problems are practically important? Let us look in to, may be 1 or 2 practical cases to see, that why such effects are important. So, we will look in to some example applications. So, it is, if you clearly see this is like a dam. So, there are forces which are exerted by water and these force components are like these are quite heavy or large forces, huge forces which are acting, and the structure must be strong enough to sustain it, and it is very common and to save us from floods or other calamities there are many occasions, where there are reservoirs in which these water supplies are stored.

So, until and unless the rain fall is very severe, it retains it is height, but once its, once the rain fall is, so strong it cannot retain it is height. So, some of the water has to be released, and when the water is not released it is under static condition, and it has to be calculated on the basis of fluid statics condition, the resultant forces and the situation of equilibrium. Of course, when it is, when the water is very dynamic then you do not consider the fluid static, but you consider even the dynamic effect of water as it is impinging on the surface.

So, you can see the shape of the. So, called dam, it is it is not really a plane surface, but it is a curved surface it is like arch type. There are fluids on different at different ends and it is important to calculate that what is the resultant force that is there. So, that is the motivation behind solving such a problem that it gives you a clue of, how to design may be dams or switch gates, where across which you have different fluid elements which are exerting forces. So, here on one side you have some fluid element as say water, on other side say there is atmosphere. So, on the right side there is water in this example, and we want to calculate what is the resultant force.

So, what we will do? We may solve this problem in again two ways; one is by looking to the horizontal and vertical components of forces, according to the principle that we have just seen, or may be just by direct integration of the forces on elements. So, if we do that the second one is little bit easy to begin with. So, let us say that we have considered a small element here, at a location x comma y. For idealization let us say that this curved surface when it is projection is considered it has the equation x equal to a y cube, which is the equation of this this curve. So, at the x comma y if you considered a small element say of size dl, then what is the resultant force due to pressure on this dl?

First of all how do we specify this dl, say at x comma y, we can consider a dl which is comprising of the resultant some displacement along x say dx, and some displacement along y say dy. So, dl may be written as good as square root of dx square plus dy square. So, if you draw a magnified figure, if this is dl, it is like the sum of dx along x and dy along y. If this is a small part of the curve, this is approximately the tangent to the curve at that point, at that x comma y and so you can consider that this angle theta designates the slope of the curve, local slope of the curve at that point, where it is aligned, the dl is aligned with the tangent to the curve at x comma y.

So, we know what is dl. So, if we know, what is the width of the fluid, or which is the, what is the width of the gate, say W is the width of the gate then W in to dl is the area on which the fluid pressure is acting. So, what will be the direction of the action of the fluid pressure, it is acting in this way, normal to the surface. So, that we will have 2 components; one is the horizontal component another is the vertical component. So, how do you calculate the horizontal component and the vertical component? First of all what is the force, because of this it is p in to dl in to W, where p is the pressure at x comma y. So, how it is related to the depth capital D? So, how can you write express p in terms of the depth capital D? Yes.

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So, the height of this is D minus y from the free surface. This is the resultant force, but if we break it up in to components that is what is our objective, and we should break it up in to components, because we cannot just integrate it like this, because the direction of such force on each element is changing. So, we cannot just algebraically or scalarly add. We should take out extract individual components and add up the components.

So, when we take the horizontal components. What is the horizontal component, because of this? Yes.

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So, this is the resultant force. So, so the dl makes theta with the horizontal. So, normal to the dl should make an angle theta with the vertical.

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So, the horizontal force is, p dl W in to sin theta. Now, if you look in to this figure dl sin theta is nothing, but dy. So, p in to W in to dy. So, this, let us call this as d F H, because it is just on a differentially small area. So, rho d W D minus y dy. How can you calculate the vertical component of the force? So, calculating horizontal component is straight forward, you just integrate it. So, when you integrate it, let us try to write the complete expression I will leave the integral on you, but at least let us write the complete expression

So, F H is integral of d F H, what should be the limit when which you integrate; y equal to 0 to y equal to D. Something will interesting it shows as if, this is independent of the function of the graph x equals to A y cube, it does not matter what is x, and intuitively it is supposed to be that way, because when you take the projection of the surface in the sight plane, does not matter how it is curved, because the projection will always be straight. So, you just required to know the extent of the depth, and you can verify that this will be nothing, but the projection on the surface which is basically our surface projected on a vertical plane, and the same horizontal component of force will come out with that exercise. What about the vertical component of the force? So, d F v.

What should be the vertical component of the force? So, it is p dl W cos theta, so rho G D minus y in to W. What is dl cos theta? dx. So, now, you may express either x in terms of y or y in terms of x, the function is given, and if you integrate you will be getting the total vertical force. So, F v is; so dx you can write as say 3 A y square dy, and you can now take the limits from y equal to 0 to y equal to D. We will not go in to the integration in details, because it is a very simple integration, and it is not worth to waste time just on that, but we will focus on something which is bit more important.

That is let us say that we want to find out the same vertical component of force, but using the method of the weight of the fluid; that is contained within that projected volume. One way to see that whether it is correct or not, is that if we find out that weight and we come up with an expression it should be same as this one. So, that you can of course, find out the weight, and check that the expression at the end is same as what you get out of this integral, but even before that how you qualitatively asses it. So, what was our method, you project from the corners, vertical lines which meet the free surface or it is extended form. So, one line is projected here, and another it becomes a point, and it should be the shaded volume of the fluid. If I were in your place, the first question that would have come to my mind is there is now fluid here.

So, how can you claim that this shaded volume of fluid will give you the, or weight of this shaded volume of fluid will give you the vertical component. So, just think in this way. Let us say that this is the curved surface, and let us consider that there is volume of fluid one on this side and another on this side. If that was the case then you can see that in a static condition it is naturally in equilibrium, because whatever is the force due to pressure from one side the same is the force due to pressure on the other side. So, the 2 sides are keeping hitting equilibrium. Now, when you do not have a fluid here; that means, that this part is like minus, it is subtracted. So, it is a condition equivalent to a deviation from equilibrium, because of a lack of presence of these rather than the real presence of these. So, the deviation from the equilibrium, is because of this equivalent volume of element, which is which is in the upper part.

So, if you calculate that volume of fluid, and find the corresponding weight of that you will see that you will get it exactly the same as this one. And you can clearly tell that what should be the direction of force, acting on vertical component of force acting on this; upwards or downwards, upwards, because the vertical component of this one is like upwards I mean directed in this way. So, it is not apparently the vertical component of the shaded volume of fluid, because it will appear that if there is a volume of fluid here it is weights should act downwards you have to remember, it is an imaginary volume of fluid and it is just giving you the equivalent volume that needs to be employed for calculation from the vertical component of force. The exact sense of the force has to be determined from the physical meaning of this type that is. So, if you subtract kind of volume like this it is as good as extra upward force, because something is missing, some weight is missing from the top.

Otherwise, also from the direction of pressure itself it will follow. So, either way whenever you are calculating, either by the fundamental method of finding force components on individual elements, summing the vector summing them up; that means, summing up scalar forms in terms of the x and the y or the horizontal and the vertical components, or finding out the vertical and horizontal components by the alternative method that we have seen. Whatever is it you must assert in the correct sense from the physical condition, and that will not always be dictated by the rule base. It will come from the consideration of where is the volume of fluid that is present is exerting the force, and what is the sense of that force when it is exerting a pressure on that element. So, that consideration should give the proper sense of the vertical component of the force.

Now, we have considered forces on or force components on plane and curved surfaces. We have seen that there are some simple ways by which we can evaluate these force components. Now, what we have assumed is that when the surface is put in the fluid, the surface is in equilibrium, and that equilibrium is not disturbed, but if there is a slight tilt, because of whatever reason, then that equilibrium may be disturbed, and if that equilibrium is disturbed what will happen we have to understand. So, we have to now go through the concept of stability of floating and submerged bodies.

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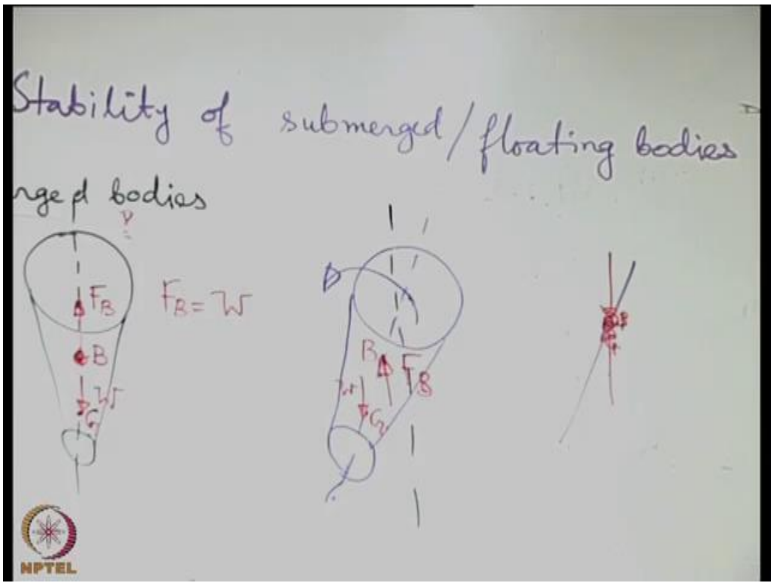

Let us say that first we consider submerged bodies. So, submerged body is something which is completely immersed in the fluid, and floating means a part is at the top. I mean above the free surface, and the part is below the free surface. So, let us take an example, let us say that we have this kind of a body looks, like a parachute type, the top is light and the bottom is heavy, because of some added mass. Initially it is completely submerged, and the resultant forces whatever are acting on this, you may write in the terms of the buoyancy force and the weight. So, the buoyancy force will be acting on acting through some point, and the weight will be acting through some points. So, the buoyancy force is based on the volume which is submerged not the mass. So, when you consider the volume; that is submerged the greater portion of the volume is at the top.

So, may be the resultant buoyancy force acts through this, but the mass is more concentrated towards the bottom. So, the weight may be, is more concentrated on the resultant, force due to the weight distribution, is passing through the point which is G of the centre of gravity, that is located somewhat below. And the resultant buoyancy force is now acting through some point B. So, what is the point B? So, B is the so called centre of buoyancy; that means, whatever is the location the centroid of the displaced volume. So, it is fully a geometrical concept. Whereas when you have G this is this depends on the distribution of mass over the body. So, this is something where for equilibrium you have F B equal to W, it is in equilibrium. Now, let us say that you have slightly tilted it.

So, when you have slightly tilted it. It has a deformed, not deformed, but deflected configuration like this. So, it is axis has got tilted from the original vertical one, may be, because of some disturbance, now the entire body is within the fluid; therefore, the location of the centre of buoyancy and centre of gravity relative to the body does not change, because the entire body is within the fluid itself. So, if you have say this as g, this still remains as G, if you have this A B this still remains as B, because it is already totally within the fluid. So, it is volume distribution within the fluid, it is mass distribution everything it does not change. So, you have F B acting like this, you have W acting like this. Only thing what has now changed is that, no more F B and W are co linear.

So, when they are not co linear they will still be equal and opposite forces, but in not passing along the same line. So, it will create a couple moment. So, what will that couple moment try to do? So, if you see the sense of this couple moment, what it will try to do. So, the weight, it will try to create a rotation like this, which is shown in the figure if you look in to the senses of the forces, and this rotation what it will try to do it will try to bring it back to it is original position or configuration. So, we call it a restoring moment. On the other hand if G was above B then this would have been downwards and then this would have been upwards, and that would have tried to increase the angular displacement even further.

So, what is the hallmark of a stability stable equilibrium; that is if you have a slight displacement, it will try to come back or restored to it is original configuration. So, this type of situation ensures that, it tends to come back to it is original configuration; however, if G was above B it would have tried to increase the angular displacement even further not restoring, but helping the disturbance. So, in that way, it will be unstable equilibrium. What would be the situation, if B and G are co incident somehow?

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So, where ever it is there, still it will be a co linear, say you have somehow an arrangement where you have say this as B this as the same point is G. So, this is B, this is as good as G. So, whatever is the weight and the buoyancy, they are always acting along the same line. No matter whether it is tilted or not, where ever it is tilted, it will locally attain equilibrium, and that equilibrium is known as neutral equilibrium. So, the stability of submerged bodies, the equilibrium condition depends on the relative location of B with respect to G. So, what we can summarize from this. If B is above G, what it will imply. It will imply stable equilibrium. If B is below G, it is unstable equilibrium, and if B coincides with G; that is neutral equilibrium so far so good, but we have to remember that submerged bodies are not the only types of bodies that we need to consider, many practical examples are cases of floating bodies like ships. So, a part is within the fluid and the part is outside the fluid.

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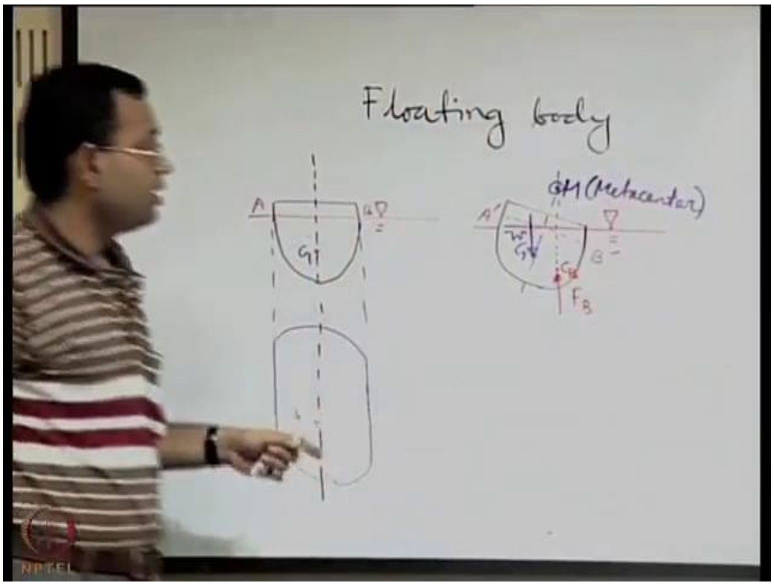

So, what will be the situation for that, let us take an example, let us consider a floating body. So, when you consider a floating body. Let us see that how is it different first from a submerged body. So, let us say that this is a free surface of the fluid. This is the body which is floating, something like a boat shape or similar to that. If you consider it is intersection with the free surface of the fluid; that is the sectional view of the intersection with the fluid, let us say that it is something like this. So, this part that we have drawn is like this one, whatever we have drawn that plane is like this.

Now this we are assuming that this is having an axis of symmetry, say this the axis of symmetry. Now let us say that this is tilted. Let us see what happens when it is tilted. We will assume that it is tilted very slightly, because when we test the stability we just give a small displacement and see how it responds to the small disturbance. So, we just tilt it like this, and it comes to this configuration.

So, whatever was the line of interface before? Now say that line of interface has with relative to the body, whatever was the line say A B. Now say it becomes A prime B prime. If it symmetrical with respect to the axis at over which it tilts, you will see that one interesting this has happened. What is that interesting thing, some new part has gone down in to the fluid, some new part has come up, and if it is very symmetric these 2 parts are of same volume. So, the volume that was earlier immersed is still the same, but the distribution of the volume has changed.

So, once the distribution of the volume has changed what has happened, the centre of buoyancy has changed. So, there is no sanctity with respect to the location of the centre of buoyancy; that is very important. So, for the submerged body when you have the location of the centre of buoyancy, relative to the body it does not change. Whereas when you have a floating body, depending on it is tilted configuration, there will be an extent of dominance of one side of the body relative to it is immersed conditions with respect to the other, and accordingly there will be a bias that there will be preferred side across which the centre of buoyancy will be moving. So, the centre of buoyancy cannot be one of the fixed parameters, with respect to which we may decide whether it will be stable or not; that is the first thing. So, the stability criteria for submerged body will not work. So, whenever we are trying to learn something new, we have to understand that why are we trying to learn it at first. I mean if the same criteria for submerged body would have worked, we would have not gone on to this exercise.

So, first we are getting that motivation that how or where is the difference the difference again I sum up is like this, the centre of buoyancy location is now not fixed with respect to the body, but it goes on evolving as the body is tilting . So, let us say that the centre of buoyancy now comes to this position, say C B centre of buoyancy. We expect that it will be coming towards this direction, because it is now more tilted towards the right. So, more part of the body is now in to the fluid towards the right. So, you have the centre of buoyancy in this way. So, you have the resultant buoyancy force like this. The centre of gravity is something which is fixed with reference to body that does not change. So, if the centre of gravity earlier was say relative to the body here, let us say that the centre of gravity is still here.

So, the weight of the body is like this. So, now, again you can see that there is a couple moment, and whether it is restoring or helping, it depends on that whether it is if, it is extended where it will meet the axis. If it meets down of this one, then it is one way if it meets above g, it is the other way. So, where it meets the axis that point is known as Meta centre.

So, in our next class we will see that what is the consequence of this Meta centre, and how the location of the Meta centre will dictate the stability under this condition. So, it is not the centre of buoyancy that is important here, but the location of the meta centre relative to the body is, what is going to decide whether the body should be stable or not for a floating body, that we will take up in the next class.

Thank you.