**Non-traditional abrasive machining process: Ultrasonic, Abrasive jet and abrasive water jet machining**

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**Lecture – 07**
**USM- Horn Design (Contd.)**

Welcome viewers to the 7th lecture of the course on Non-traditional Abrasive Machine Methods. And we have gone well into the discussion of horn design in ultrasonic machining. So, we will quickly finished that and take up some numerical problems and finish this part. So, what we were discussing was we were having this particular expression that is from the final solution of the velocity due to the vibration.

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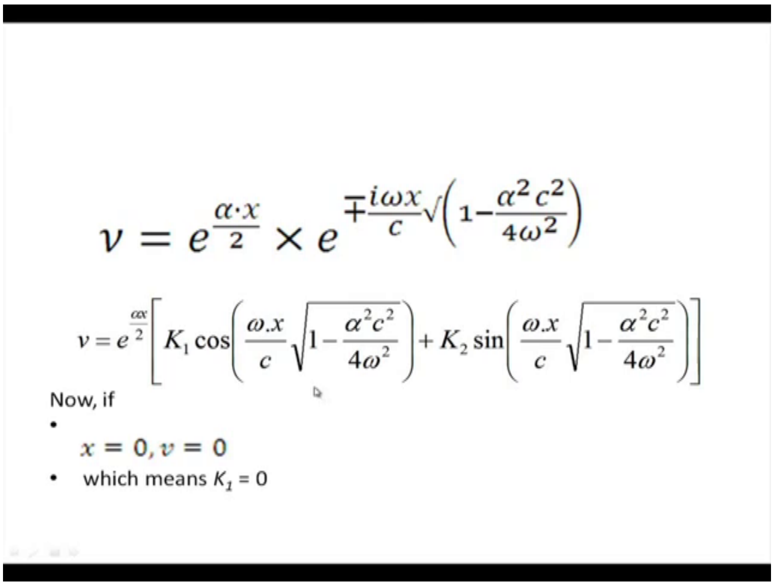


We have it is expression as this one. And displacement will also have a similar expression. So, this means what is this displacement as we have discussed that we were talking about? It is a displacement due to vibration, it is going 2 and fro this is the vibrational displacement and velocity that we are talking about.

So, at x equal to 0 we had discussed that the velocity is 0 since it is no. At x equal to 0 suppose we consider that velocity equal to 0, why say why is in that the section in which where connecting the transducer? So, how can the transducer end of the horn have zero velocity? To this my answer is we are taking the general case now then we draw figure to you know discuss this point.

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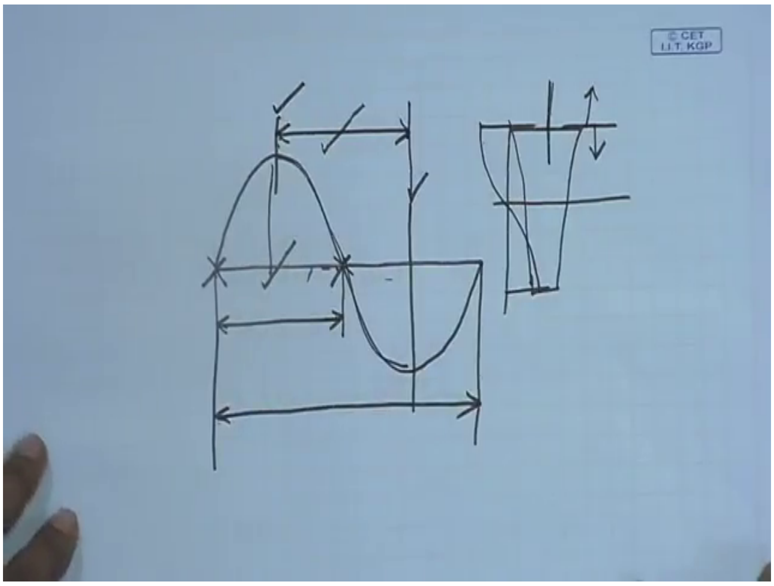

This is the typical sin wave and if I take the distance between 2 points which are you know completely non-moving, it is it is this pair of points. If I take 2 points which are always I mean which are having the maximum movement it is this pair of point what is the displacement I mean what is the distance between them the distance between them is basically half the wavelength, half the wavelength. What is the full length of the wave? The full length of the wave is this one.

So, if I am asked to find out this particular length I can also do one thing the basic wave equation that I am dealing with I can put conditions of 0 displacement to or 0 velocities are these 2 points, solve for it and say that this length as it is equal to this one I can well solve for this these 2 conditions and apply it as the length between these 2. I will solve for this length between these 2 points I will say that is equal to this one that is what we are going to do. And how does it you know apply in our case. Our case is, in our case say at this point is undergoing motion, is undergoing vibrational motion here, at this point connected to the transducer. Therefore from here there is a point at which there is no motion at all.

So, it be draw the graph of motion verses displacement it can be like this. Same as this one same as this one let me draw clear figure if I say that ok.

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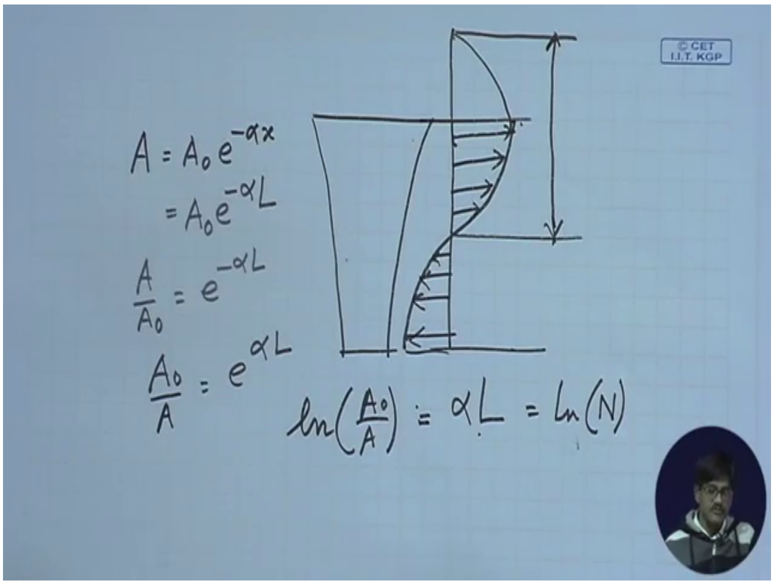


I am drawing the motion against distance. So, they are corresponding displacement suffered at these points. So, basically is vibrating in this manner. And this is the middle point at which it is it is a note point has no motion. So, if it is vibrating this way. So, I can solve I can find out this length to be half the wavelength, half the wavelength and I can solve it for this one also. I will say I will solve it for these 2 points. No motion points, note points. I will solve it for this and I will say it is equal to this one be. So, let us proceed. So, I put x equal to 0 velocity equal to 0 I will put this velocity equal to 0 at 2 successive places.

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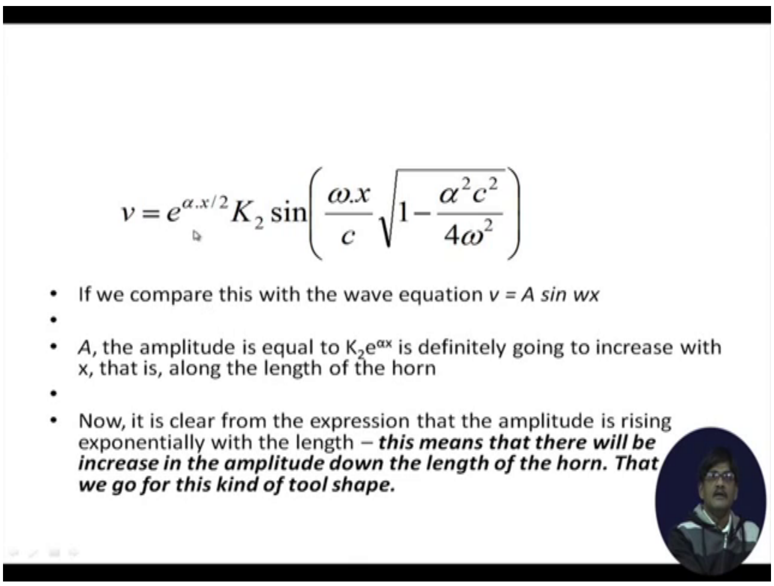

So let see. We will say that if we do that if we do that k 1 will come out to be 0. Why? Because if we put 0 here and if we put x equal to 0 here. So, it becomes cos 0 cos 0 is one. So, it survives, but sin 0 is 0. So, this one gets cancelled out, this is gone and we have the have the expression after x equal to 0 and v is equal to 0 has been put in 0 is equal to e to the power something into k 1 that is all. So, k 1 becomes equal to 0 and k 1 vanishes. So, if k 1 vanishes let us see what we have here, we have this expression. It is interesting to see that the amplitude of vibration this term it is having an x dependent expression; that means, the amplitude is dependent upon the distance from the transducer, in what way as we move away from the transducer x will increase.

So, will the amplitude. The amplitude goes on increasing as you move away from x and this means that if you take the exponentially decaying horn; that means, a horn whose cross sectional area is exponentially decaying you will have this expression for displacement and velocity and therefore, the amplitude will definitely go on increasing as you go down the what you call it go down the length of the horn. And we have achieved what we started with amplification of amplitude ok.

Now, what do we do this part we will say if we put once again v equal to 0. If we put once again v equal to 0. Let us put that and find out. So, our first thing is achieved amplification of amplitude is taking place. Next let us see if I have to design such a horn suppose someone says go and make a horn of exponential type ok.

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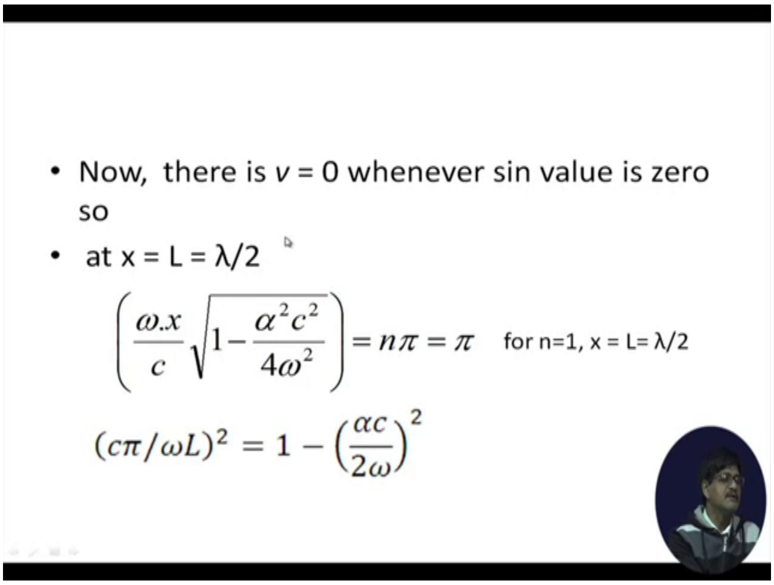

You make it you know that it will amplified, but first question is how much length of the horn will choose.

So, let us go for that now there is v equal to 0 whenever sin value is 0. So, what we do is let see where sin value is taking. So, it is basically you know a into sin something. So, let us put the sin value 0 therefore, we will definitely have v equal to 0 amplitude cannot be 0. If amplitude is 0 everything is 0 everything is peaceful it you do not have to bother about that. So, whenever in this particular expression the sin value become 0 let us have a look. So, at x equal to L equal to lambda by 2 at x equal to L it could be lambda by 2 as we discussed it how the wave length once again we are going to have that particular condition let us see.

So, we understand that sin 0 sin pi sin n pi and sin 2 pi etcetera etcetera all these places the value of sin is going to be 0. So, let this be equal to we have already considered this to be 0 now let this be equal to pi the second case; the second successive case. So, if it is equal to pi then let us put it to be pi this side you have pi and in place of omega where is omega I have lost. Oh right, we have sent c to the other side. So, it c pi divided by omega x and in place of x I have put length whatever be the length and this movement we are simply putting length.

So, put omega to the other side in goes to the denominator, see goes to the numerator. So, numerator c denominator omega denominator L replaced replacing x whole square because you are taking away this root over sin and therefore, you have 1 minus alpha square c square divided by twice omega f o square omega square was equal to this that is good, that is good.

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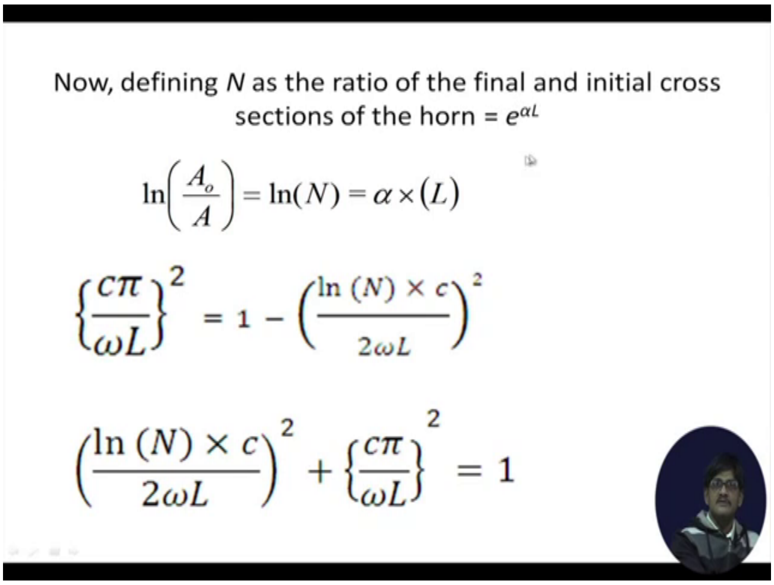


Let us see what we get out of that now we bring in another particular variable, what is that variable? That variable simply is n and what is n? n is the ratio of the final length initial cross sections of the horn and it is equal to e to the power alpha L now what is that? If we take let us quickly do that calculation here e to the power sorry, we know e is equal to a 0 into e to the power minus alpha x. And in our case it is equal to I mean the final cross section is equal e to the power minus alpha L where L is the length of the horn, how much is it I do not know I have to find out in order to design ok.

So, what do we do? We can say to A by A 0 is equal to e to the power minus alpha L and therefore, we can see e 0 by L A 0 by A is equal to equal power alpha L. Which means if I take long I will get ln A 0 by A equal to alpha l. So, here I am making a replacement this is equal to L n of n that is all. So, I get ln of n is equal to alpha n this is the thing that we have used. Why is this one is this is quite you know frequently used term. So, we are introducing that terminology nothing else no new idea.

So, looking at the previous expression we have a quick look at that this was our previous expression. So, this is probably going to be intact and we are going to simply replace alpha, that is it we had found out that n is equal to e to the power alpha L from that we had got ln n is equal to alpha L. And therefore, we have alpha replaced by ln n divided by L ln 0; that means, natural log of n divided by L and therefore, we have c pi whole square etcetera etcetera all the terms previously. And so, therefore, this is the expression that we get and we start algebraically manipulating it; that means, taking this term on that side etcetera etcetera.

This we can recognize this has been brought here this side one has remained on the outside and c pi by omega L is here. So, that we can easily recognize that c common, L can be taken common, omega can be taken common and therefore, it means that this will be depleted of hardly any of the terms pi can also be taken common and taken out.

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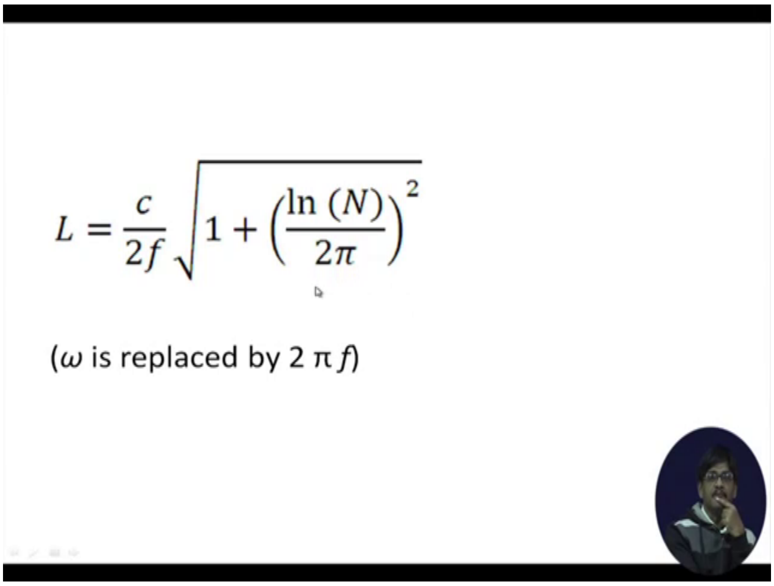


So, that we have the final expression is this one. What is the change that has taken place? From that other term simply we have replaced omega by twice pi f. We replace omega by twice pi f what are we getting from the previous expression let us do it, and finally put a seal to it yes.

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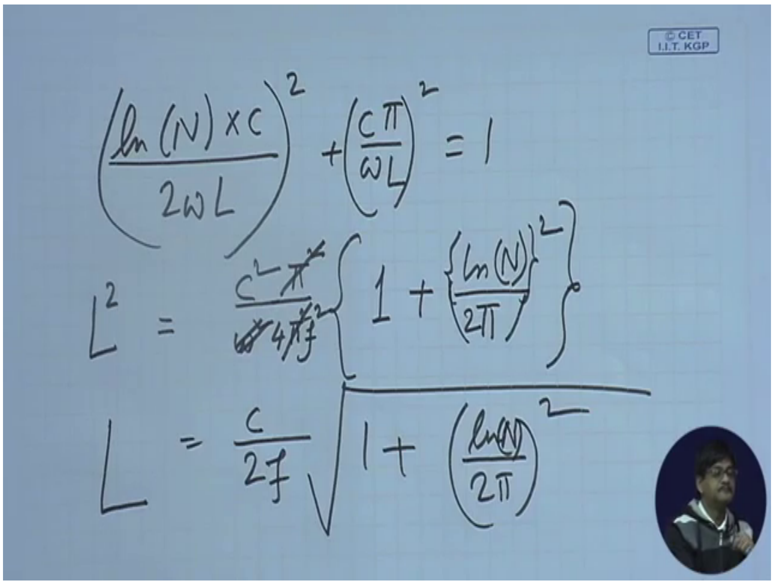


So, we have if you have if you look at the expression ln n into c divided by twice omega L whole square plus c pi by omega L whole square equal to 1. Now what can we do here? We can L common here. And so, that the other side will have L square that is good equal to. So, I am writing this site on this one, L square is equal to c can be taken common, c is taken common c square by omega square and suppose I take pi square also common. And therefore, I will have this one is depleted of everything. So, it has 1 plus ln n is still there holding the citadel, by pi whole square does that look correct. Let us see c was gone, omega was gone, 2 is there still 2. So, what can you do over this let us replace omega square by 4 pi f square.

So, n will be equal to in that case c, pi pi will cancel 4 pi square omega square 4 pi square f square c by twice f root over 1 plus ln by 2 pi whole square, seems to be correct, let us see. Exactly, you have got the expression that we had previously noticed. So, this means that after all that calculus and all that analysis, you are ultimately coming to the conclusion that the length of the horn with exponential you know exponential exponentially decaying cross section can amplify the amplitude that we have seen. And it is length should be this one, it is length should be this one this is called you know half wave horn; that means, it is length is go to half the wave length. We can go for different lengths also we need not necessarily stick to half the wave length, but it is one of the most common ones.

So, let us take a few numerical problems on this.

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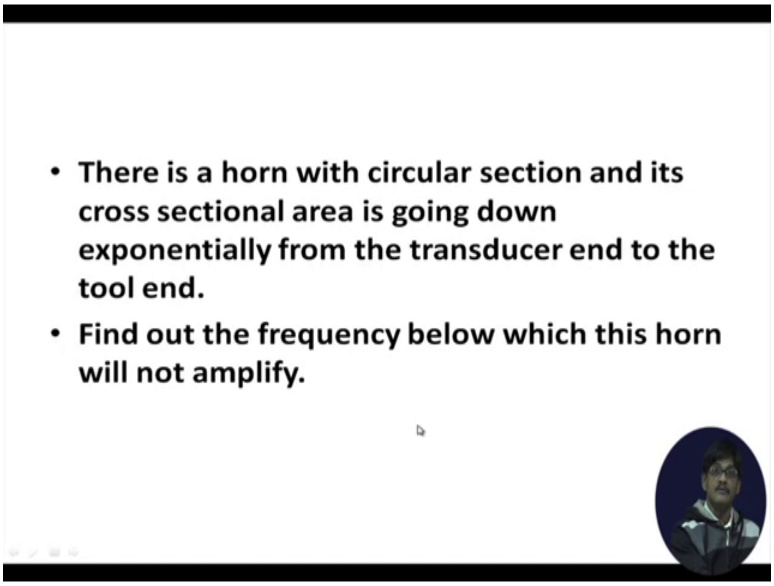


There is a horn with circular cross section and it is cross sectional area is going down exponentially from the transducer end to the tool end find out the frequency below which this horn will not amplify. So, it is a negative statement the frequency below which this horn will not amplified.

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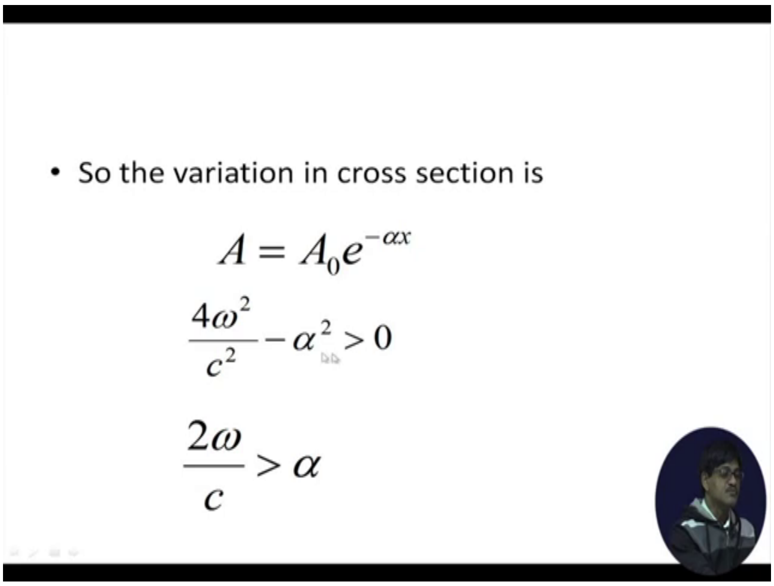


So, basically it means that we will simply find out the frequency for which it will have you know negative roots, and for app we will say that above this particular frequency is going to have you know vibration; that means, cyclic motion and therefore, beyond below this frequency we are not going to have.

So, variation in cross section is this one, and therefore, we have previously seen if you remember this would have vibrational motion; that means, cyclic motion; that means, sinusoidal motion only if this term is greater than 0 this was the discriminant. So, naturally if you if you arrange the terms ultimately it boils down to if twice omega by c is greater than alpha, in that case you can have sinusoidal motion vibration, cyclic motion, periodic motion.

So, just let us have a quick look at the previous case. This omega can be replaced by twice pi f. So, that we have 4 pi f 4 pi f by c is greater than alpha, that is it.

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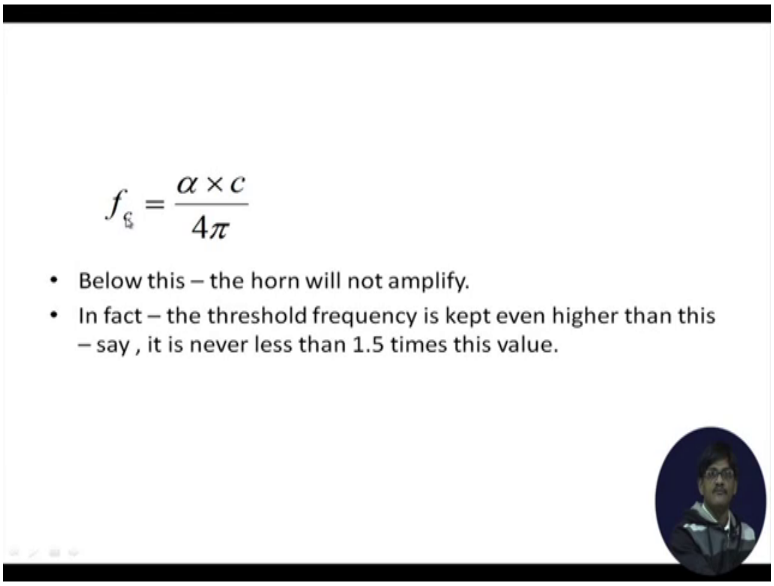


That 4 pi goes down and alpha climbs up I mean c climbs up. So, at you have frequency is equal to or greater than alpha c by 4 pi if you have this is the least frequency for which you are going to have periodic motion. Below this any frequency will not amplify the amplitude because simply it will not be periodic.

And In fact, when we are designing we generally make a check that the frequency that is being used it is definitely you know at least 1.5 times this frequency or maybe 2 times this frequency like that, let us take a numerical problem.

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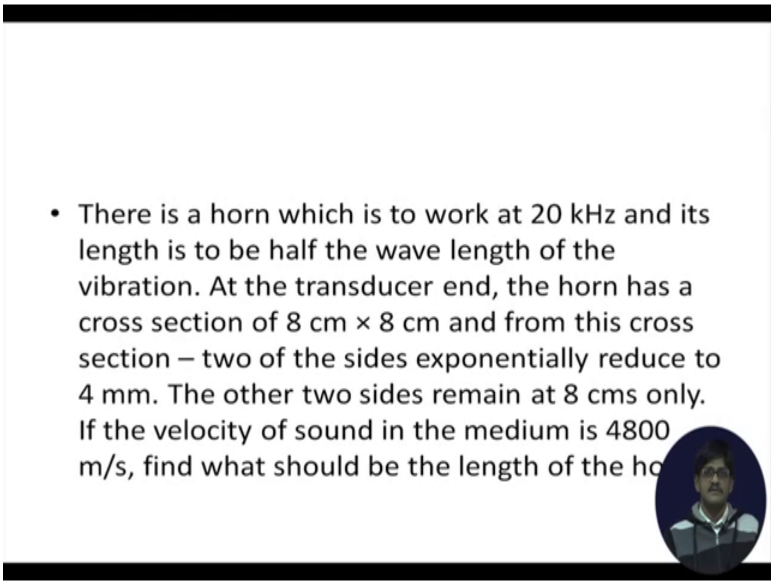


There is a horn which is to work at 20 kilohertz and it is length is to be half the wave length of the vibration that is good. So, the frequency is 20 kilohertz, the length of the horn is half the wavelength of vibration. At the transducer end the horn has a cross section of 8 centimeters by 8 centimeters and from this cross section 2 of the sides; that means, it is it is a square. So, it has 4 sides and from this particular cross section 2 of the sides are exponentially reducing to 4 millimeters, 4 millimeters.

So, it is it is it is reducing like this and become a (Refer Time: 20:38) it is moving like this and reducing this way. What does it read further? It reads that the other 2 sides remain at 8 centimeters only. If the velocity of sound in the medium is 4800 meters per second find what should be the length of the horn. So, we have to find out the length of the horn. So, let us draw picture how it looks like.

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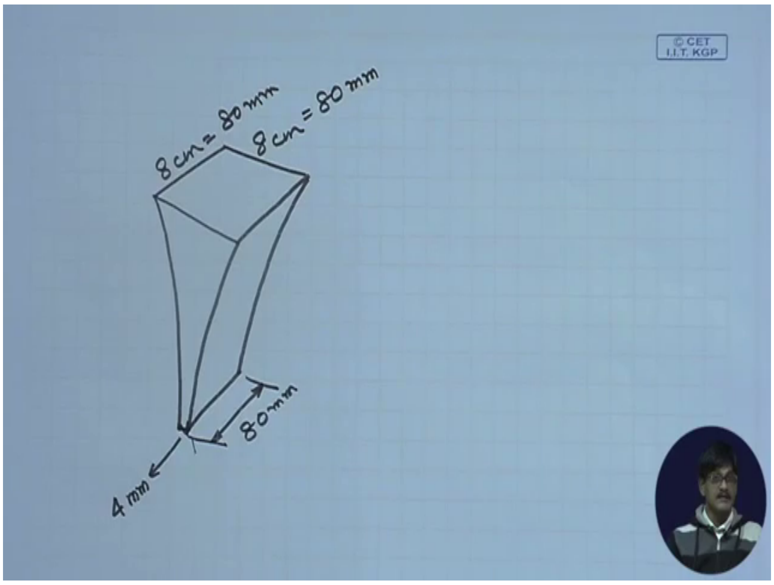


That is it the horn look like this. The transducer is at the top that I have drawn down, but the horn is you know you know the (Refer Time: 22:02) which is tapering of and this side is 4 millimeter mind you this edges still it. So, once we are understanding this is the shape now comes the question of designing it such that we can find out it is correct length so that it is a half wave horn.

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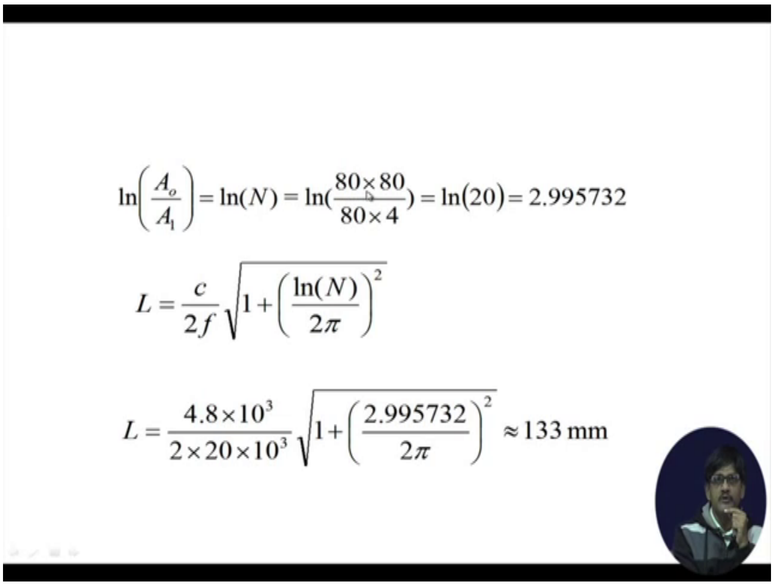


So, let us go on finding out the terms I mean the dimensions that we have come across like for example, the natural logarithm of n, because as we know in a second expression that we have this simply is the expression of the length of the horn. So, c A 0 and A 1, these are given to us a 0 is the original cross section and the transducer end and this is the cross section at the at tool end. So, this is equal to the natural logarithm of n and if would be put in values is the area at the start and this is the area at the end 80, 80 cancels of the basically ln. Ln 20 ln 20 is equal to 2.995732 ok.

So, this has been found out. So, this by 2 pi whole square plus 1 just let me see, into twice into c by 2 f is equal to the length. What is the speed of sound in that particular medium? If the horn if the horn is made of steel the sound will be traveling much faster than it travels in air and it has been given let us have a quick check. 4800 meters per second in there is a velocity. And 20 kilohertz is the frequency that is all. So, we put 4.8 into 10 to power 3 divided by twice f 2 into 20 into 10 to the power 3, 10 to the power 3 cancels and therefore, after all these calculations we will have 133 millimeters. 133 millimeters, I will very much prefer that you carry out this calculation yourselves to cross check this one, they might be simple you know (Refer Time: 24:44) mistake.

So, it will be good practice to follow the calculations by yourselves once and if you find some discrepancy please report it to me So that I can also correct it, and then convey to everyone. So, the first part of the problem is done, 133 millimeters second.

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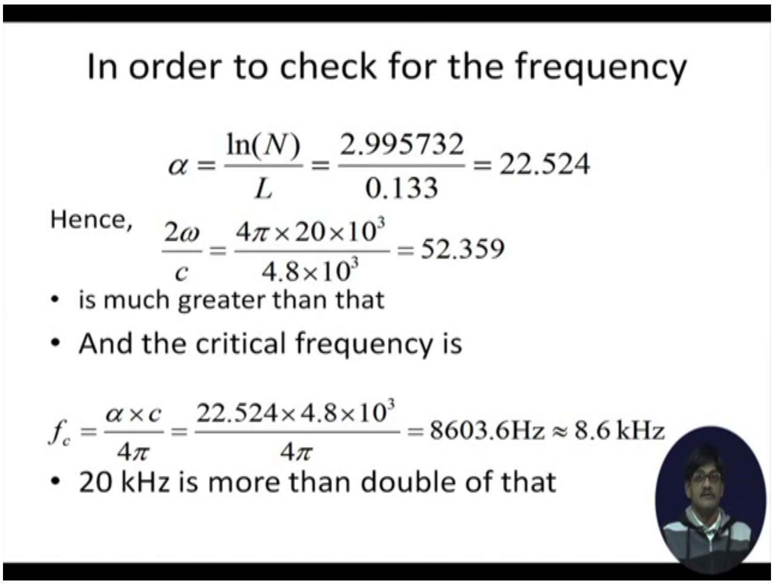


Now, we have to make a check if you remember that expression what was that f is equal to alpha c by 4 pi, what does it mean? That means, that we have designed it for 20 kilohertz, but what is the threshold frequency that is a frequency value which below which there will be no periodic motion and we should generally be 1.5 to 2 times that particular frequency in order to have a safe region of I mean safe selection of frequency. So, alpha is equal to we have already found it out 2.995732 divided by the length. And that gives us 2.995732 divided by 0.133, this was the length if you remember we are expressing into the meters.

So, this comes out to be 22.524, that is good. So, once we have this therefore, twice omega by c is equal to what is twice omega by c, twice omega by c is the this was the check that we were making, this was the check that we were making in order to you know be sure I will I will just go there. In order to make it sure that periodic motion is definitely going to take place twice omega by c should be greater than alpha. So, twice omega by c is equal to you know 4 pi into 20 into 10 to power 3 2 pi f. So, 2 into 2 pi f divided by c 4 point into 10 to power 3 is equal to 52.35 time.

And therefore, this particular term this particular term twice omega by c is much greater than alpha. We can look at it in another way let us find out the critical frequency it is much easier to understand it in terms of critical frequency; that means, a threshold frequency below which there will be no periodic motion. And for that we use the next expression if you remember f c was equal to alpha c by 4 pi.

So, we put the values of alpha we put the value of c and we put the value of 4 pi here and it comes out to be 8.6 kilohertz. So, are we more than are we at more than 8.6 kilohertz, yes we are working at 20 kilohertz. So, even if you double 8.6 kilohertz we are you know we are even more than that 20 kilohertz is more than double of that. So, we are in a safe zone of frequency. So, we have found out the length of the horn and we have checked that we are working in a particular range of frequency which is you know safe range in comparison to the threshold frequency below which they will be no periodic motion. [FL]

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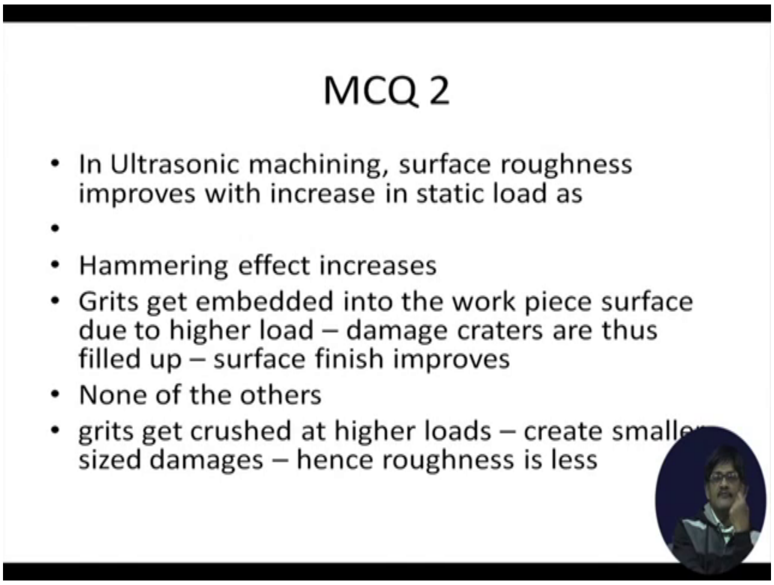


So, I have wanted 2 minutes now left of this lecture. I would like to discuss one MCQ. This is actually connected with something that we will be discussing in more detail later on, but I am sure some of you will be able to you know understand what is the answer. In ultrasonic machining surface roughness improves with increase in static load static load. Means the load with which you load which you applying on top of the what you call it, machine head in order to keep the keep the tool and work piece pressed together.

So, if you increase the load the surface roughness improves, I mean roughness becomes less. Surface finish improves actually we should have said. So, one possibility is because hammering effect is more and then grits get embedded into the work piece surface due to higher load and the damage craters are all filled up. And therefore, surface finish improves. None of the others and the grits get crushed at higher loads create small smaller sized damages and hence roughness is less.

So, the answer to this is the last one. Actually grits get crushed at higher loads and therefore, they are rendered into smaller grits and these smaller grades give raise in turn to lower values of roughness. And surface finish improves, but you are going to lose in MRR. MRR is going to come down.

So, with this we come to the end of the 7th lecture. In the next lecture we will be dealing with the other parts of the ultrasonic machine.

Till then, thank you.