Introduction to Fluid Mechanics

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Lecture – 17
Force on a surface immersed in fluid- Part-II

We continue with our discussion on force on a plane surface submerged in the fluid and we take up the same example that we took in the last class and we will try to find out the force on the surface by direct integration without going into the standard expression for force. So, here if you recall what is the objective of this problem is to solve that what is the force that is required to keep this gate stationary and to do that what we required is to find out the movement of the distributed forces acting on this semicircular plate with respect to the hinge point o that was the one of the objectives for writing the equations of equilibrium with respect to the rotational equilibrium.

So, to do that what we can take we can take small element. So, when you take a small element here you have to consider a radial and a circumferential element simultaneously.

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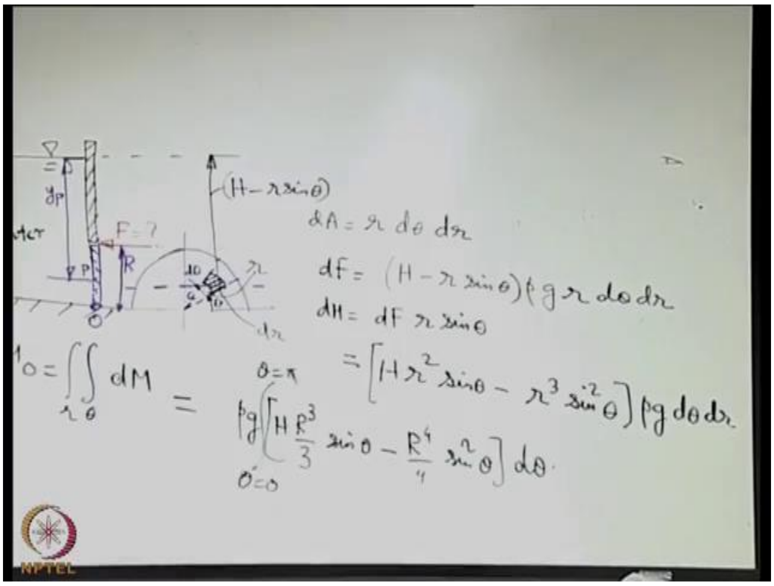

So, we can consider a small element shaded like this what is the specification of the small element which it is located or centered around R comma theta; that means, we considered that this is located at a radial location of R and angular location of theta the angle subtended by the element is d theta, the radial location is small r which is the radial location of the element the radial width of the element is d r.

So, what is the differential area that is being represented by the element? Yes.

Student: (Refer Time: 02:18).

No question of 2 pi, that is a just shaded element the shaded area.

Student: (Refer Time: 02:27).

So, it is like roughly like a rectangle one of the sides is r d theta another side is d r. So, r d theta d r what is the force that acts on this element due to pressure what is the pressure acting on this element? It is related to the local height of the element, what is this local height?

Student: (Refer Time: 03:00).

So, capital H is the total height minus small r sin theta. So, the force acting on this d a that is d F is equal to H minus r sin theta into rho into g into r d theta d r.

What is the moment of this force with respect to o? So, this into r sin theta, so, if we can break it up into 2 terms multiplied these by sin theta r sin theta. So, it will be H r square sin theta minus r cube sin square theta that into rho g d theta d r. Now the total moment of this distributed force with respect to the point o is integral of this d m now when you consider the integral the integral is with respect to both theta as well as r it is a double integral.

So, you may evaluate the integral with respect to r first or theta first it is irrelevant let us say that you want to evaluate the integral with respect to r first. So, you have first the integral with respect to r. So, when you have integral with respect to r small r varies from 0 to capital R. So, the first term becomes this is r square d r; that means, r cube by 3. So, this we are integrating with respect to r sin theta minus this is r cubed d r. So, r to the power capital R to the power 4 by 4 sin square theta d theta now integral with respect to theta, what is the limit of theta?

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0 to pi, so, the remaining work is very simple I need not complete this one. These are very simply integrals and you can complete yourself. So, once you complete this integral you will get an expression for resultant moment of the distributed force with respect to o in terms of H capital R and of course, rho and g and this resultant moment is equal to what capital F into capital R? So, that will tell you what is the value of capital F we have seen that why it is so, because all other forces acting through it will pass through o. So, they will have no moment with respect to o. So, this is just like these are the 2 counter acting moments.

So, this example shows that whenever you have force on a surface, it is not necessary that you have to go by the formula that we have derived you may as well take a small element considered pressure distribution over the small element and find out what is the resultant force due to that pressure distribution resultant moment and so on by fundamentally integrating it over the entire area without going into the formula.

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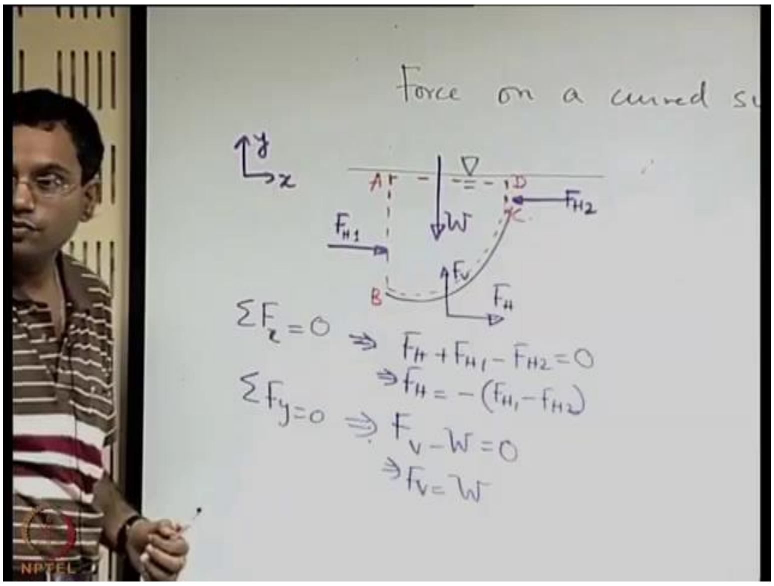

Next what we will consider is force on a curved surface now can you tell me that what is the fundamental difference or what you expect the fundamental difference to be as compared to the force on a plane surface how you expect force on a curved surface to be different conceptually force on a curved surface again what is this surface this is immersed in a fluid at rest. So, that we are not repeating. So, the surface may be something like let us say the surface is something like this. Again if you see this surface is actually something like this and there is some fluid on one side and that fluid is exerting some force on the surface. So, this is again H view of the surface, earlier we were concentrating on force on a surface like which a plane surface now it is a curved surface.

So, you can understand the geometrical difference that is fine fundamentally in terms of basic mechanics, how do you expect this to be different as compared to that for force on a plane surface?

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Force direction changes as you move from one point to the other point; that means, it is not a system of parallel forces. So, the plane surface has to deal with a system of parallel forces. So, you can treat it like a scalar addition problem or a scalar summation problem whereas, in this case, it is having the pressure always acting normal to the boundary normal to the boundary is a direction that is changing from one point to the other and therefore, the resultant force is been dictated by a varying direction of the surface.

So, you no more have a system of parallel elements which is giving rise to the resultant. So, whenever you are adding here that to make the resultant force it has to be a true vector addition it is not just by adding it up in a scalar way integration is nothing, but summing up the individual components and you could see that very very simple integration could give the same result for force on a plane surface for a curved surface. Therefore, fundamentally the principle is the same that is you take element of the curved surface you find out what is the force acting on it the force acting on it will be normal to it you may break it up into components horizontal and vertical components in this way for each of the elements you can find out the horizontal and vertical components algebraically sum them up and make the vector addition.

So, this is a something which is very trivial now it is possible sometimes to reduce the calculation a little bit by taking some help from the concept of force on a plane surface and let us see how just considered that there is a fluid column like this. This is a volume of fluid which we are considering to be located within the projected part of this curve surface. So, if this curved ends of the curved surfaces are projected to the free surface whatever volume is contained within that that contained volume is enclosed by this dotted line. Now let us say that we are interested on the forces which are acting on this volume of fluid.

So, this volume is now having 2 plane surfaces C plane; plane surfaces, but forget about the top surface just for the timing considered the side surfaces. So, if you considered the volume say A, B, C, D, the surfaces A B and C D are plane surfaces and let us see that what are the forces which are acting on this plane surfaces? So, we are essentially trying to draw free body diagram of the volume element which is enclosed by this dotted line. So, what are the forces which are acting on this? So, when you have the left phase there is some fluid towards the left of it that exerts the force due to pressure what will be the direction of that force let us say we setup coordinates like this x and y.

So, what will be the direction of that force x. So, we call it a horizontal force assuming that x is a horizontal direction let us say that F H or H F 1 is the horizontal force acting from the left towards this element similarly if you have some fluid again on this side there will be a horizontal force say F H 2 acting from the other side towards these remember this force is due to pressure its compressive in nature. So, whatever is the fluid element located on the other side it is having a tendency to compress it and that is how the sense of these vectors are there these are the horizontal components of the forces on this sides what are the additional forces on this element it should have its own weight.

So, whatever water or fluid is contained here that will have its weight say W, any other force, yes.

Student: (Refer Time: 13:50).

There is a reaction between the surface and the fluid and that reaction again is likely to have 2 components, one is the horizontal component another is the vertical components. So, let us say that it has horizontal component F H and a vertical component F V, what are these? These are the components of the reaction forces exerted by the curved surface on the fluid now the fluid is in equilibrium. So, when the fluid is in equilibrium you must have a resultant force along x equal to 0. So, you have F H plus F H 1 minus F H 2 equal to 0; that means, F H is equal to minus of F H 1 minus F H 2 then resultant force along y equal to 0 what it means F V minus w equal to 0; that means F V equal to w.

From this apparently very simple calculation, we come up with a very interesting result that is if you have a curved surface still whatever forces which are acting on it you may resolve it into 2 components for the horizontal component it is basically the resultant force on the horizontal projections or on the projections of these curves ends up the curves on a vertical plane or so called horizontal components of the forces on vertical projections of the end of the curve; that means, if you have a curve like this when you considered the n this n when it when you considered its projection the projection is just like this. So, we are basically having to considered a horizontal component of force for the left or horizontal component of force for the right and the difference of these 2 is actually giving the horizontal component of the net force for the vertical component it is just the weight of the fluid that is being contained within this extended volume.

So, if you somehow can calculate the weight of the fluid and that is as good as calculating the volume of the fluid because then you can use the density to calculate the weight of the fluid. So, whatever fluid is contained here within this dotted line weight of that fluid is the vertical component of the force and whatever is the horizontal component of force or whatever is the force because of pressure on the projections from the taken from the sides of this curve that contributes to the result and horizontal force and remember that these are the forces exerted by the curve surface on the fluid we are interested in the other thing the opposite thing that is what is the force on the curved surface. So, by Newton’s third law those are just negatives of this one. So, minus F H and minus F V are the forces which are exerted by the fluid on the curved surface.

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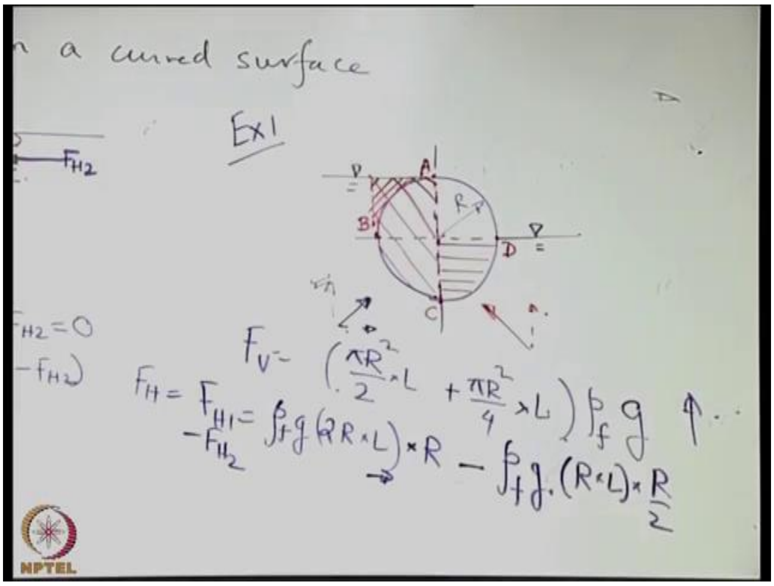

Let us take an example to see that how we calculate it. Let us say that we have a long circular cylinder and you have the free surface of the fluid in this way. So, on the left side the this is the free surface on the right side this is the free surface and the solid material is a circular cylinder long its length is perpendicular to the plane of the board we are interested to find out what is the result and horizontal and vertical component of force exerted by the fluid on the cylinder. So, what we will do is we will give some names of or we will give some markers to important parts of the surface say the 4 important points A, B, C and D and we will consider the forces acting on this curve surface one by one.

So, first let us consider the force acting on A B, first let us consider the vertical components of forces then we will do the horizontal component. So, for vertical component what we want we should raise projections from the end of the surface till |it reaches the free surface whatever is the volume of the fluid contained within that it is the weight of that fine. So, for the part A B, it is like this, next let us consider the part B C. So, when you consider the part A B and you know the vertical component of force, is it downwards or upwards?

Student: (Refer Time: 19:40).

It is downwards and some common sense it is clear that the pressure is being exerted in such a way that its vertical component is downwards. Now consider B C. So, for B C, what we do? Again we raise the projection from one and it is like this from C we raise the projection up to the free surface. So, whatever is the volume that is contained within this, so what volume is contained within this?

Student: (Refer Time: 20:10).

This is the volume that is contained between B C and its projected parts up to the pre fluid surfaced. So, this is an imaginary volume of fluid right and what will be the direction of the vertical component of force acting on it upwards or downwards how do you make it out just see that if you have B C like this you will have the pressure acting on it in this way its vertical component will be up words.

So, it is such a distributed pressure over B C. So, look into the fundamental origin that will give you the guideline whether the resultant force is upward or downwards on that part of the surface now when you consider these 2 together you can see the common part which is shaded once it has come downwards another it has come upwards. So, they have cancelled out. So, what remains is the fluid equivalent to the volume of half of the cylinder for this half part. So, what is the vertical component of force acting on say a B C it is nothing, but the equivalent to the weight of the volume of half of this cylinder weight of what weight of the fluid equivalent to the volume of the half of the cylinder.

So; that means, as if it has displace the fluid equivalent to its volume and that is exerting it and up thrust. So, this is nothing, but the Archimedes principle that you have learnt in high school physics. So, in effect what is happening what when the solid is being immersed in a fluid it tends to displace a volume of fluid which is equivalent to the volume which is immersed and that tends to exert an up trust net up thrust and that up thrust is nothing, but same as the weight of the displaced volume of fluid by that particular volume of solid now if you come to C D. So, for C D, how you calculate the force vertical component of the force? So, you extend it up to the free surface. So, for this part the free surface is up to the level shown.

So, it is nothing, but the volume of this much. So, is it upwards or downwards it is upwards. So, if you have the pressure acting in this way its vertical component will be upwards. So, the resultant is upwards with what magnitude if is the radius of this cylinder. So, what is the volume corresponding to the left part that is pi r square by 2 into say L where L is the length of the cylinder then this part is pi r square by 4 into l. This is the volume that multiplied by the density of the fluid not density of the cylinder, but density of the fluid into g and acting upwards. So, this is the result and vertical component of force.

How can you calculate that what is the location through which the resultant of this force passes. So, it will definitely be passing through the centroid of the displaced volume and then it boils down to the calculation of the displaced centroid of the displace volume we are not going into that you can do it by simple statics how do you calculate the horizontal component of force. So, again you have something in the left and something in the right. So, when you have this A, B, C on A, B, C, what is the horizontal component of force? Let us say that is F H one just following the symbol and on C D there is a horizontal component of force. So, that is F H 2.

So, F H 1 is what?

Student: (Refer Time: 24:52).

What is its projected area on which you are considering that because this is now equivalent to force on a plane surface?

Student: 2 R.

So, 2 R is the height and L is the length. So, its projection on the side view is 2 R is its height because 2 are is the height of the cylinder and L is its length perpendicular to the plane of the figure. So, if you recall what is the resultant force rho of the fluid g into a what is a 2 R into L that is the equivalent projected area into H C what is H C H C is just R. So, when you say H C, H C is the location of the centroid of the projected area not the curved area.

So, it is the location of the centroid of the projected area from the free surface. So, that is F H one is it towards right or left it is towards right because again you can you can see that its horizontal component is towards right. So, this is towards right and then on C d what is F H first of all it is acting towards left its horizontal component. So, we put a minus sign again rho g what is the projected area into r into L into r by 2 which is the location of the centroid of the projected area vertical location from the free surface and the. So, this is F H one minus F H 2 that is F H. Hence you can calculate, what is the resultant? So, these are 2 individual components, you can find out the resultant by vector addition. So, that is trivial.