Introduction to Fluid Mechanics

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Lecture – 16
Force on a surface immersed in fluid-Part-1

When we are discussing about fluid statistics, one of our objectives will be that to find out because of that pressure; what is the force that is acting on a solid surface, the solid surface may be a plain one or may be a curved one.

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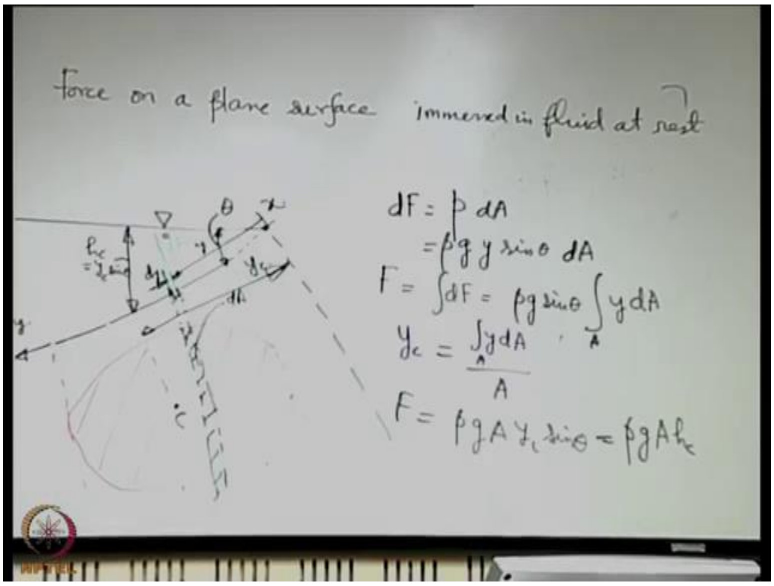

We start with an example of force on a plain surface. What is the special characteristic of the plain surface that is such a surface we are considering that is immersed in fluid at rest we will try to make a sketch of the arrangement, let us say that this is the free surface of the fluid; that means, on the top there may be atmosphere and a bottom there may be water as an example there is a surface the edge view of the surface is like this it is a plain surface.

So, how does it look? Let us try to have a visualization of this assume that the surface is like this. So, this is a plain surface and when you are seeing its projection in the plain of the board it looks like just the edge view that is the line. So, whenever we draw a line do keep in mind that it is some kind of arbitrarily distributed flat surface the edge of which is represented by this line. So, such a surface is there in a fluid and we are interested to find out what is the force on this surface because of the pressure distribution in the fluid.

With that objective in mind, let us try to may be draw the other view of the surface it just may be very very arbitrary.

So, it is a plain surface of some arbitrary geometry. So, let us say that this is the section of that surface when looked parallel to it we will set up certain coordinate axis let us say we extend this and it meets the free surface at this point. So, we will call this as y axis and may be an axis perpendicular to that as x axis our problem is actually a very simple problem it is a problem of finding out the resultant of a distributed force because pressure distribution gives rise to a distributed force why it is a distributed force because pressure varies linearly with the height different elements of the plate are located at different heights from the free surface. So, it is a distributed force.

The advantage of handling with a plain surface is that this is distributed force is a system of parallel forces. So, if you have for example, if you consider a small element at a distance y from the axis let us say the thickness of the element is d y. So, what is the force due to pressure that acts on this element? Let us say that d F is the force that acts on this element due to the pressure distribution this is the fluid at rest. So, what is d F? D F is the local pressure on the element times the elemental area let us say that the elemental area of which we are talking that can be represented in the other view completely let this be d A, it just corresponds to this d y. So, d F is the local p at the location y into d F what is a local p? Yes.

Student: (Refer Time: 04:50).

Rho g.

Student: y.

Into y sin theta also if you call it; call this angle as theta because y sin theta is nothing, but the vertical depth from the free surface to the location under concern that multiplied by d a. So, what is the total force that acts on the plate it is integral of d F. So, it is integral of. So, rho g sin theta those are like invariants with respect to the integration. So, you are integrating with respect to d A integral of y d A and integral is over the entire area say capital A is the shaded red colored area.

Student: (Refer Time: 05:52).

Pressure at the free surface is the reference pressure say if the water was not there at the bottom just consider this example let us say that this is a surface. Now there is no water say it is surrounded by air from all sides if it is in equilibrium; that means, air pressure is cancelling the effect from all the sides together the rate effect is that the sum total force is 0. So, whatever force is acting on the surface is because of the difference from the atmospheric pressure. So, whenever you are calculating the resultant force on a surface remember what that you are implicitly dealing with the gauge pressure not the absolute pressure because the atmospheric pressure if atmosphere was existing otherwise it would have kept it in equilibrium.

Now, you are having some pressure over and above that why atmospheric pressure any other pressure if there is a uniform pressure acting on a close surface we will see that later on that if you have uniform pressure acting on a close surface then the resultant force of that is 0, that may be proved by a very simple mathematical consideration that you have a distributed force which is always normal to the boundary then integral of that over a close boundary is 0 if the intensity of that pressure is uniform throughout. So, if you have if you want to find out what is the resultant force you may eliminate that common part and consider only that part which is over and above that that is why we are considering only the water effect?

Now, you can clearly see that what does integral y d A represent?

Student: Movement about x axis.

Movement.

Student: About x axis.

About x axis movement of what the movement of area.

Student: Area.

So, to say, the first movement of area and first movement of area gives; what gives the centroid of area. So, if you recall the formula for centroid say y coordinate of the centroid of the area that is integral of y d A divided by A, we will use the formula here and we can straight away write this as rho g A y c sin theta let us say that the centroid is somewhere here. So, what we are talking about we have some distance y c and this height which we may give a just a name say h c this is y c sin theta which is vertical depth of the centroid of the plain surface from the free surface. So, we can just write this as rho g A h c.

Now, this gives the resultant force, but since it is the distributed force we also need to find out what is the point through which the resultant of the distributed force passes. So, the point of application of the distributed force to find out that let us say that there is a point p, we give it a name p and say that p is the point over through which the resultant of this distributed force passes. So, what we can do to find out what is the location of p location of by location of p we mean the y coordinate of the point p.

So, our objective is to find out what is the y coordinate of the point p through which the resultant of the distributed force due to pressure passes for that we will just use the very simple principle which we have learnt in basic statics that, if you consider an axis with respect to which you take the movement of forces then the movement of the resultant force with respect to that axis is nothing but the summation of the movements of the individual components of that forces with respect to the same axis this is known as Varignon theorem and we will try to use that for finding out the location of y p.

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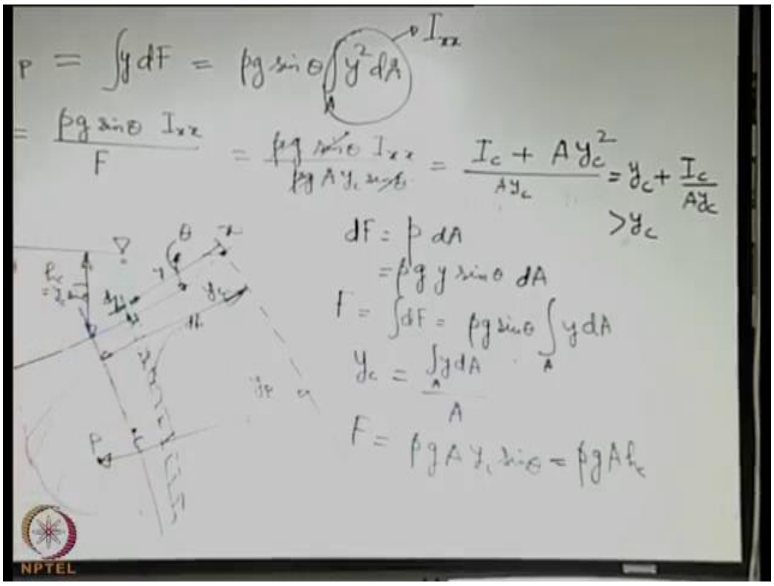

So, if F is the resultant force and the moment that we are trying to take this movement with respect to the x axis, then F into y p gives the magnitude of the movement of the force F, this is same as the sum of the movements of the individual component. So, individual component is like you have a d F that is an individual component. So, what is the movement of d F with respect to x? So, that is just d F into y local y. So, the total movement is integral of y d F. So, rho g sin theta integral of y square d A. So, these are second movement of area sometimes also loosely called as movement of area movement of inertia just by virtue of a similarity with mass movement of inertia this is not really a fundamentally movement of inertia it is better to call a second movement of area.

So, we can write this as the second movement of area with respect to the x axis. So, I xx, so we can write what is y p rho g sin theta I xx divided by F what is F? F is low g into a into y c sin theta. So, we cancel the common terms I xx we present the second moment of area with respect to some arbitrary axis it is more convenient to translate that to an axis which is parallel to x, but passing through the centroid and because centroid is a reference point with respect to a particular surface and to do that we may use the parallel axis theorem to translate it to see. So, if we consider an axis which passes through c and parallel to x with respect to that axis we can write that I xx is nothing, but I c where by c we mean this axis which is passing through c and parallel to x that axis is totally visible from the view parallel to the surface.

So, this is I c plus A into y c square that divided by A y c. So, from these we get a very important expression that y p is y c plus I c divided by A y c this point p is given a special name in consideration of fluid statics and that name is center of pressure. So, center of pressure is the point through which the resultant of distributed force due to pressure passes that is known as center of pressure. We can clearly see from this expression that y p is greater than y c because y p equal to y c plus a positive term; that means, the center of pressure in terms of depth lies below the centroid right and that is a very important observation.

So, the 2 things that we learnt from these simple exercise one is to find out what is a resultant force on a plain surface which is immersed in a fluid due to the pressure distribution and where is the point through which this resultant force acts. We will consider a simple example to begin with to demonstrate that how we may calculate this.

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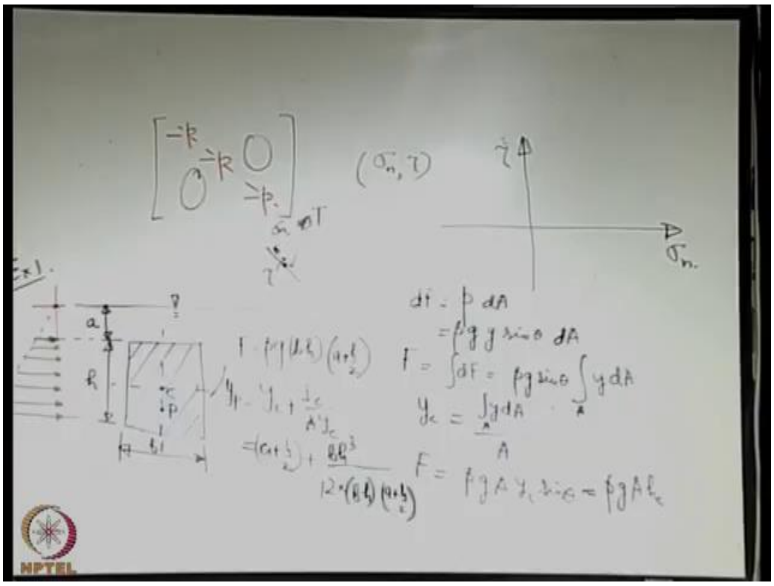

Let us say that you have a surface which in its sectional view is like this and this is the vertical surface immersed in a fluid. So, this is the fluid and the subject is a or like this object is a rectangular section with the dimensions b and h. What is the resultant force due to pressure acting on this?

Let us give a dimension of this say. So, this is example one the question is what is the resultant force on this shaded surface because of pressure. So, take an example like this. So, this is like the surface water or fluid some other fluid is acting on it from one surface and you are interested to find out what is the resultant force because of pressure distribution on this. So, this is a special case of the inclined situation. So, the inclined situation was like this now we had made it vertical. So, it is also an inclination with theta equal to ninety degree. So, for such a surface now if you want to find out what is the resultant force that acts on this surface, what is that? You look at the formula this one rho g A h c. So, what is A? A is b into h what is h c. So, c is the centroid of this area.

So, if it is a homogenous area it is a plus h by 2 what is the location of center of pressure y p is equal to y c plus first let us write the formula and then we will substitute the value what is y c here y and h are the same because it is just a vertical one. So, a plus h by 2 is y c plus I c. So, you have to now figure out first that it is second movement of area with respect to this vertical axis or this horizontal axis. So, try to recall that when this was the inclined plate the second movement of area was taken with respect to this axis right. So, when it is vertical you have a second movement of area with respect to these axis and that is translated to the centroidal axis so; that means, the correct axis should be this one. Just try to visualize it will be it is trivial, but it is very very important because based on the orientation you have to figure out that with respect to which axis you are having to calculate the second movement of area because centroidal axis is not something which is unique you have different axis passing through the centroid.

So, I c will be based on this axis what will be that? B h cube by 12. So, this is the expression, if you want to find out or if you just make a sketch of how this force is distributed let us try to make a sketch of the pressure distribution. To make a sketch of the pressure distribution what we note, the pressure varies with the depth. So, here the pressure is 0 which is the reference pressure not 0 in an absolute sense, but relative to atmosphere and then it will linearly increase with the depth. So, at this height this will be the pressure at the bottom height this will be the pressure and it is a distributed force like this which varies linearly with the height and you can clearly see that the area under this loading diagram will eventually give you what is the force these kinds of examples you have already gone through in basic engineering mechanics and you can verify it for the case of fluid at rest very very similar.

Now, what is the state of stress in which fluid at different depths they are the fluid elements at different depths they are subjected to. To consider that we will refer back to the stress tensor which we introduced earlier when we were discussing with the traction vector I am trying to relate the traction vector with the stress tensor. So, whenever you have a fluid element at rest let us say that we are interested to write the 6 independent components of the stress tensor. So, now, you tell. So, we have the diagonal elements and off diagonal elements if you recall the diagonal elements represent normal components of stress and the off diagonal elements represent the shear components of stress.

So, what will be the off diagonal components? They will be 0 because it is fluid at rest. So, it is not subjected to shear because with shear fluid will deform. So, these are all 0es when you come to the diagonal element you have only the state of stress dictated by the normal component which is just pressure and that acts equally from all directions. So, the all the 3 components will be minus p minus p minus p tau 11, tau 22, tau 33 and the reason of putting minus is obvious the positive sign convention of normal stress is tensile in nature where pressure by nature is always compressive.

Now, let us say that we have the task of drawing a mode circle of distribution of state of stress if you recall what is the mode circle? So, if you consider that there is an elemental area which has an inclination say theta with respect to some reference that has a resultant force and that resultant force is given by the traction vector components per unit area you can decompose it let us say that traction vector component is like this you can decompose it into 2 parts - one is a tangential component another is a normal component let us say we call it sigma n and let us say we call it tau.

So, depending on how you orient the area you will get different combinations of sigma n and tau if you draw the locus of that then that is what constitutes the mode circle right. So, mode circle gives us a visual appeal or a visual feel of what is the state of stress of different locations or at different locations based on the choice of different choice of orientation of the area or may be at the same location with different choice of orientation of the area. So, when we are drawing up one particular mode circle we are concentrating on one particular point, but changing the orientation of the area to get the feel of the normal and tangential components of the forces on that.

So, we are having 2 axis, one is sigma n another is tau our locus is or our objective is to find the locus of sigma n versus tau or tau versus sigma n. So, how will the mode circle look like for such a state of stress? This state of stress is a very unique one and this is known as hydro static state of stress the reason is obvious it is represents a hydro static physical situation. So, how will the mode circle look like? See at a particular point you have the normal component of stress that is because of pressure and it acts equally from all directions; that means, if you change the orientation of the plain sigma n will not change sigma n will be unique and what will be tau? Tau will be 0; that means, no matter whatever plain you choose sigma n is equal to minus p and tau is equal to 0.

So, the locus of all states of stress converts to a single point with coordinate minus p comma 0. So, the mode circle becomes a point say minus p comma 0. So, this is just a point not a circle I have encircled it, but it is just like to show that it is a point. So, this is a very important interesting limiting case when a mode circle shrinks to a point signifying that there is no change in state of stress with change in orientation.

Next we will consider another example on force on a plain surface.

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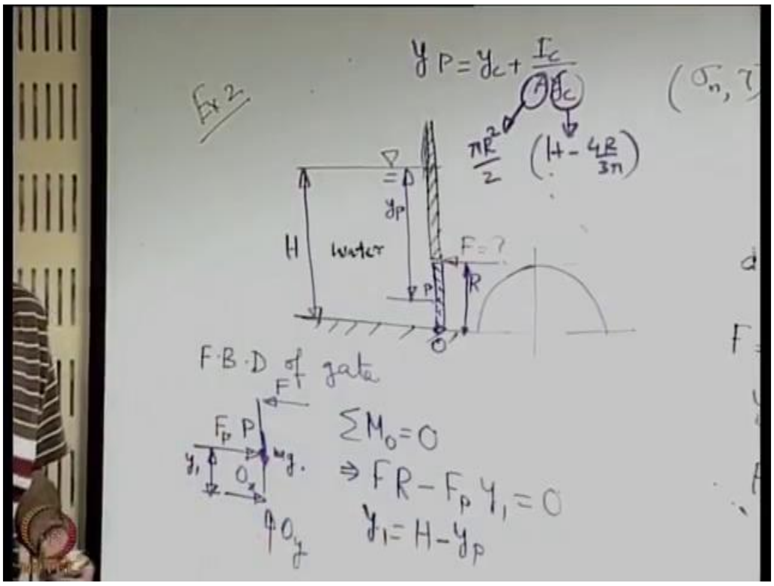

Example 2: we will make this example a bit more involved than the previous one. Let us say that there is again a free surface there is some fix structure there is a base and there is a gate like this. So, it is something like a say a Swiss gate. So, on one side there is water and it is. So, that this water is being confined in a particular place and these gate has a tendency to move because there is a resultant force of water from the left side and there must be some mechanism some holding mechanism by which this gate is kept at rest at equilibrium. So, there is some force which is acting on the gate say the force is this F by

some support or whatever there is some force F which acts on this to keep it in equilibrium this gate may be of different shapes and let us take an example with the gate shape as a semi circular one like this. So, this is the section of this gate.

Student: (Refer Time: 27:32).

This is like I mean it is fixed with something. So, it is there is a there is a structure that goes with it. So, we are not going to concentrate. So, much on the upper one we are I mean it may be extending even beyond the free water surface or so on, but we are more interested with the gate. So, on this gate there is a force due to water and that force you may calculate by considering some dimensions one of the dimensions this is capital H, let us say that the radius of this semi circular gate is capital R and the density of the fluid off course is given and g this gate is hinged at this point say o.

So, this is a situation where if you want to find out what force F should keep it in equilibrium you must calculate what is the resultant force due to pressure acting on the gate this is a plain surface shape of the plain is a semi circle, but still it is a plain surface and you may use the formula for force on a plain surface. So, let us try to draw the free body diagram of this gate. So, our objective is that what should be this force to keep the gate in equilibrium. So, we draw the free body diagram of the gate you have a force F what other forces you have? You have.

Student: Hinged.

Hinge reactions say let us say O x and O y the 2 components these have its own weight. So, some m g and the force due to pressure distribution in water let us say F p which passes through the center of pressure. So, for equilibrium the resultant movement of all forces with respect to O should be 0 the reason of choice of O as moment center is obvious it eliminates all unknowns except the F that we are interested to find out. So, what it will give? It will give capital F into r minus F p into say this distance y one equal to 0 and how can you calculate y 1? Y 1 is capital H minus y p where y p is a location of the center of pressure from the top surface right, fundamentally you can calculate y p by the formula that is y p equal to y c plus I c by A y c. So, what is y c? Y c is the y co ordinate of the centroid of this semi circular area.

So, this part the location of the centroid from the bottom is 4 r 3 pi. So, y c should be H minus 4 r by 3 pi to that extent find A, what is A?

Student: pi r (Refer Time: 31:56).

Pi r square by 2 that is also quite clear I c something which we always forget, if you ask me to recall I mean I will be in a great tension I have really forgotten what is I c? I mean second movement of area with respect to an axis of a semi circular thing which passes through its centroid of course, you may derive it, but we will see that this derivation is not necessary because we will try to avoid this root of this formula best determination of this and we will just do it from the fundamental method of integration of which you find out the resultant of a distributed force the resultant movement of distributed force and. So, on and the entire reason is that there are certain simple areas for which we may remember the expressions for the second movement of area with respect to the centroidal axis quite easily, but it is not. So, convenient for many complicated areas this is not complicated as such, but even for that we should not tax our brain by remembering that I mean that it is not very special information that we should remember.

So, what we will do is in the next class we will try to see we will keep this problem in mind, we will see the alternative way by which we will be solving this problem you can of course, solve this problem by substituting the value of I c here expression and this expression is given in the appendix of the textbooks in statics. So, you can find out the expression or you may even derive it if you want and just substitute it to get what it y p and from that you can get f, but we will see that whenever possible and whenever convenient it may also be alright if we just find it out by simple integration of the distributed forces. So, that we will do in our next class let us stop here.

Thank you.