Non-traditional abrasive machining process: Ultrasonic, Abrasive jet and abrasive water jet machining

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Lecture - 06
USM- Horn Design

Welcome viewers to the 6 lecture of the online course on Non-traditional Abrasive Machining Practices. And today we are going to continue with the discussions on Ultrasonic Machining, and we will take up Horn Design today.

As we have discussed previously the horn happens to be the connecting element between the transducer and the tool. And the main function of the horn is to amplify the amplitude of vibration. First of all it forms a physical connection between the transducer and the tool. Secondly, it provides this amplification of amplitude, because and the transducer itself we cannot give the full amplitude because if we try to do that we will be unnecessarily putting the transducer material to a lot of stress and fatigue and it might fail.

So, instead of doing that we get a small amount of amplitude at the transducer and amplify it with the help of this mechanical device called this horn. And we get higher amplitude our desired amplitude level at the tool end of the horn.

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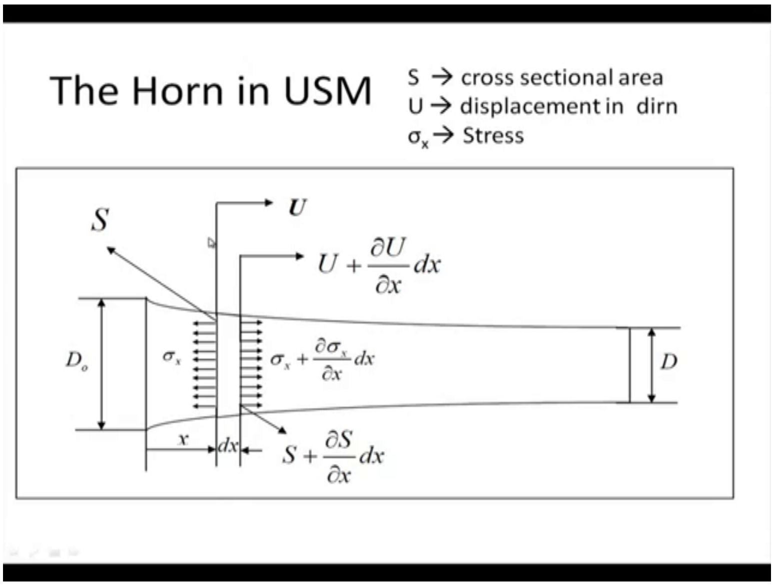

So, we have drawn a free body of the horn; first of all a schematic within that free body of an infinitesimal horn element. What do we have here? First of all the horn is extending horizontally, but generally it is placed in a vertical position. And on the left side we have the transducer end. So, this is the place where it is fixed with the transducer at this section, at this end. And thereafter it is shown to you know decay the cross section reduces continuously so that after that it comes to a particular section shown here. We have shown diameters, but the horn does not necessarily have a circular cross section it might have something different as well.

So, what do we have here? Here we have taken a cross section and a two cross sections in fact at here we have put x equal to 0 plane and from there this is at a distance of x a variable value and this particular element that we have shown here its having a thickness of dx. So, this infinitesimally thin specimen, this particular small body that we have drawn it is undergoing a displacement given by U at this particular section, and its undergoing a section; I sorry and going and displacement given by this particular expression at a section shifted by dx value from this particular section.

Why this change in displacement? This is because displacement itself might be having a gradient, it might be having a change with x. And this particular derivative of displacement with respect to x it is multiplied by dx this infinitesimally small distance, and therefore we get the actual value of displacement at this particular section with respect to U at this section. You might ask me what is this derivative d U by dx del U by del x; I will say I do not know, at this moment I do not know but we can related this way and we can later find out its relation with the other known components I mean known parameters. So here, this cross sectional area of this particular section we are designating as S, and this cross sectional area which is shifted away from it by dx we are designating as S plus del S del x into dx.

So, now I think viewers can well understand that our idea is this we are designating some values here and when we are shifting by dx, along the x axis we are simply relating this value to the values at the previous section by the derivatives of these variables with respect to x and multiplying it by the infinitesimal thin dimension dx. So, in the same way if stresses are built up due to vibrations as sigma x, the normal stress parallel to x axis; so in the same way stresses sigma x plus del sigma x del x into dx will be developed at this particular section. So, let us take them one-by-one.

So, we understand that at this moment we are talking about we are dealing with displacements due to vibration and cross sectional areas and also stresses built up due to this vibrational motion. But first of all comes the question: in which direction is this vibration taking place. This body is vibrating this way ok, the horn is vibrating this way at the node points we have no motion and at the antinodes we have maximum motion. So, first question that would be like to the antinodes to be; the antinodes should be at the connection point between the transducer and the node and the horn.

And I ideally we would like to have this particular point, the position of the cutting section or the cutting tool this should be an antinode points so that it will be vibrating with the maximum amplitude. At some point in between definitely we will have at least one node point which will be not having any particular displacement, and at that point we can hold this horn because physically we have to hold it at some point.

Why is displacement taking place? Because, as it is executing this sort of vibration displacement is definitely going to take place. Why are stresses being built up? Stresses are being built up by virtue of this particular motion, because just like in wave like motion the motion is being transferred from energies being transmitted from one point to another and its building up stresses.

So, let us start with.

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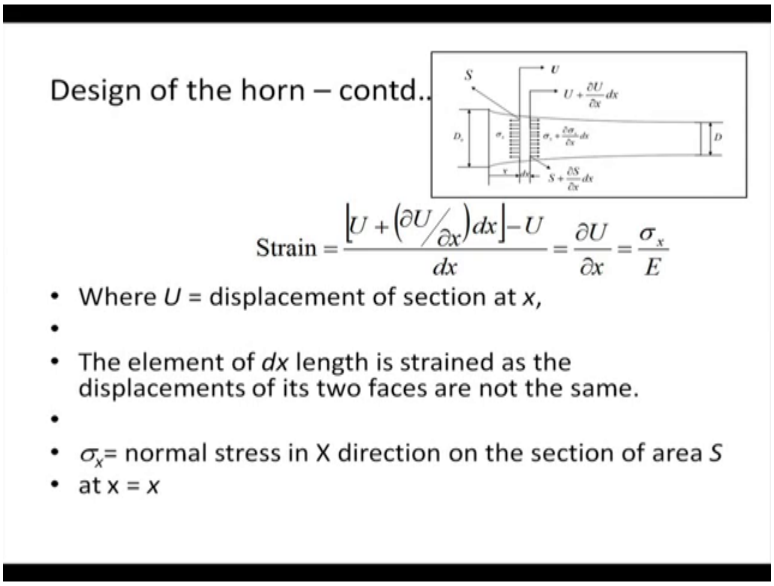

First of all let us take the question of strain. So, what is strain? Strain will occur if the length of a particular section of this horn changes due to the stresses applied on it within the elastic limit. We are talking of elastic strain at this moment. So, how can we find it out? Suppose you ask me, ok that is very good I am really impressed by this idea. So, tell me is this particular disc undergoing in it strain, this small disk of dx thickness is it under going any strain. In that case I will say that I will find out whether it is changing its length along x axis and then I will divided by its original length equal to dx. And that is exactly what we have done here.

That means, the displacement at this particular section is U and the displacement at this particular section is U plus del U del x into dx. So, if these values are the same it is not suffering any displacement, but if these values are different definitely these sections are travelling different distances or they are suffering different displacements in the same time. And therefore it is definitely resulting in a change in this particular thickness.

So, what we do is we simply take their difference. So, U plus del U del x into dx is the displacement of this section, while U is the displacement of the previous section. So, we just take the difference in this particular expression and divided by the original length. And therefore, if it comes out to be del U del x. And we say that del U del x must be the strain. And since stress by strain is equal to elastic limit, therefore we equate this to stress divided by elastic limit. That means, elastic limit can go upstairs and strain can go downstairs so that you will have E is equal to sigma x by epsilon or a strain.

So, up till this point we are simply resorting into stresses and strains and their relationships and your satisfied that strain is equal to del U del x. That is fine, and that is equal to sigma x by E. So, all these things that we have discussed they are formally defined here U is equal to displacement of section at x, the element dx length is strained as the displacement of its two faces are not the same, etcetera, etcetera, etcetera. So, let us pass on the next step.

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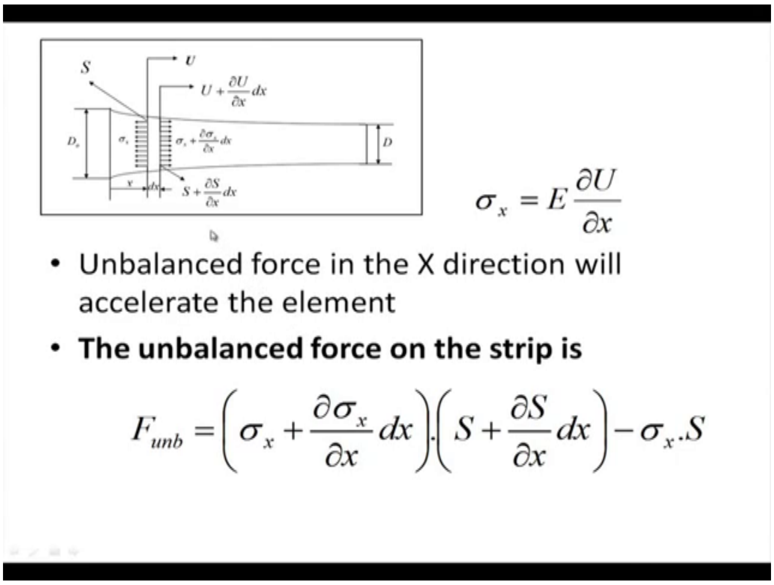

In the next step we are saying that if there any unbalanced force on this particular element. So, why should unbalanced forces come? Before when sharing on that particular debate let us find out simply its connected to its connected to its neighbouring material simply by forces occurring on these faces. And let us compute the forces which are occurring on these faces take their resultant and find out its 0. if its 0 then there is no unbalanced force acting on this body.

But if there is an unbalanced force it will definitely cause acceleration. So, first of all on these sections what are the forces acting; the normal forces. The normal force is first of all: one is the stress multiplied by the cross section on this one and stress multiplied by the cross section on that one; that is very good. So, we have on one of the faces sigma x plus del sigma x del x into dx, and the corresponding area is also S plus del S del x del S del x into dx minus; this is the force on one of the sections. This is the force one of the sections and the corresponding force on the other section is simply sigma x into s. That means, sigma x the normal stress on that face multiplied by the cross sectional area S. So, this will gives us the balanced force.

So, this we can you know just multiply these and get the products for example: we will have sigma x into S we will have sigma x into del S del x into dx then del sigma x del x plus a del sigma x del x dx into S like that. So, let us see how it comes out. A number of terms will cancel out. For example, sigma x into S will cancel out with this sigma x into S etcetera.

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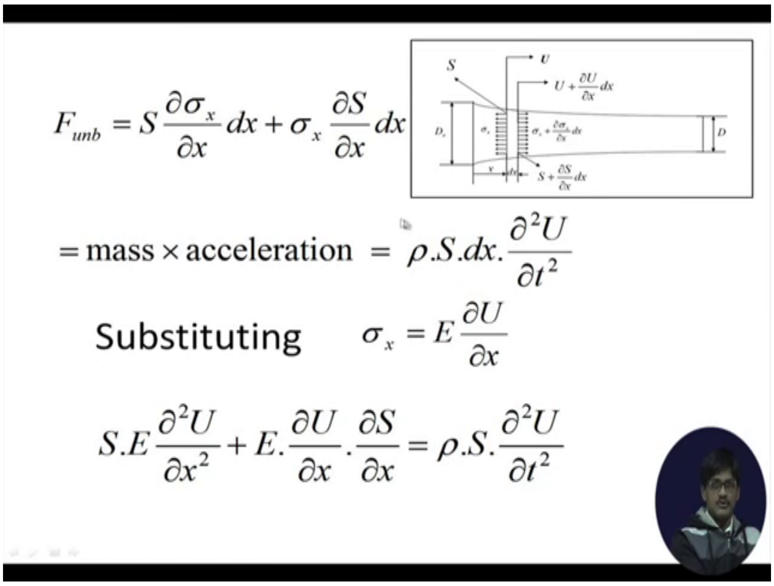

So that ultimately we will get this particular expression. I think it will be a good idea to just have a quick look and the calculations if I do it here.

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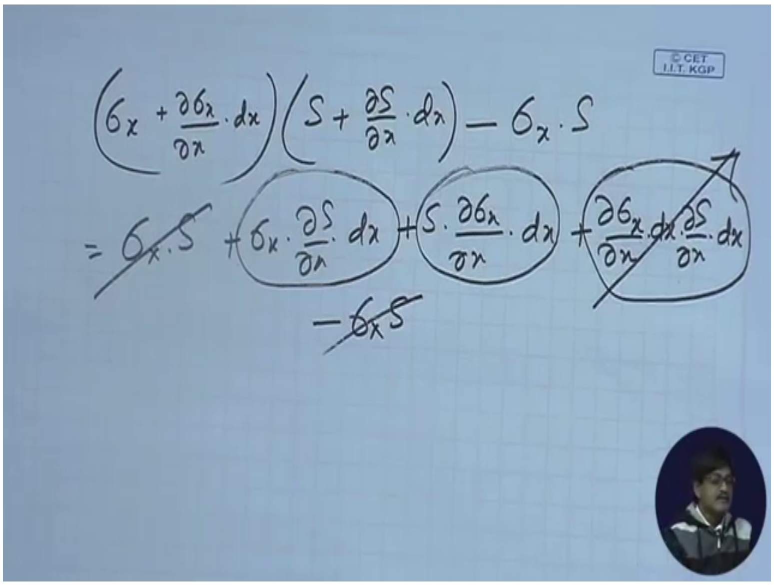

Sigma x plus del sigma x del x into dx multiplied by S plus del S into del S del x into dx minus sigma x into S. And if we expand it we will have sigma x into del S del x into dx plus S into del sigma x del x dx plus- oh my, so there are two difference terms; do we have space for that, yes del x dx del S del x into dx minus sigma x into S that is good. This one and this one will cancel out, this one is you know its fate is sealed because it has two difference terms ds dx it is gone; it will be too small for our consideration. So, we have only two terms surviving, only two terms and it is shown here; only these two terms will survive.

And therefore, we come to the conclusion that- yes there is an unbalanced force on this small I am in on this thin element. So, in this thin element this unbalanced force will cause acceleration. Now why would you do that? If it has to obey Newton's laws it has to do that. And therefore, we use a term acceleration; I mean p is equal to mf p being the force, force we are already computed and mf is mass into acceleration. And therefore, some people use f is equal to m a and some people use p is equal to mf it means basically the same thing.

So, on this side we have an expression of density to volume; cross sectional area multiplied by the thickness will give us the volume multiplied by rho gives us the mass. So, this is the mass expression and this is d 2 U del 2 U del t 2. That means, the second derivative of displacement which all of you know is acceleration.

So, once we have this you can straight cut see that if this is equal to this term then dx will first cancel out; and do we have S cancelling out? No, why, because S is here, but S is not present here. So, S will not cancel out. So, it seems that apart from dx nothing else cancels out, but we are making a substitution which is sigma x. Sigma x was you know it was present here del sigma x del x and also sigma x is present here. So, we were not knowing what to do with them. But now there is a good chance of getting rid of sigma x by replacing it with E into del U del x with this we have proved just now. Stress by strain and strain has an expression of del U del x. So, this is going to replace. And therefore, sigma x once its replaced and once dx is cancelled whole equation we will have S meaning the surface I mean the cross sectional area E being the elastic limit multiply it by del 2 U del x 2.

Now where does this come from? This comes from the fact that we are already having del sigma x del x here, not sigma x. So, derive this with respect to x once and l you will get what we are having here. Naturally, E is a constant so it comes out unscathed, but del U del x gets derived once and therefore you have this particular term. So, we are satisfied that this has been transferred, it has been represented correctly in the next expression and we go to the next term.

What does the next term have? It has sigma x. And therefore, you straight away you can put E here E is here. Del U del x; now, what we do with del U del x? We do nothing we simply bring it here and after that you have del S del x that is good; del S del x is also here. All the terms which were causing as lot of worries unfortunately they have been transferred unchanged to the to the next expression. Rho was here, we cannot do anything with rho. S was here, so S has been transferred.

So, only dx which has been cancelled from all the expressions that is gone and we still have del 2 U del t 2. Now that is a problem because you do not have any t containing terms here. So, what do you do with this, because this going to add to the unknowns and the variable has come in crept in what we do with this one. So, for that you resort to an assumption.

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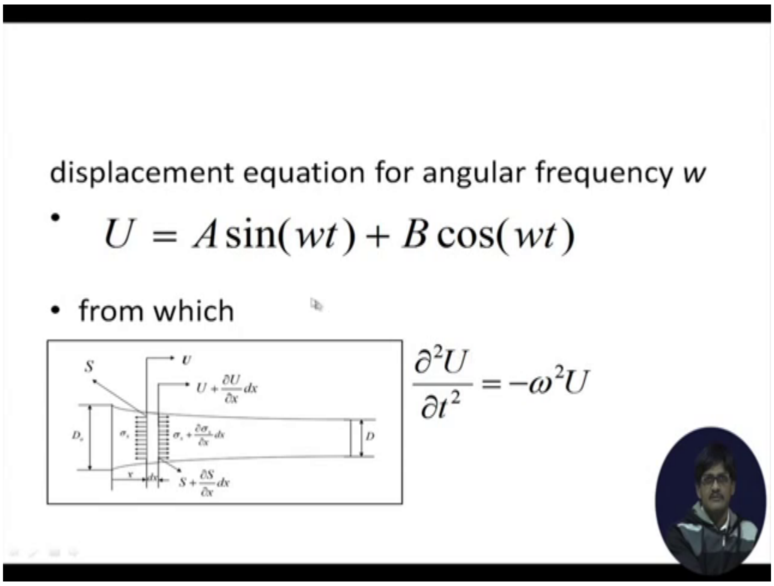

We say that the displacement is going to be simple harmony and its going to be additional these two terms to represent the general simple harmonic motion. These are displaced I mean with the phase difference from each other by 90 degrees. So, we are saying the displacement is simple harmonic; whom I say, why, I will say because it is vibrating all cyclic motions are they are ultimately simple harmonic motions and therefore we represent the displacement by this particular expression with general expression. That is good.

But what comes out of it? What comes out of it is del 2 U d t 2 del t 2 comes out to be minus omega square U. This is the standard derivation in case of simple harmonic motion. Simple harmonic motion states that the acceleration of the body is such that you know it is always directed towards the mean position, when it comes to the mean position the acceleration is 0, when it is goes to the you know extreme positions its velocity is 0 but the acceleration is maximum. And you see equal to this one: why is the you known sin negative because it is always directed towards the mean position ok.

So, now comes the question: how did we get this? See if you derive this with respect to d twice you will find that the same thing will come back and you can replace U in that particular place. So, I am not deriving this I am sure you can do it. You can derive it twice and you will find that the same thing its coming back replace U with that you will get minus omega square U. Now how does this omega square come? That is because its sin omega t, it does not look fully like omega. I am sorry about that looks more like w, but its omega. And therefore if you derive it, it will become A omega cos omega t minus B omega sin omega t. Next derivation it will become A omega square sin omega t plus I mean minus A omega square sin omega t minus B omega square cos omega t. And therefore, you have omega square coming out coming from them minus sin coming out common then what is left is the displacement term which is this one only, ok.

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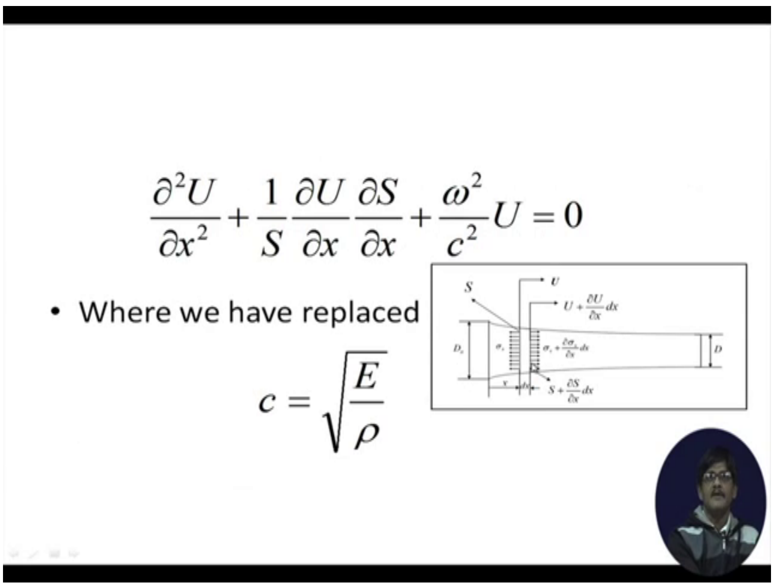

So, the last a t containing term is successfully removed and you have minus omega square u. So, what is the expression look like? The expression looks like this. That means on the first term we have del 2 U del x 2, this was coming from the derivative of sigma remember and then comes the middle term which was unchanged derivatives that we have had to resort to in order to relate displacement and area to the x direction.

That is there their gradients with x with 1 by S. And omega square by c square is coming, because we have made a replacement now as the velocity of sound in the medium of the horn being root over E by rho. So, let us have a quick look at the expression before this. E is here, rho is here so that you will get E by rho here E by rho here. So, 2 E by rho is here, and therefore you can also say that I have 1 by E by rho here.

So, having replaced root over E by rho by c you will find that we will have a c square here. Where does this particular term come from? Remember we had minus omega square U from assumption of simple harmonic motion. So, it comes from there and we have a c squared term coming from this particular substitution. That is good; so far so good. But what do we do with this? We have x we have u, so this is one variable this is another variable; S is yet another variable; what do we do with this? We make now an assumption.

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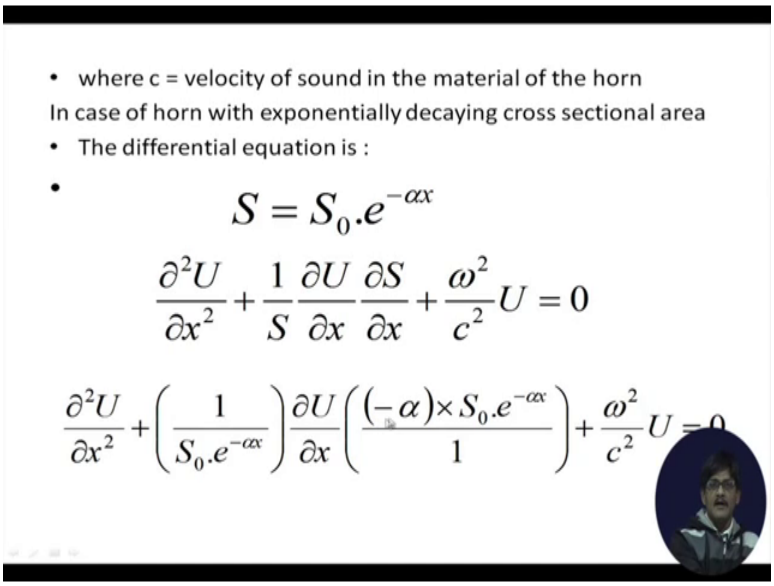

Assumption is this: we say that if the cross section is changing with x in that case let us take the case of a horn with an exponential decaying cross sectional area. So, the cross section itself is coming down, I do not know whether its circular, I do not know whether its rectangular, but simply I know that the cross section is decaying with x by this expression S is equal to S 0 in to E to the power minus alpha x. What is alpha? Alpha is the way in which you know its decaying it is a constant value, ok.

So, one we have made this assumption or once we are considering such a horn we simply replace the value of S by this expression. So, what do we have? We have 1 by S. So, for that in the second term in this place we have replaced S by this expression; S 0 E to the power minus alpha x. So, what is S 0? S 0 is the cross sectional area where it connects up with the transducer. And the transducer end we are having S 0; sorry.

So, when we are deriving del S del x and as if having E to the power minus alpha x E to the power minus alpha x will come. Once again and it has appeared here. So, a minus alpha will be coming out due to due to the derivation. And therefore, we have all the terms unchanged only S value has been put in and it is a good thing that this will completely cancel out from the numerator and the denominator so that we will have let see the final expression of this cancellation; that is it.

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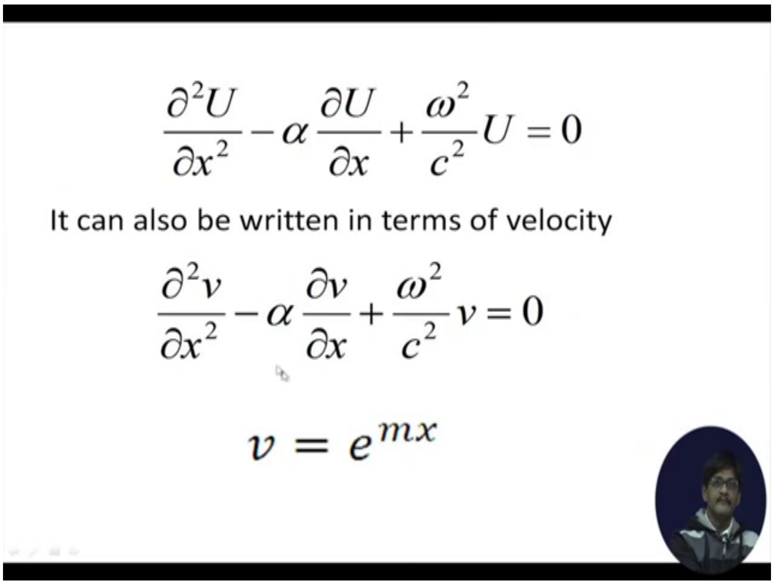

Del 2 U del x 2 minus alpha into del U del x plus omega square by c square into U equal to 0. Now just like its true for a displacement it is also true for the velocity, if we derive this whole thing with respect to time del 2 U del x 2 l; sorry you can get derived inside and give us velocity. Therefore, we can also have this particular expression valid for our discussion.

So, in this case what do you ultimately have? We have differential equation in which the velocity or here the displacement they are related to the distance along x and there are number of other contestants appearing. And if we solve it we will be getting an expression of the displacement along x. What is this displacement for? Due to vibration.

So, in this case there are number of ways in which this particular differential equation can be solved. One way to solve it, one of the easiest conventional ways is just take the dependent variable to be equal to E to the power m x where x is the independent variable, and m is a constant, ok.

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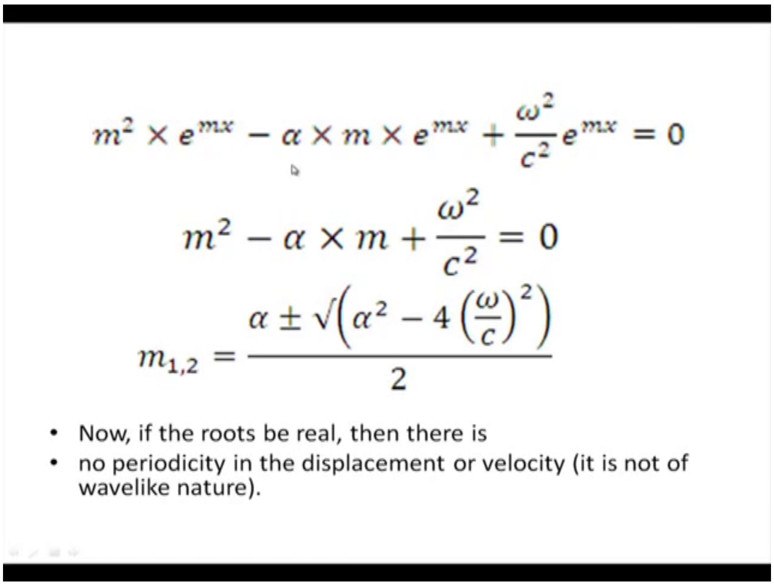

So if you do that, let us just go to the previous expression once again. This was the previous expression. In this case E to the power m x when derived twice it will give us m square and E to the power mx when derived once it will give us m. So, it should be m square into E to the power mx minus alpha m into E to the power mx plus omega square by c square E to the power mx equal to 0.

That is it: m square into E to the power mx minus alpha into m into E to the power m x, because E to the power mx after derivation will give E to the power mx once again multiplied by that constant m into omega square by c square is constant into E (Refer Time: 28:03) equal to 0.

Once we have this expression, if we have to you known solve it E to the power mx can be canceled out now from the expression if it is assume to be nonzero. And after that we have this particular expression: m a quadratic equation in m. If you have this quadratic equation in m we can simply find out its root as minus b plus mine b square minus 4 ac; sorry minus b plus minus root over b square minus 4 a c by twice a. And that is what we have to done.

So, here what we say is that if the roots are real that mean if alpha square is greater or equal to 4 omega square by c square, in that case the root will be real and you know you will not get any wave like motion corresponding to that; ok understood. So, what we do about that? We can add that if however 4 omega square by c square is greater than alpha square then you will have imaginary roots and once you have imaginary roots it will exhibit wave like motion. Now why is this so?

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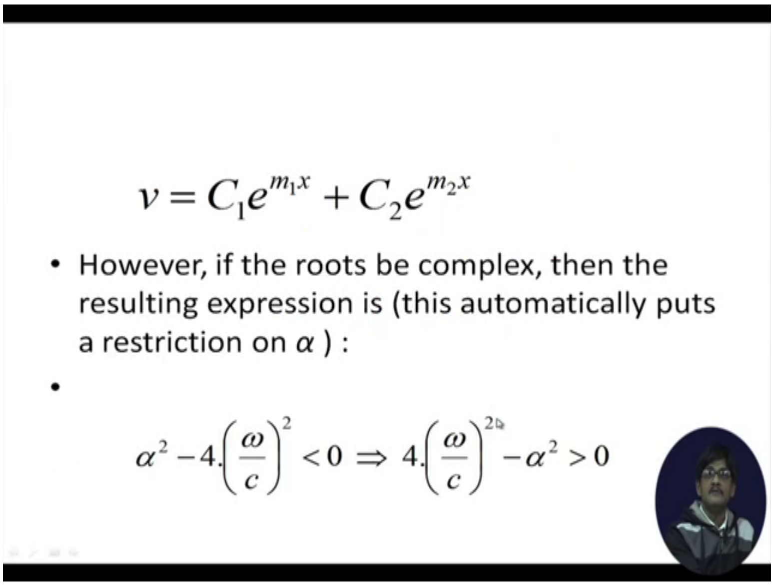

This is because if we are having this sort of a solution, in this sort of a solution if the roots are complex; that means if as we discussed previously four omega square by c square into minus alpha square is greater than 0.

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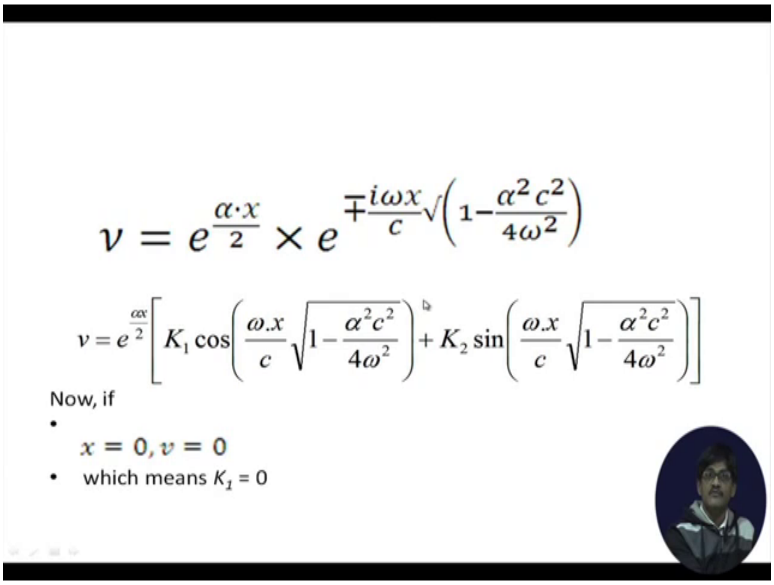

In that case we will have a solution of this type and this one will give rise to sin and cos waves; sin and cos solutions. Imaginary if you remember this, this when expanded will give rise to sin and cos and therefore what we have is this part comes out to be the real parts so it is multiplied as a coefficient to the amplitude, and this part give rise to the wave like motion.

So, so far so good, now we put initial condition. But before that I think our time today is over. So, we will continue this in the next discussion, ok.

Thank you very much.