Introduction to Fluid Mechanics
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Lecture – 10
Problems and Solutions

Let us consider an example one.

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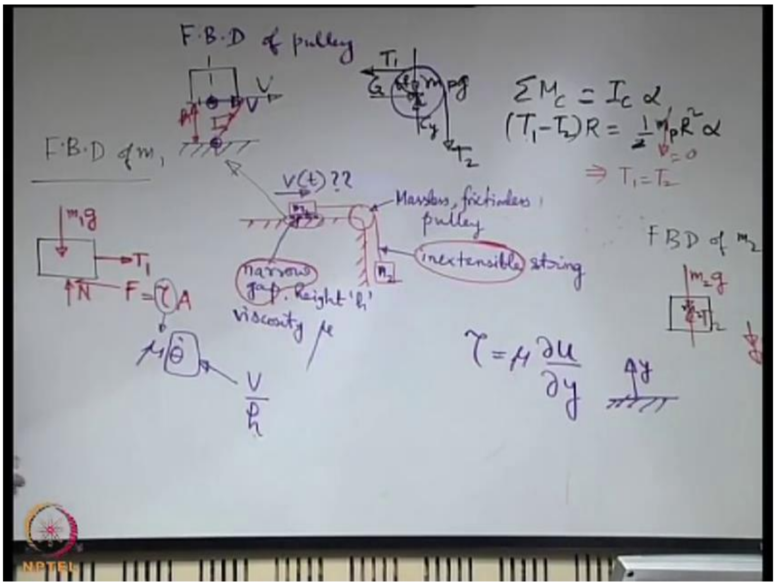

Let us say that you have a system with a mass and pulley and string; the kind of system that you always love to deal with in mechanics problems. So, we start with such an example because it will have a good transition from your earlier studies in mechanics to the mechanics of fluids. So, where we make a change is like we put some fluids, say oil in the narrow gap between the block which is there on the plane and the plane.

So, this qualitatively is a narrow gap of say height small h. Let us say that mass of these is m 1 and one that is hanging is m 2. We make further statements that these pulleys massless and frictionless, and also this string is inextensible. Assume that the fluid which is there in between the block and the plane is Newtonian with the viscosity mu. Our objective say is to find out what is the velocity of this block of mass m 1 as a function of time. So, this is the question that we would like to answer.

This is the very simple problem, but we will try to go to as much fundamental depth as possible for such a simple problem. We may start with the expressions, and we will see the expressions are very intuitive we may write those easily. But we will start with the free body diagram of different elements which are present in the system to have a more thorough insight on what happens. Let us draw first the free body diagram of the pulley, most of the times you will never draw it, but if required we should. The pulley is hinged to this surface, so the center of the pulley at the center there is a hinge. Now what should be the forces which are acting on this?

Student: Normal reaction between.

Normal reaction if you say normal reaction by whom to whom?

Student: At the center from the hinge; sir, from the center.

One by one; so first say we are talking about the hinge. So, what type of reaction you expect for the hinge?

Student: (Refer Time: 04:21).

So, there are contact forces it is something which is occurring in a plane. So, in general you will be having general force in a plane which may be dissolved into two components. Let us say that we are talking about two components C x and C y which are like arbitrary orthogonal components of the forces which are there at the hinge; then?

Student: Tension.

Tension in the string; so here you have some tension T 1 here you have some tension T 2. For the time being let us forget about these assumptions which are being given; massless pulley, friction less pulley, in extensible string, and then we are not exactly using the consequences of these in drawing the free body diagram. So, in the free body diagram we were keeping everything as general. So, let us consider that the pulley might have a mass also.

Just to see that if it has a mass; when we say massless we do not mean that it is literally massless, we mean that effect of that is not significant that is all. So, whether the effect of mass is significant or not how will we understand. These types of statements are very important like massless, frictionless, inextensible, and so on. Most of the times you think that these are for beautification of the problem statements, and at end you come up with the conclusion which sort of abstracts you from the path by which you have come to the conclusion.

For example; if we say T 1 equal to T 2, we will of course see T 1 equal to T 2 all of you have learn from your mechanics that it should be like that. Now can you tell that whether T 1 equal to T 2 is because of massless pulley, frictionless pulley, inextensible string or what?

Student: (Refer Time: 06:30).

See now I am getting 3 different answers from.

Student: (Refer Time: 06:35).

So, like if it is a multiple choice question massless, frictionless, inextensible all the above none of the above or many choices are given to you. Let us see.

Student: (Refer Time: 06:49).

Let us see which one is the fundamental and which one is dominating. So, if you think that the massless is the thing that should be putting question. So, let us consider that it has a mass and see that what is consequence; it will help us in understanding that what would be if it is massless. Let us say that m p is the mass of the pulley, so it has its weight also. Now what we do is we want to get an expression between T 1 and T 2. So, if we take moment of all forces with respect to C the 3 C x C and m g these get cancel it, helps us in obtaining a relationship between T 1 and T 2.

So, what is that basic equation that we are looking for? Resultant moment of all forces with respect to an axis passing through C perpendicular to the plane of the board is the moment of inertia of this pulley with respect to an axis which is the same as the axis with respect to which you have got the moment times the angular acceleration. So, if say capital R is the radius of the pulley then you can write this straight away as T 1 minus T 2 into R. Then the pulley is like a disc as an example we call it half m R square. So, if the mass of the pulley is neglected, then automatically it will give rise to T 1 equal to T 2.

So, it does not matter whether its friction less or not so far as this goes. Is it a totally correct statement? Now another question I am asking you.

Student: (Refer Time: 08:57).

You should try to understand it through a contradiction say; you have encountered cases where you have a pulley with the string or a build around that and there is a difference between these tensions. And the difference is given by say- if it is an impending sleep T 1 by T 2 is t to the power coefficient of friction into the theta angle of rap. This you have learnt in the statics. I mean here we are seeing something different right, so what is the anomaly? You think about it, I will ask you next time. Let us continue with this problem, because for this particular problem this that is not going to be important so much I can tell you.

Now we come to say the free body diagram of m 2 maybe. So, you have the weight then you have the tension which is T 2, of course T 2 is equal to T 1. And then, if the system is released what would be the direction of motion that you expect?

Student: Downwards.

So, this is coming downwards and this is expected to go towards the right. So, understand the physics of this problem; when it is moving downwards it is tending to go towards right, there is a resistance at this interface which does not want to make it go towards the right so easily. So, it is somewhat like a friction. And that friction here is sort of lubricated it is not a direct contact between two solids, but there is a thin film of liquid in between. That means, if that is the case then let us say it comes down with an acceleration a.

So, if this comes down with an acceleration a, which is you may say this a is like d V 2 dt. So, when it comes down with an acceleration a you expect that m 1 also moves right towards right with an acceleration a. Again by which assumption it is there?

Student: Inextensible string.

Inextensible string; so inextensible string had a role to play here; massless pulley we have seen had a role to play here, and yes frictionless pulley also has a role to play here. Because the belt is being wrapped around the pulley, and because of friction there may be a difference in the contact forces between the delta and the pulley. So, that will be more apparent if you draw the free body diagram of the belt and see its interaction with the pulley.

So, when you are thinking of interaction between the belt and the pulley that is not considered here. And that is not considered implicitly because it is frictionless, so important interaction is through friction. Normal reaction will always get nullified even if it is considered because we are taking moment with respect to the center. So, all these have been used in some way or the other. So, let us draw the free body diagram of m 1 which is the important matter so far as the understanding of viscosity goes. So, if we draw now the free body diagram of m 1. Now you tell what are forces which are acting on m 1?

Student: (Refer Time: 13:06).

Yes, one is T 1. There is some normal reaction to the; what is the origin of this normal reaction? This is not direct contact between the solid boundary and the block. Yes?

Student: (Refer Time: 13:34).

There is some fluid is there, and there is a pressure distribution at the interface between the fluid and the solid. So, the resultant of that pressure distribution gives rise to this normal reaction. Of course, the same normal reaction we have drawn, but we have to understand that physically where it originates. Then there is some resistance also. So, what is a resistance? There is let us call it F, just with analogy with the problems involving standard mechanics of solids. So, this F is sort of friction force, but here again the origin is not the direct contact between the plane and the blocks. So, what is the origin of this friction force?

Student: Viscosity.

Viscosity: so it is viscosity or viscous affects at the interface between the block and the fluid. That should be expressible in terms of the shear stress. And the shear stress is expressible in terms of the rate of deformation through the Newton's law of viscosity because we are assuming it is a Newtonian fluid. So, what we expect here is that this should be some equivalent shear stress at the interface times the area. So, let us say that A is the bottom surface area of the block; that is another input to the problem.

Now only work that is left is to relate the tau with the rate of deformation. Now the rate of deformation is something which takes place; if you look at the magnified view of what happens in this thin film. So, in this thin film you see that there is a solid boundary at the bottom which is stationary at the top there is a block that is moving, so this is the block. So, this block is moving with the particular velocity. And this block is idealized as a particle. So, when you when you idealize sort of rigid body as a particle? When you do not have rotational effects; the fundamental difference between a particle and a rigid bodies are rigid body has rotation or it is capable of having different rotational components, but particle cannot rotate.

So, these of course, we are not considering that this could rotate in this type of situation, sot is just like a particle or a point mass. So, everything is moving towards the right with the particular velocity which is changing with time. Let us say that this has the velocity V. So, when this has the velocity V, of course that is a function of time that you have to understand.

Now if you draw the velocity profile at any section which includes the fluid between the block and the plane. At the wall, because of no slip boundary condition it is 0. Here, what should be the velocity of the fluid?

Student: V.

V, it should be V that is also because of no slip boundary condition. So, no slip boundary condition is not 0 velocity of the fluid, but 0 relative velocities. So, if the solid moves with that velocity fluid will also move with that same velocity. Now see the catch word; again a very nice catch word is there is a narrow gap. So, narrow gap means this thickness h is so small; it is so small that this variation of velocity between 0 to V may be assumed as linear.

Now if you see that this shear stress let us now complete the expression for the shear stress. It is mu times the rate of deformation. Rate of deformation could be different at different y locations, but if it is a linear profile it is same everywhere, because rate of deformation is related to d u d y; that we have seen. So, rate of deformation in such a case is d u d y, where u is the velocity. So, if u versus y is a straight line d u d y is a constant. So, this for our present case will become.

Student: V by h.

V by h; so remember what are the approximations of simplifications which have led towards this, if this is not arrow gap it has to be what is the rate of change of u with respect to y at the interface between the block and the fluid not at anywhere. But since now it is a straight line it is as good as it is anywhere.

So, the very important understanding is we are using that tau equal to some mu, it is better to say that it is a partial derivative of u with respect to y because u could be function of many other things. Here of course u is a function of time, but no space code in it. So, still it is like a partial derivative. If you write d u d y, which we wrote for the first time when we wrote such an expression just for simplicity in understanding, but it is in general of partial derivative- it assumes that there is only one component of velocity. And the partial derivative comes from the fact that, that one component of velocity itself could be a function of many things x y z time like that.

And it could itself be variable. And what is y? The important thing to keep in mind is that if you have a solid boundary y is the coordinate which is normally directing outwards from the solid towards the fluid. So, y is a genetic thing, it is not that any y axis you define it is that d u d y. So, this y has a special meaning. This y is the axis which is normal to the surface into the fluid that we are considering. So, if you are having a different type of co ordinate axis then you have to adjust this with the plus or minus sign.

So, that you have to keep in mind that this is the orientation that we had considered for describing the Newton’s law of viscosity that is entity should be preserved. With that understanding now we write the equations of motion by Newton’s second law which should be straight forward. So, for the block m 1, of course along the y direction it is equilibrium, so N equal to m g that is not a part of the requirement of the problems.

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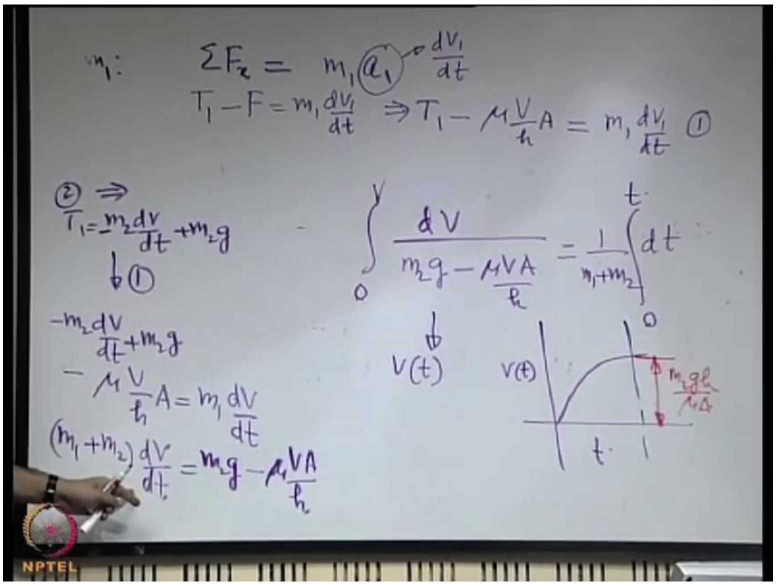

So, we are just writing the resultant force along x for m 1 is equal to m 1 into the acceleration of a that is a 1 that is nothing but d V 1 dt. So, resultant force along x is T 1 minus F. Since it is an inextensible string we will have V 1 equal to V 2 equal to V. So, very soon we will write in place of V 1 as V or V 2 as V where V is the common velocity with which the system is moving.

So, you can write T 1 minus mu V by h into A is equal to m 1 d V 1 dt. Let us say this is equation number 1. We write next the equation number 2, which should be for the mass m 2. So, let us draw the free body diagram of the mass m 2 or that is already there, so we just write its equation. It is now along y we are considering. So, this is coming down. So, m 2 g minus T 2 is equal to m 2 d V 2 d t. From the inextensible string we can write V 1 is equal to V 2 say equal to V. And we have already seen T 1 equal to T 2. So if we call this as equation number 2, in equation number 2 we have this T 2 same as T 1. So, we can get what is the expression for T 1 and substitute in equation number 1; let us do that.

So, from 2 we get T 1 is equal to m 2 d V d t, sorry, T 1 is minus into d V d t plus m 2 g. And that we substitute in 1. So, minus m 2 d V dt plus m 2 g is equal to that is T 1 minus mu V by h A is equal to m 1 d V dt. So, you have m 1 plus m 2 d V d t is equal to m 2 g minus mu V A by h. It is very easy to integrate it; you can just separate the variables. So, you bring this whatever function of V is there on one side and whatever function of t on the other side. So, you have d V divided by m 2 g minus mu V A by h is equal to.

So, you can integrate it with respect to times say at time equal to 0, the velocity was 0 or V 0 in general may be. If it was having an initial velocity and at time equal to t say the velocity is v.

From this if you integrate you will find an expression for V as a function of time. Of course, in an exponential form it will come, because it will be in a logarithmic form the integral that will appear. So, it will be some function of time. And, if it is released from rest is velocity should increase with time or decrease with time increase.

Student: Increase.

Increase right, because the m 2 g that is falling down and making it to move towards the right. At the same time there is a viscous resistance, so it will come to a sort of asymptotic state when it has a balance between these two forces. And then there is no further change in velocity. So, without working it out may be the velocity versus time characteristic could be something like, maybe like this.

Student: (Refer Time: 26:02).

And let us say that it comes to a sort of steady state at some time which is last large time theoretically tending to infinity practically large time. So, what should be these velocities? So, can you tell that what should be these velocities?

Student: M 2 g h by mu a.

Yes, m 2 g h by nu a. So, this is; and the reason is straight forward that when you have a steady state there is no more change of velocity with respect to time. And therefore it should be equilibrium between these two forces so that could give rise to this type of expression. So, whenever we work a problem it is important that we also try to get a physical feel of what is happening, try to formulate it in terms of the physical feel, try to get a sketch may be of what is the variation. And then it will be so easy to interpret whatever results you are getting, whether they are correct or not, or whether they are having some physical sense or not.

With this we will have short break and then we will continue with the discussions on viscosity and surface tension in the next part of the lecture.

Thank you.