Introduction to Fluid Mechanics

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Lecture – 14
Governing equation of fluid statics

With this background now we will move into more general considerations for equilibrium of fluid elements. Here whenever we were discussing the surface tension force we were assuming that the pressure is being distributed in a particular way and we were intuitively using some concept of high school physics that if you have a depth of h then what should be the variation in pressure because of that depth of h.

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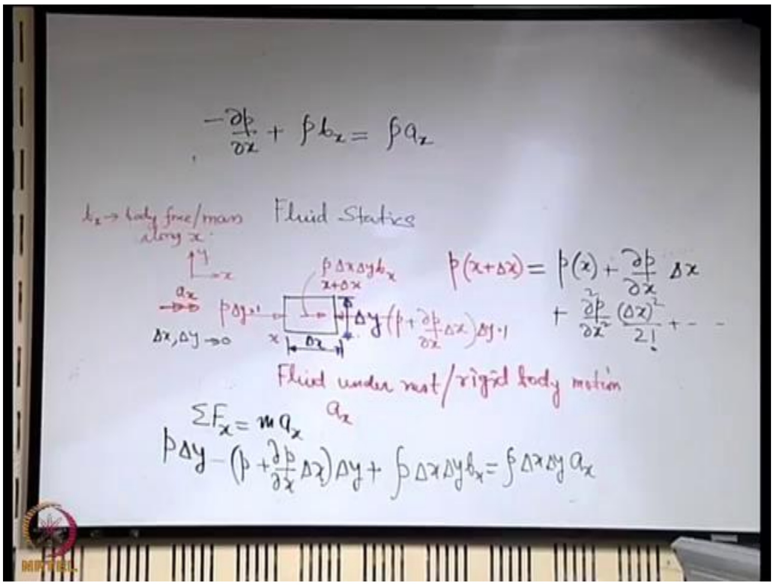

Now, we will look into it more formally. So, we will go into the understanding of fluid statics. We start with an example or a fluid element which is in static equilibrium and we take an example of a 2 dimensional fluid element just for simplicity, we have taken such examples earlier what these examples signify that you have a uniform width in the other direction perpendicular to the plane of the board.

Let us say that delta x and delta y are the dimensions of this fluid element remember that this fluid element is at rest when this fluid element is at rest; that means, we are sure that it is non deforming because deforming fluid element is definitely not a fluid element at rest and when it is non deforming, we are clear that there is no shear which is acting on it; that means, there is only normal force which is acting on all the phases of the fluid element. So, we can designate the state of stress on each phase of the fluid element by pressure. So, let us try to do that.

Let us say that we are only writing forces along the x direction just for simplicity similar equations will be valid for the y direction. When you have say the left phase under consideration just like this, let us say that p is the pressure on the left phase and the force corresponding to that is p into delta y into say one which is the width of the fluid element. When you come to the opposite phase we are bothered about these phases right now because we are only identifying the forces acting along x because we will write equation of equilibrium along x not that forces are not there on the other phases. So, this is not the complete free body diagram it only just shows the forces along x direction.

So, if the pressure here is p what should be the pressure here that is under question, will it be p, will it be something different from p. In general we are not really committed to what are the other forces which are acting on it there may be any other body force which is acting on it along x and y. So, if the pressure here is p the question is will the pressure here be p or something else in general special case of course, it may also be p here, but we are talking about a more general consideration mathematically speaking what question we are trying to answer we have a function here say p, we want to find out the value of the function at a different location say this location is x we are interested to now find out the value of the function at x plus delta x in terms of what is the value of function at x the function here is p. That means, we want to see that hat is p at x plus delta x in terms of what is p at x and that we can easily do by using a Taylor series expansion. So, that we will do we will write this as this as p at x plus the first order partial derivative of p with respect to x into delta x plus and so on.

There are infinite number of terms, but as you take delta x very small may be you may neglect the higher order terms in comparison to the dominating term and the gradient keeping that in mind that we are treating with cases where delta x delta y are very small. So, delta x delta y all tending to 0, this will become from the expression that we have here what we can write this will be p that will be the pressure here we will keep this in mind. So, later whenever we encounter any function we will use the Taylor series expansion to identify what is the change that is taking place across different phases of fluid elements because that we will have to do very commonly many of our analysis. So, this multiplied by the area on which it is acting is the force due to pressure on this phase.

Let us say that there is a body force which is also acting on the fluid element. So, the body force let us say that b x is the body force per unit mass acting along x, so b x is body force per unit mass along x. So, what will be the total body force which is acting on this along x fist you have to find out what is the mass of the fluid element what is that it is the density times the volume of the element that is delta x into delta y. So, this is the mass of the fluid element that times the body force per unit mass gives the total body force along x. So, these are the forces which are acting on the fluid element.

Now, let us try to answer another question. Are this still the force only forces which are acting if the fluid element is under rigid body motion that is the fluid element is moving like a rigid body there is no internal deformation, but as a whole it is just like a solid that is getting displaced that may be displaced linearly or angularly, but it is having a motion, but the motion is a rigid body motion if that is the case then are these the only forces see what forces we have identified surface forces we have identified body forces we have identified. So, the question was down to that are these the only surface forces even if the fluid is under rigid body motion, the answer is what? See what is the difference between a fluid element at rest and fluid element are under rigid body motion the only difference that when it is under rigid body motion it might be having like a velocity acceleration and so on, but in terms of the surface forces which are acting if the fluid element is non deforming then there is no shear component of force. So, for a non deforming fluid element there is no difference between the surface forces which are presented in this diagram and the surface forces which are there when it is say moving with acceleration.

So, this type of forcing description is equally valid if the fluid is under rigid body motion. So, we the identify this situation not just at fluid under rest, but also rigid body motion we will see such examples were the rigid body motion of the fluid will be very interesting like you may have a rotation of a fluid element like a rigid body and we will see that what kind of situation it creates. So, broadly this is also studied under the category of fluid statics not because it is a static condition, but in terms of the characteristic of the fluid the deformable nature is not highlighted here and that is why we may use broadly similar concepts and we will learn these concepts together under the same umbrella because they are very very related in one case it will it may have an acceleration in other case it may not be otherwise it is very very similar.

So, let us say that it is under rigid body motion and therefore, let us say it has some acceleration along x; say a x is the acceleration which is there along x. So, we can write the Newton's second law of motion for the fluid element and when you do that what do we get the resultant force which is acting along x is equal to the mass of the fluid element time x times acceleration along x. So, it is p into delta y minus the other term which is there on the opposite phase plus rho into delta x into delta y into b x is equal to rho into delta x into delta y that is a mass times the acceleration along x delta x into delta y will get cancelled from both sides these are small, but not equal to 0 these are tending to 0. So, you can cancel from both sides.

At the end what final expression you will get. So, this will be the expression which relates the pressure radiant with the body force that is acting and if there is any acceleration that acts on the; I mean that the fluid element is having similar expressions are valid for the motion along y. So, we are not repeating it again with this kind of a general idea. So, this is a very general expression this general expression just considers that there is a body force and the fluid is having some acceleration in a particular directions subjected to the body force, but it is a non deformable fluid element.

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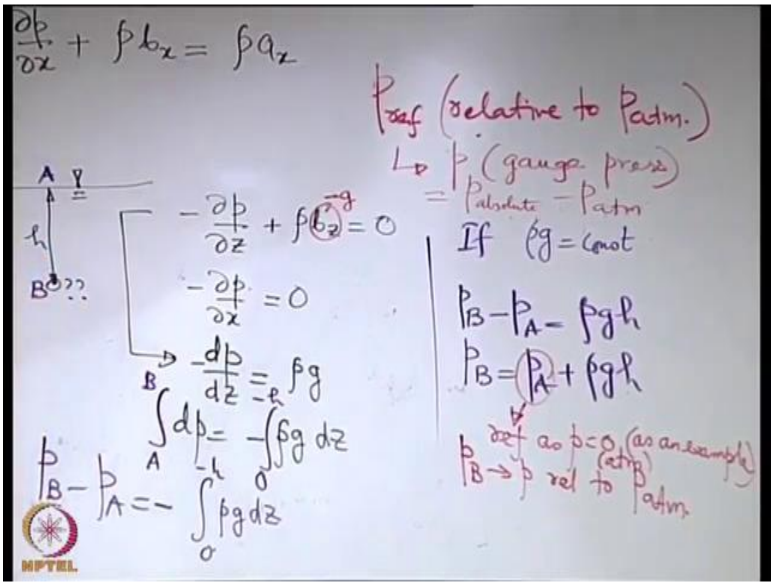

With that understanding we will try to identify that what is the variation of pressure just due to the effect of gravity as a body force in a fluid element at rest we consider that there is a free surface of a fluid this is a symbol in fluid mechanics that will be using to designate a free surface these are triangle with 2 horizontal lines very short horizontal dashes or lines at the bottom this is a kind of a technical representation of a free surface. We consider that we are interested about some depth usually the direction in which depth varies is typically taken as z direction this is just a common notation in most of the books that the vertical direction across which the gravity is acting of course, the opposite to the action of gravity because gravity u will be vertically downwards and opposite to that is considered as a z axis just as a common notation.

So, let us try to write this kind of equation for this fluid which is at rest it is of a substantial depth. So, we are interested to find out what is the pressure at this point which is at a depth h from the free surface this there is no acceleration of this fluid it is under absolute rest. So, a x or here a z term will be 0. So, you will have minus this one when you have the z direction you also have a horizontal direction like x and for x you can write again similar expression. So, if you write it for x what is the body force which is acting along x there is no body force which is acting along x because only body force to which this is subjected is the gravity. So, there is no body force along x and there is no acceleration that is it is having along x.

So, the second expression is even more simple, but it gives as a very important insight what is that? That if you are not having a body force along a particular direction and the fluid is under rest then pressure does not vary with in the same fluid along that direction; that means, for a horizontal along a horizontal line you are not having any pressure variation in a continuous fluid system and this is one of the basic principles that we use for measurement of pressure differentials as you have seen in examples of manometers earlier. So, this is something which is of great consequence, but it is a obvious conclusion what we get from the first equation. So, we let us try to replace the b z, what is b z?

Student: g.

See this is the z direction and this is acting in the opposite direction. So, this;

Student: minus g.

Minus g, so you have from this and since pressure is not varying with x. So, you can write this as d p d z in place of partial derivative because it is now just a function of a single variable. So, you can write this as minus d p d z is equal to row g. So, d p is minus rho z d z. So, if you want to find out what is the pressure difference between say 2 points A and B, you have to integrate it with respect to the z variation from say A to B when it is a say we take our reference such that the origin is located here; that means, at a z is equal to 0 at B it is minus h. So, you can write p B minus p A is equal to minus integral of; now it is important to see that what is the length scale that we are considering over which this variation is taking place if this h is quite large there may be a significant variation in density over it just like consider the atmosphere which is above the surface of the earth as you go more and more above the surface of the earth you expect the density to change because the temperature changes and so on and therefore, the density in many cases may not be treated as a constant. So, if it is treated as a constant then it can only come out of the integral.

Similarly, you also are probably working on length scale over which g is not changing if you are taking a large height like if for people who are dealing with atmospheric sciences for them the length scales are large lengths scale over which you may have even a change in acceleration due to gravity, but if you consider that such a situation is not there just for simplicity. So, if rho into g is a constant that is the best way to say because a very tough mathematician will say that I do not care whether rho is varying whether g is varying I am happy in bringing this term out of the integrals.

So, long as rho into g is a constant. So, may be mathematicians way of looking into it is that rho varies in a particular way g varies in a particular way, but those variation effects gets cancelled out somehow. So, that rho into g is a constant may be very hypothetical, but for our case to bring it out of the integral the product being a constant that will serve our purpose. So, then what you have then you have p b minus p a is equal to rho g h; that means, p b is equal to p a plus rho g h which is your very well known expression.

Now, important thing is that see we are not expressing the pressure at b just in an absolute sense we are expressing it relative to the pressure at a many times this pressure at a say this is atmosphere. So, that may be taken as a reference. So, if this is taken as a reference as p equal to 0 as an example. So, whatever is the atmospheric pressure say we call it 0; that means, any other pressure we are expressing relating to the atmospheric pressure. So, then in that case p b is the pressure relative to p atmosphere if p equal to 0 is the atmosphere it is not definitely equal to 0, but if you if you have p equal to 0 we I mean that is that is just the choice of your reference. So, that you expressed any other pressure in terms of that as a reference.

So, any pressure which is expressed in term of that atmospheric pressure that is a relative way of expressing the pressure it is not that always you have to express relative to atmospheric pressure, but atmospheric pressure being a well known standard under a given temperature. So, reference with respect to atmospheric pressure is something which is a very standard reference that we many times use. So, reference pressure relative to atmosphere. So, we are talking about a reference where the reference is the atmosphere then what ever pressure is there at any other point we call that as a gauge pressure. So, this is just a terminology. So, gauge pressure means that any pressure relative to atmospheric pressure so; that means, it is nothing but p absolute minus p atmosphere just the difference between the absolute pressure and atmospheric pressure that is as good as taking the atmospheric pressure as 0 reference and mentioning the pressure relative to that.

So, this is a very simple exercise, but from this we learn something what we learn something. So, whenever we have an expression we should keep in mind what are the assumptions under which it is valid. So, we will develop the habit of not using any formula like a magic formula this is very very important formula based education is very bad. So, whenever you have a formula and you want to use, it try to be assured that it is valid for the condition in which it you are applying if not exactly, but at least approximately. So, when you are using pressure equal to h rho g, what are the assumptions under which it is valid?

Student: (Refer Time: 22:13).

So; obviously, rho into g is a constant and there is no other body force which is acting on it and fluid is at rest that is these are the assumptions that are there with such a simple expression. Now with these kind of a concept one may utilize device one may utilize this type of concept in making devices for measuring atmospheric pressures just like you have barometers.

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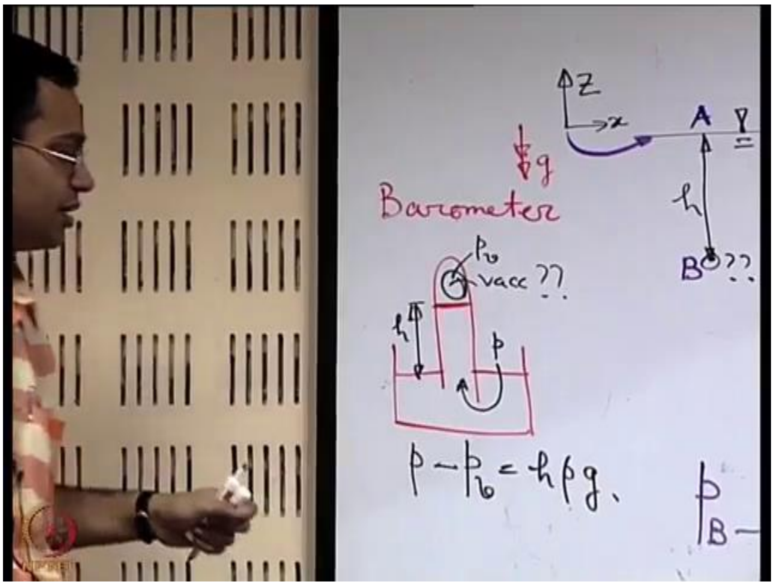

Whenever we will be learning a concept we will try to give examples of measurement devices which try to utilize those concepts as all of you know a barometer may be utilized to measure the atmospheric pressure. So, how it is there you have say inverted tube and this inverted tube is say put in a bath of some fluid say mercury and let us say that it is they are up to this much height. Now this much of height is there; there are various which are acting on these. So, one is you have we are of course, neglecting the local surface tension effect and the capillary formation you must keep in mind that as this radius becomes smaller and smaller the effect of the curvature may be more and more important because surface tension effect will be more and more important and there may be significant errors in reading because of that.

Now, if we just neglect that effect for the time being then you have atmospheric pressure acting from this side if you assume that there is a vacuum here there is a big question mark whether there will be vacuum or not, but let us for the sake of simplicity assume that there is a vacuum then what ever pressure is there which is acting from this side that is balanced by the height of the liquid column which is there on the top. So, from that you can get estimation for what is the pressure here let us say that p is the atmospheric pressure. So, p into the surface area on which it is acting is the force that is being sustained by the weight of the liquid column. So, that is nothing but what?

Student: h rho g into area.

So, it is like h rho g that into the area and area gets cancelled from both sides. So, you get this p if it is vacuum as the h rho g, but if it is not a vacuum let us say that there is some pressure here which is the vapour pressure of the fluid which is occupying this and it is common that such vapour pressure will be there why because if it is a saturated liquid it is likely to have its own vapour on the top of that and that that will always exerts some pressure. So, it is never a vacuum in an ideal sense. So, we can say that p minus p vapour is actually what is being balanced by this weight. So, that is the h rho g.

So, if there is the vapour pressure you cannot just use h rho g for the estimation of the atmospheric pressure, but you have to make a correction because of the presence of the vapour and that is a function of the temperature because vapour pressure varies with temperature very commonly the mercury is one of the fluids that is used for this purpose and why mercury is being used; obviously, because it is quiet dense it will not occupy a very large height for representing the atmospheric pressure if you use any other fluid it may occupy a great height. So, it may be an unmanageable device unmanageably long device also the vapour pressure of mercury is quite small in most of the temperature ranges and therefore, this correction is not that severe these 2 are the important reasons there are many other reasons which are always into the picture when you when you select a fluid for measurement of a pressure like in a barometer.

A barometer is a very interesting device we have discussed something quite seriously, but I would just like to I mean share a kind of a story associated with barometer a very well known story and I am sure that many of you have heard about it long time back in a high school examination there was a question it was a physics examination and the question was that how can you measure the height of a building using a barometer now although all of you or most of you have heard about this story we will try to get a moral out of that story and we will try to keep that in mind whenever we are going to learn something.

So, what the student replied in the answer the answer was that you just have a thread you connect the barometer with the thread go to the roof of the house just drop that barometer with the thread and then like the total height that the total height of the thread that is required to bring the barometer to the ground.

Student: (Refer Time: 28:08).

Maybe plus whatever is the portion of the barometer will give the height of the building.

Student: (Refer Time: 28:12).

Now, as in most of the examination systems this student was given a big 0.

Student: (Refer Time: 28:17).

And the expectation was it was justified that why he was given a big 0 because it was expected that the answer should reveal some basic concept in physics it was a physics examination, but these does not reveal any basic concept in physics. So, he was given a 0, but then the student went for an arbitration, it was a democratic system even in that time.

Student: (Refer Time: 28:41).

So, the student said that no like I mean let my answer be reviewed. So, there was a panel the panel said that may be you were not aware that what kind of expectation that we are having from your answer. So, we give you another chance. So, you think about a solution for this question which reflects your understanding in physics and we will evaluate you from that. So, student said that let me be given some time. So, he was given 5 minutes to think about that. So, he was thinking for 5 minutes and when he was thinking for 5 minutes and he was still not coming up with an answer then like the evaluators were very happy that he might be failed again. So, they said that no, you could not come up with an answer.

So, we are sorry, then he said that no actually there are lots of answers have come to my mind. So, I am not sure that what should I say and that is why I was not giving a response and then he was asking permission that I mean am I going to be allowed to speak of that remaining answers then they said that fine I mean whatever you have thought you just tell then he said that I will what I will do is I will drop the barometer from the top.

Student: (Refer Time: 30:06).

Of the building and I will measure the time that it takes to reach the ground.

Student: (Refer Time: 30:08).

And h equal to half g t square, so from the time I can measure the height it reflects some understanding of physics, but it is a bit distractive because the barometer.

Student: (Refer Time: 30:22).

Like it may be damaged and like and so on then you said that no then if you want a different answer may be what I will do? I will try to make a pendulum out of the barometer.

Student: (Refer Time: 30:33).

Swing it one in the bottom ground level another on the top level and we will measure the time period.

Student: (Refer Time: 30:41).

And these time period difference will give the difference in g between the 2 heights.

Student: (Refer Time: 30:46).

And since g is a function of height.

Student: (Refer Time: 30:48).

It will tell us that what is the height difference between the 2 and I mean still the examiners were not happy and they, but still they were ready to pass him because these were like a reflecting some of the basic concepts in physics then he said that even if I am given different opportunity what I will do is I will climb across the stair case and in the side of the stair case you can just put the barometer one after the other till.

Student: (Refer Time: 31:20).

You traverse the entire height, the number of times you put it you multiply with the length it is a basic length measurement principle. So, from that you get that and he said that there are many answers which are coming to my mind, but given me a entire freedom, what I will do? I will not really put this much of effort; I will go to the house master and tell that see now I have a beautiful new barometer for you I am giving it to you please tell me what is the height of the house.

Student: (Refer Time: 31:50).

And the house master will; obviously, tell that because it is like a gift free gift that the housemaster is having at the end he said that perhaps you are not expecting all these answers from me you are expecting to me to give a very structured answer that the barometer measures that atmospheric pressure. So, from the difference in the level of atmospheric pressure in the 2 heights we can easily say that what is the difference in height between the 2 and since this is the most structured answer I hope that you will be happy with that and then of course, the evaluators passed him and he was quite successful and the name of the student is Niels Bohr, I mean who later on I mean like discovered many many beautiful phenomena in physics.

So, the whole idea is that I would always encourage you may be I mean none of our self like Niels Bohr, I mean we are not born we those special abilities, but at least whenever you are having an opportunity to think of solutions do not always go for a structured solution try to think about different possibilities whenever you are thinking about designing a measurement principle whenever you are thinking about a solving a particular problem just try to think about various possibilities some of the possibilities may not be very very encouraging very very welcome. But at least these possibilities will give us some kind of clue that what could be alternatives some of the alternatives may be discarded they may not be very smart, but they will at least give us a scope of thinking laterally and that is how one may improve in science technology and research and I mean such a simple example like barometer I mean one is always reminded of that kind of a story and I feel that it is not something too bad to share with you.

So, what we will do is we will not go further ahead today we will stop here in the next class we will just make a plan of what we will do in the next class now we have found out a particular way in which you have a estimation of variation in pressure because of a body force which is acting and for fluid element which may be at rest or subjected to acceleration. So, we will utilize this principle to calculate 2 important things one is if there is a plane surface which is immersed in a fluid what is the total force which is there on the plane surface because of the pressure distribution now you have realized that pressure is like a distributed force because it varies with the depth. So, at different depths you have different pressures.

Therefore it will be like a simple statics problem where you have a distributed force on surface to find out what is the total force which is acting if you have a curved surface we will see that there are technique may be a bit different, but broadly we can utilize some of the concepts of pressure distribution on a plane surface even to calculate force on curved surface.

So, in the next class we will we will see what are the forces on plane and curved surfaces which are there in a fluid at rest and then to see that what are the consequences and will work out some problems related to that. So, we stop here today.

Thank you.