**Introduction to Fluid Mechanics**
**Prof. Suman Chakraborty**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Kharagpur**

**Lecture – 11**
**Problems and Solutions**

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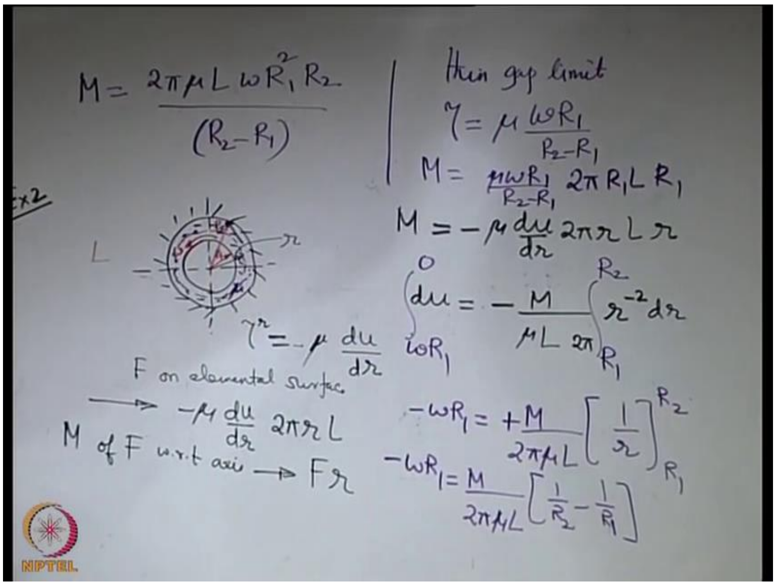


We continue with our discussions on viscosity. So, let us consider second example, let us say we have concentric cylinders and the specialty of this system is that may be one of the cylinders is having a relative motion with respect to the other. So, let us take an example where the outer cylinder is stationary outer one is stationary and the inner one is rotating with the particular angular speed omega.

There is a fluid that is present in the gap between n the 2 cylinders this kind of visual example that we have seen in our previous lecture. This type of example is important in many ways I will give you 2 examples for application of these type of situation. One is if you consider an industrial application in industry, there are shafts cylindrical shafts which transmit power from say one point to the other. So, think about the inner cylinder like a shaft. So, that shaft is rotating and it is rotating, but it has to be housed in a certain position at the same time if you constrain it with the direct metal to metal contact that will give rise to a lot of wear and tear.

So, there is a lubricating material which is typically like an oil which separates that from a outer housing which sometimes is called as a bearing and the whole idea is to create a lubricating layer in between these 2 which prevents wear and tear because of the metal to metal contact. So, the shaft bearing type of arrangement in industries is very common and that is where you will find lot of application of this kind even if you are looking from a more fundamental consideration of fluid mechanics there is often a necessity to measure the viscosity of fluids and this kind of arrangement may be utilized to measure viscosity of fluids that is the case where it is known as a rotating type viscometer.

Viscometer for measuring the viscosity and rotating type because of it is particular nature of motion that you can easily appreciate.

We will try to see that what is the physical situation that is going behind this type of example when the inner cylinder starts rotating it will try to move the fluid with it because of no slip boundary condition the fluid immediately in contact with that will be rotating with the or will be moving with the same linear speed at different locations as you go radially outwards let us say the inner cylinder has a radius of R 1 and the outer has a radius of R 2. So, as you go from R equal to R 1 to R equal to R 2 what you will find you will find that the velocity in the fluid goes down and the velocity is 0 at the outer radius R equal to R 2 that is also by no slip boundary condition.

Now, because of the presence of the fluid the cylinder which is rotating it is not rotating in an unhindered manner it is being subjected to some resistance it has to overcome that resistance and maintain it is motion. So, it requires an external power to be imposed or so to say a torque to be there which is continuously rotating it overcoming the viscous resistance and let us say that we are interested to find out what is that torque or may be power necessary to make it rotate with a uniform angular speed that is the objective of analyzing this and therefore, if we apply that particular torque and if we see that it is rotating with the uniform angular speed that may be measured by something like a tachometer then it is possible to relate these 2 in terms of the viscosity of the fluid.

So, everything other everything else being measured from that expression we should be able to evaluate what is the viscosity of the fluid that is the basic principle by which one may measure or evaluate what is the viscosity of the fluid that is there in between typically these gap is very narrow and we will see what is the consequence of that narrowness. Now let us say that we are interested about a section of the fluid, let us at a radius at some intermediate radius say r which is a local variable small r, let us say that the length of the cylinder or both the cylinders is L which is perpendicular to the plane of the board and mu is the viscosity of the fluid which is occupying the annular space. So, when we consider at a location R, we have an imaginary surface of fluid which is having a surface area of what 2 pi R into L that surface of fluid is a surface on which there is relative resistance or there is a relative motion between the fluid layers one is towards the inner and another is towards the outer, whatever is towards the inner tends to move faster whatever is towards the outer tends to move slower.

So, that is a location where there is a shear stress that is present which is related to the rate of deformation. So, if we want to write what is the shear stress at the radial location r or we should; may be use a superscript because it is not really tau with r as a subscript, subscript meaning we have preserved for something else. So, if we write these then what would be its corresponding expression in terms of say Newton's law of viscosity? There is a mu, there is some sort of d u d y type of term. So, what is let us call it some d u; d u say now we are using a coordinate of R and if we had used a coordinate of y the only difference would have been that y is from the wall from the solid from the 0 velocity wall towards the inside towards the inner one. So, that y is just oppositely directed to R. So, whatever is d u d y is just adjusted with a minus d u d r, there is no other difference because y direction is preserved for the direction which is from these 0 velocity to the fluid and this is the R direction is just opposite to that that is why this minus sign is there to adjust it.

And you may think also in a different way as you are increasing with radius, you are having a reduced velocity. So, this is negative if you want to make it positive you want to add just it with the negative sign that is just a matter of sign convention, but we have to be consistent with the sign convention whatever we have followed till now, we will preserve that. So, that is the shear stress, if it is a Newtonian fluid then what is the shear force which acts on this elemental surface? Let us say d F is the shear force on elemental surface. we have already identified what is the elemental surface that is the surface of the imaginary fluid with the dotted line as it is radial envelop. So, d F on that 1 is minus mu d u d r, we may just call it F, there is no necessity to call it d F because it is it is not like an elementary small volume that or area that we are talking about so minus mu d u d r into 2 pi r l. So, this force is a tangential force. So, this force is like typically you will have this type of force which is tangential to these elements. So, this force will have a moment with respect to the axis of the cylinder. So, what is the moment of F with respect to axis that is F into r, we are just writing it in a scalar form not bothering about the vector nature because the moment vector is perpendicular to the plane of the board that we can understand very easily.

So, this is something now you have to understand physically what is happening, there is a particular power that is impose a motor is driving this; that means, there is a torque that is being input to the system and the same torque is transmitted across different fluid layers otherwise it will not be able to rotate with the uniform velocity. So, what it means is that if you call this as say M then M is something which is a which is a sort of an input and it is balanced with the resistance movement that takes place at various sections. So, that you have a particular number a particular value associated with that and that is dictated by the input; input power of the motor. So, you have M is equal to minus mu d u d r into 2 pi r L into r.

So, now, you can separate the variables in the 2 sides. So, you can write d u equal to minus M by mu L 2 pi into r to the power minus 2 d r. I am very bad in algebra. So, whenever I am mistaken, please correct it. Now when you integrate this, you can get a sort of variation in u, remember one very important thing; this u is the velocity in the fluid. So, it has a variation from the inner to the outer, for the inner cylinder within the cylinder there is no variation in velocity because it is a rigid body. So, of course, linear velocity is varying, but angular velocity is the same. Now the outer cylinder also is stationary, but in between there is a difference in linear velocity because the fluid is deforming it is not a rigid body. So, at the inner radius that is at R 1, what is the velocity? Sorry R 1 on the right side we should write. So, R 1 equal to at the velocity at this is omega R 1 and at R 2 this is 0.

So, very quickly we can write that minus omega R 1 is equal to minus M by 2 pi mu L this is that minus sign will get absorbed, you can simplify this and write as say M equal to 2 pi mu L omega R 1 then another if you just simplify this another R 1 R 2 divided by R 2 minus R 1. If there is any mistake please let me know now. So, you can see that there is a difference between R 1 and R 2 and here you are actually having R 1 square R 2 in this expression, but if you neglect the variation of or you if you neglect the velocity profile variation from the inner to the outer and assume that the gap is very thin. So, that it is a linear profile.

Then what would be the difference in expression that you get? So, if it is a linear velocity profile that is taken that is taken that is the velocity is varying from omega into R 1 from inside to 0 outside in a linear manner, if the gap is thin then that is valid. So, for in a thin gap limit, in a thin gap limit, you have the tau equal to mu into omega into R 1 divided by R 2 minus R 1 omega R by h d v d y; if it is a linear velocity profile it is just the ratio of the change in that 2.

So, if this the tau then what is the moment? So, M is equal to mu omega R 1 by R 2 minus R 1 that should be multiplied with a 2 pi now because the gap is thin you can write 2 pi R 1 L 2 pi R 2 L or if you want to be a little bit more accurate may be R 1 plus R 2 by 2 L or whatever it will not make a lot of difference, let us write may be 2 pi R 1 L into R 1. So, you can clearly see that as the difference between R 1 and R 2 tends to 0 these 2 expressions lead to almost the same thing. So, if the gap is narrow then the second approximation will give you a very quick estimation of what is the situation and from these or the more involved expression, you can clearly see that if you now know, what is the power input to the shaft the powered input is this M times the omega?

So, if that is known; that means, M is known the dimensions will always be known. So, R 1 R 2; that means, R 1 R 2 minus R 1 L that will be known omega can be measured with a tachometer. So, that can give you what is mu from this expression. So, if you are having a careful experiment where you are having the proper estimate of the power input as well as what is the angular velocity at which this inner cylinder is rotating it will give you some good estimation of what is the viscosity of the fluid that is there inside provided it is Newtonian and that is how you may estimate the viscosity of an unknown fluid that is a fluid for which you do not know the viscosity.

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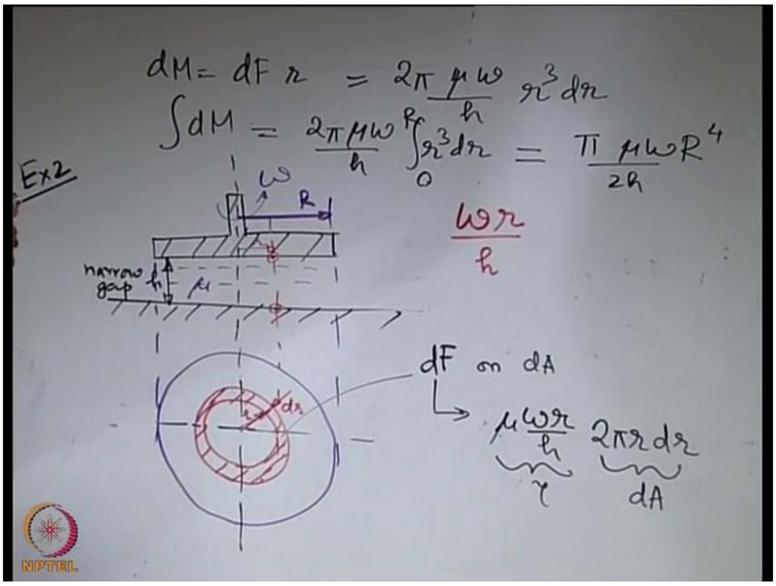


Let us consider a third example; we considered that there are 2 plates for example, 2 circular plates. So, these are the sectional views if you draw the other view this will be a circle. So, if you draw the other view of say the top plate it will be something of a circular nature the problem is whenever I want to draw a circle it becomes an ellipse whenever I want to draw an ellipse it becomes a circle. So, assume this a circle although looks more like an ellipse or may be not even an ellipse. So, this is the other view of the plate there is a fluid which is there in between and our objective again is to see that what is the torque or power required if we want to rotate this one with a given with the particular angular speed the bottom one is stationary, situation is quite similar to the previous one.

So, we should be able to work it out quite quickly. We have assume that this gap is narrow the viscosity of the fluid is mu and let us say radius of the plate is the top plate is R because the gap is narrow; obviously, it is expected that we may approximate it with a linear velocity profile from the bottom to the top, but a key factor here is that that linear velocity profile is now radially changing. So, if you consider say particular radial section like this, here you have 0 velocity here and what is the velocity that you will have here omega into local r. So, let us say small r is the local r. So, omega into local r will be the velocity therefore, the velocity gradient at section R will be omega r by h and this is because of linear velocity profile assumption from the bottom to the top.

So, if you take different radial sections these will be different. So, if this was a constant we could have easily calculated the shear stress by multiplying whatever constant it was with the total surface area of the plate that is being exposed to the fluid, but now these being a variable we must take it as summations of constants over small small elements and that is how or that is why we have to choose small elements and integrate over that elements.

So, we take a small element at a radius r of thickness d r. So, whenever we are solving any problems these are very common situations many times because of systematically practicing problems we are habituated in taking elements certain cases doing integration and. So, on, but many times we forget why we are doing it and it is very important to keep in mind that why we should do it. So, here since it is continuously wearing we are interested to obtain estimation for the shear stress or the shear force which is our objective and that shear force is locally wearing because the shear stress is locally wearing we should take a small element at least over which it is a constant.

So, with in this d r it does not vary significantly therefore, it may be treated like a constant over d r. So, if we consider this area we can multiply the local shear stress with that area to get the local shear force. So, what is that local shear force? So, let us say d F we call as local shear force on this d A, what is that? So, first you write the expression for shear stress mu into omega R by h that is tau times the area d A. So, what is that area 2 pi r d r; 2 pi r d r. So, with this as the shear elemental shear force this elemental shear force will have a moment with respect to the axis. So, what is that elemental moment? This into r, so that is d F into r, so that will be 2 pi mu omega by h r cube d r.

So, the total resistive moment should be the integral of this from 0 to r. So, that will become pi mu omega by 2 h r to the power 4 no, why it should be 2 pi r? No, no, no, it is distributed over the entire surface. So, it is like it is when you consider a radial location it is not a point it is like entirely distributed and that distributed force has a moment with respect to the axis. So, it is like over the entire element you can think it even more fundamentally do not thus consider the full 2 pi r, but consider a small angular element with between theta and theta plus d theta and between R and R plus d r and then if you integrate that from theta equal to 0 to 2 pi that 2 pi term has automatically being taken care of. So, you should not take it take care of it doubly by considering 2 pi here also.

So, this is a very simple expression, but it again tells that like there can be situations of variable velocity profiles and those may be taken care of in this way. So, what we will do we will post you some assignment or homework problems on viscosity may be very much related to these types of problems or may be slightly different and you will find that in the course website the homework problems and maybe we will give you a deadline in the next class that when to submit those problems.

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So, we cannot sum up with our studies on viscosity we will study more effectively viscosity later on in one of our related chapters that is equations of motion for viscous flows when we will learn viscosity effect more mathematically, but just now we can sum it up to see that here there is some highly viscous gel and this highly viscous gel is being stirred and you can see that when it is being stirred it tends to get broken and separated in parts. So, when it is doing that of course, it is a highly viscous gel and there is an important additional force that is coming into the picture which is making it to behave in that type of way and that force is nothing, but a surface tension force.