

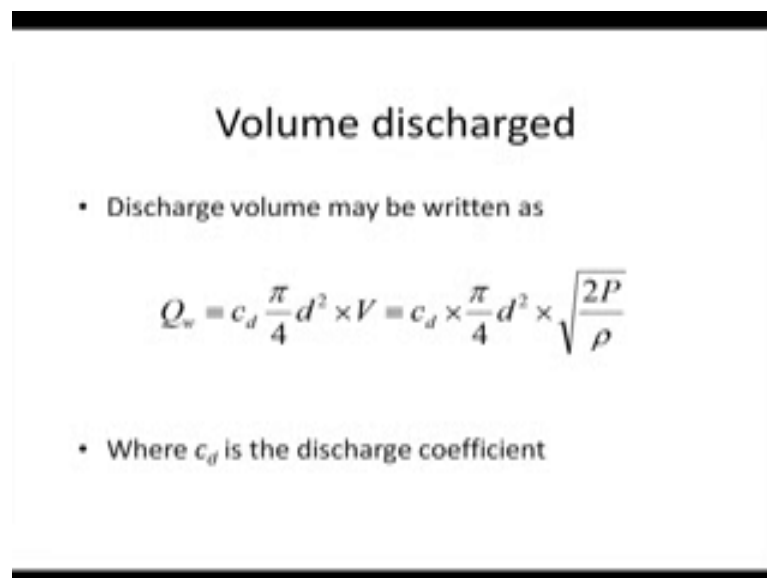
Non-traditional abrasive machining process: Ultrasonic, Abrasive jet and abrasive water jet machining

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**Lecture – 17
AWJM- Equipment**

Welcome viewers to the 17th lecture on Non-traditional abrasive machining processes. And we had started the discussion on water jet machining, and abrasive water jet machining. So, let us start with the discussion right away. So, in abrasive water jet machining we were discussing this part.

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Volume discharged

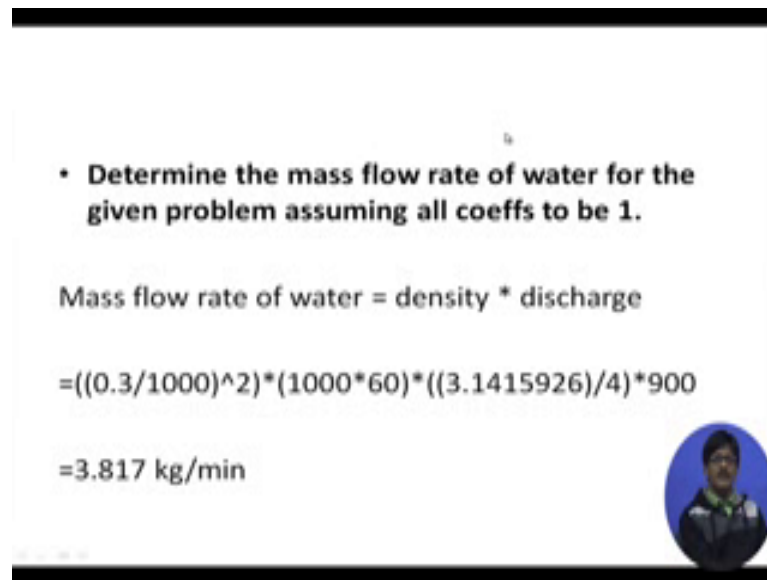
- Discharge volume may be written as

$$Q_v = c_d \frac{\pi}{4} d^2 \times V = c_d \times \frac{\pi}{4} d^2 \times \sqrt{\frac{2P}{\rho}}$$

- Where c_d is the discharge coefficient


This is very simple to understand where is once we have computed the velocity to be around 900 meters per second. If we multiplied with the cross section and the discharge coefficient we will be getting the discharge volume, the amount the volume of material volume of water coming out in 1 second.

(Refer Slide Time: 01:02)



• Determine the mass flow rate of water for the given problem assuming all coeffs to be 1.

Mass flow rate of water = density * discharge

$$= ((0.3/1000)^2) * (1000 * 60) * ((3.1415926)/4) * 900$$
$$= 3.817 \text{ kg/min}$$


So, let see this determine the mass flow mass flow by the way mass flow, rate of water for the given problem assuming all coefficients to be 1. That is good if the coefficients have one we have to worry less [FL]. So, mass flow rate of water is equal to density multiplied by discharge. And discharge calculation we have already done, it is equal to the cross sectional area and the discharge coefficient which is one here into velocity. So, let us search out the terms one by one because I have skipped the expression, but I am sure you can find out yourselves right. This is velocity 900 meters per second. This one is pi by 4. So, pi by 4 a must be a d square somewhere around yes d square shall we quickly check what was the discharge orifice diameter. The outlet diameter of the nozzle not the orifice sorry the outlet diameter of the nozzle yeah, in this case orifice because it is water jet machining in case of water jet machine once the water comes out of the orifice that is it we do not have to further you know, send it through the mixing chamber because there is no mixing chamber to talk about in water jet machining.

So, the outlet diameter here is 0.3 millimeters that is good. So, 0.3 divided by thousand converse it to meters, square of that. So, pi by 4 into d square multiplied by the density. Now what is the 60 doing here? 60 is you know 900 meters per second is being converted to 900 meters per minute why so because probably we have to give the mass flow rate of water in terms of kg per minute. So, this calculation I have made it in excel we can directly copy and paste once you get hold of the ppt file. And it is going to give you a result of 3.817 kgs per minute. So, what to do we have here it means that the water


jet water jet equipment is going to consume water at the rate of 3.817 kgs per minute. Now is that an issue? It can be an issue and I have set and numerical problem later on way in which will find that you know the use of water is restricted for a company due to severe drought, and it has to economize and accordingly it has to decide what sort of parameters it has to set in order to circumvent that problem. So, this is calculation of mass flow rate of water from values of velocity discharge coefficient orifice diameter and density.

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Power

- Hence, the power carried by the jet would be

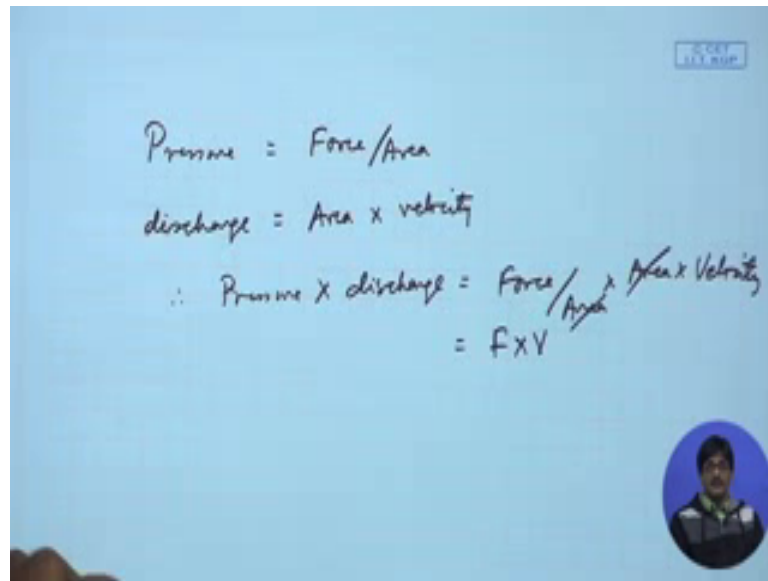
$$P_w \times Q_w = P_w \times \frac{\pi}{4} d^2 \times V = P_w^{\frac{3}{2}} \times c_d \times \frac{\pi}{4} d^2 \times \sqrt{\frac{2}{\rho}}$$



Now power, how much is the power carried by the jet?

So, power is equal to pressure into discharge. Now why should you might only ask me, why should power we go to pressure into discharge? This is because the power is forced by unit cross sectional sorry pressure is equal to power sorry let me write it down.

(Refer Slide Time: 05:12)


$$\begin{aligned}\text{Pressure} &= \text{Force} / \text{Area} \\ \text{discharge} &= \text{Area} \times \text{velocity} \\ \therefore \text{Pressure} \times \text{discharge} &= \text{Force} / \text{Area} \times \text{Area} \times \text{Velocity} \\ &= F \times V\end{aligned}$$

Pressure equal to force by area and discharge equal to area of cross section area into velocity. Therefore, pressure into discharge becomes equal to force divided by area into area into velocity and therefore obviously, it is equal to force into velocity, which we know to be power force into displacement is energy and force into velocity is power.


So, this is how coming back to our discussion, this is how we can compute or calculate the power. Pressure into discharge giving pressure multiplied by cross sectional area into velocity and ultimately this gives one you know pressure to the power half from the velocity. And we put it here So that we get pressure to the power 3 by 2 then comes discharge coefficient then comes the you know cross sectional area, and what is what is remaining of that velocity term root over 2 p by rho has been taken out. So, we have to by 2. So, this way we can calculate the power carried by a jet.

(Refer Slide Time: 07:00)

MRR is proportional to the power of the jet

$$MRR \propto P_{wj} \propto c_d \times \frac{\pi}{4} d_0^2 \sqrt{\frac{2p_w^3}{\rho_w}}$$
$$MRR = \left(\frac{1}{S_{job}} \right) \times c_d \times \frac{\pi}{4} d_0^2 \sqrt{\frac{2p_w^3}{\rho_w}}$$

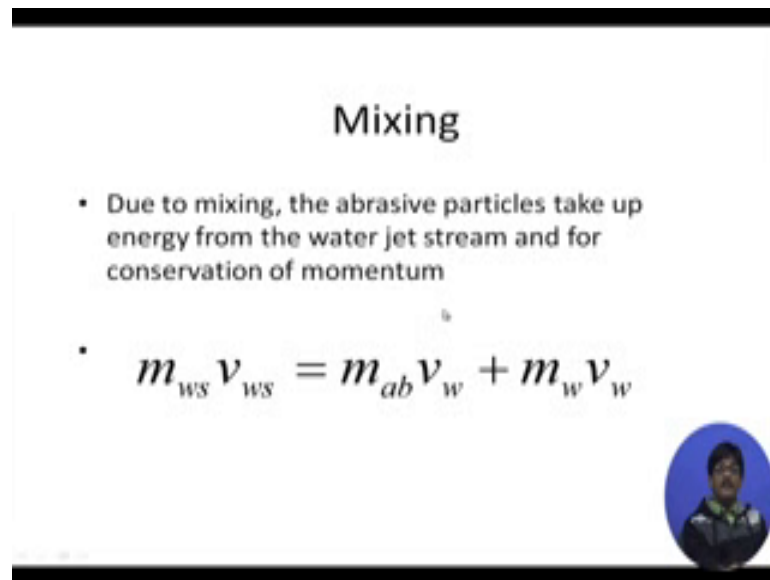
- S_{job} is the specific energy requirement for cutting of the work material by AWJM



Now, we are here stating sort of making a statement that material removal rate is proportional to the power of the jet. Material removal rate is proportional to the power of the jet. And we have the constant equal to 1 by specific energy requirement for cutting the work material by abrasive water jet machining.

So, we have found out the expression for power just now and I have simply reproduced it. MRR is proportional to the power of the water jet. And that is proportional to you know all these terms that we have come across previously. And ultimately e to the power I mean pressure to the power 3 root over divided by rho; that means, density root over etcetera. All these things we have discussed you know derived just now and therefore, once we remove the sin of proportionality we bring a constant 1 by specific energy requirement of the for cutting the work material by abrasive water jet machining. And therefore, therefore, this one is now characteristic of the particular material which is being cut. If it requires lot of specific energy MRR will be removed reduced and vice versa.

(Refer Slide Time: 08:45)



Mixing

- Due to mixing, the abrasive particles take up energy from the water jet stream and for conservation of momentum

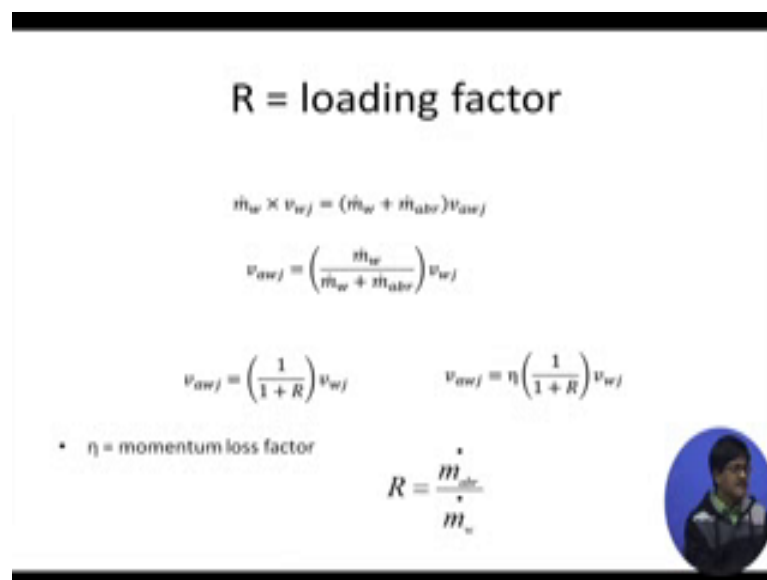
$$m_{ws} v_{ws} = m_{ab} v_w + m_w v_w$$

Now mixing, we have discussed the aspect of mixing of abrasives with water. Abrasives being at you know in the static condition and water coming in a very high velocity and we have mixing. So, it is a sort of you know collision. In this collision if it had been as a perfectly elastic collision they would have been no lots of energy, but; obviously, it is not because after sometime you know when it is coming out of the mixing chamber they are sticking to each other; that means, that they are having the same velocity, they take up the same velocity after the collision. And therefore, this is inelastic collision and therefore, even though momentum will be conserved some energy will be lost, we cannot really avoid that.

So, in this inelastic collision we are having only conservation of momentum. So, we have to first see in the control volume of the mixing chamber, what is the momentum being brought in by the liquid? And what is the momentum which the abrasives are gaining from the liquid? And ultimately what if we make the balance? What is the final velocity of the combined jet? Let us have a look at this. What have been written here? We are saying the mass of you know water m_{ws} means only the water coming in, with a mass flow of \dot{m}_w I do not really understand why I have written m_{ws} it actually differentiates the, what is stream? It means water streams mass of the water stream mass flow rate of the water stream, into the velocity of the water stream this is the momentum which comes in. Once it comes in with this particular momentum it gives it shares this momentum with the abrasives.

So, when the abrasives are finally, coming out carried in the water stream at the same velocity I mean at the focusing tube only some momentum transfer takes place. So, at the end of that when they are coming out of this control volume of mixing chamber plus focusing tube. The mass of abrasive which has been you know accelerated to the final velocity v_w . And the water is decelerated somewhat, because it has to give up some of its energy to the abrasives and some energy is lost. So, mass of the water with the final velocity of the water this product, this gives us the right hand side gives us the final total momentum. So, if momentum is to be conserved we have initial velocity of the water stream multiplied by the mass flow rate of water equal to mass flow rate of abrasives multiplied by the final velocity plus mass flow rate of water multiplied by the final velocity.

(Refer Slide Time: 12:21)



R = loading factor

$$\dot{m}_w \times v_{wj} = (\dot{m}_w + \dot{m}_{abr}) v_{awj}$$

$$v_{awj} = \left(\frac{\dot{m}_w}{\dot{m}_w + \dot{m}_{abr}} \right) v_{wj}$$

$$v_{awj} = \left(\frac{1}{1 + R} \right) v_{wj} \qquad v_{awj} = \eta \left(\frac{1}{1 + R} \right) v_{wj}$$

- η = momentum loss factor

$$R = \frac{\dot{m}_{abr}}{\dot{m}_w}$$

So, after this let see what happens? So, naturally we have I have use slightly different terminology here, but I think it is more you know more easy to understand. So, mass flow rate of water multiplied by mass flow rate of water jet is equal to mass flow rate of water plus mass flow rate of abrasive multiplied by abrasive water jet velocity. So, w a w j means abrasive water jet velocity. So, it is very easy to understand. Now you are simply taken it common So that abrasive water jet velocity can be written as you know just by rearranging the terms mass flow rate of water divided by mass flow rate of water plus mass flow rate of abrasive, multiplied by mass multiplied by water jet velocity; that means, the initial velocity with which the water entered the mixing chamber.


Therefore, we bring in one term called loading factor which is simply equal to mass flow rate of abrasive by mass flow rate of water and write re write the expression as, velocity of abrasive water jet is equal to 1 by 1 plus R multiplied by velocity of the water jet. Abrasive water jet velocity is equal to 1 by 1 plus R of initial water jet velocity. And there is also a particular term brought into account for any momentum losses that might be occurring ok.

So, ultimately we have this expression existing between abrasive water jet velocity and water jet velocity.

(Refer Slide Time: 14:13)

Power of the abrasive phase of the jet

$$P_{abr} = \frac{1}{2} \times m_{abr} \times v_{awj}^2$$

$$P_{abr} = C_d \frac{\pi}{8} \times d_0^3 \rho_w \times R \times \left(\frac{\eta}{1+R} \right)^2 \left(\frac{2p_w}{\rho_w} \right)^{3/2}$$


So, power of the abrasive phase of the jet. If you kindly remember previously we had discussed about the power which is contained in the water jet, water jet and the one which did not contain the abrasives. And now we are we are talking about the power which is contained by the abrasive phase of the jet only. And ultimately you will find we will be able to relate the power of the abrasive phase of the jet to the power of the water jet. We can show that this is it is proportional, if it is proportional. Then we can say since MRR is proportional to the power of the water jet. So, if power of the abrasive phase is proportional to the power of the water jet, then MRR will be proportional to the abrasive phase power. MRR will be taken as proportional to the power of the abrasive phase of the jet. So, let us have a look what we are talking about.

First of all the power of the abrasive phase is equal to half $m \dot{b}$ square. Half $m \dot{b}$ square is the energy and since the mass is not mass, but it is mass flow rate therefore, it expresses not energy, but power. So, we have mass flow rate of abrasives. We have velocity of the abrasive jet square of that. So, let see what it ultimately boils down to.

So, what I have done is I have written down the final expression. And let us quickly have a look whether we can derive the essential parts of this So that we can make this particular statement. First of all let us have a look at mass flow rate, what have we derived mass flow rate of abrasives. If I remember correctly this was equal to 1 by 1 plus R was it?

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$$\frac{\dot{m}_{abr}}{\dot{m}_w} = \frac{1}{1+R} = R = \frac{\dot{m}_{abr}}{\dot{m}_w}$$

$$\boxed{\dot{m}_{abr} = R \cdot \dot{m}_w}$$

$$V_{abr} = \frac{1}{1+R} \eta \cdot V_{wj}$$

$$\text{Power of abr. jet} = \frac{1}{2} \times R \cdot \dot{m}_w \times \left(\frac{V}{1+R} \right)^2 \cdot V_{wj}^2$$

$$= \frac{1}{2} \times R \left(\frac{\eta}{1+R} \right)^2 \times C_d \times \frac{\pi}{4} d^2 \times V_{wj} \times V_{wj}^2$$

Just a moment no, I am sorry this was equal to we can write this way by mass flow rate of water, was equal to R. Let me write it on this side \dot{m}_{abr} divided by \dot{m}_w .

So, we can say we can replace \dot{m}_{abr} as R into \dot{m}_w this is the replacement that we can make. What else is there? Velocity of abrasive water jet this was equal to 1 by 1 plus R into eta into velocity of water jet. This we have derived just a few minutes back. Therefore, we can say power of abrasive jet was equal to half \dot{m}_w which means R into \dot{m}_w multiplied by 1 plus. So now, we are writing v square 1 plus R eta whole square into velocity square v_{wj} all right. This one is this one can be you know written this way half into R into eta by 1 plus R whole square multiplied by now comes \dot{m}_w , what is \dot{m}_w ? Is the mass flow rate of water.

So, mass flow rate of water can easily be replaced by you know C_d into π by 4 into d square into what is that yeah, velocity of water jet multiplied by sorry, velocity water jet. Multiplied by velocity square of water jet that is it. So, we can quickly see that we have reached this particular expression. We have velocity to the power 3. Velocity to the power 3 is going sorry, we have volume into density it is it is mass. So, we have to have a density term into just a moment I will into ρw . So obviously, we are going to have R into $1 + R$ whole square into etcetera, etcetera, etcetera. So, ρw and v cube this is what is our worry now.


So, let us quickly reproduce this. And we are going to have you know power of abrasive equal to half into π by 4 into d 0 square into C_d into R into what you call it efficiency not efficiency really η square into velocity cube. So, velocity cube is essentially going to give us you know $2 p$ by ρ to the power 3, multiplied by ρw that is it. So now, we can see that we are going to definitely have $2 p$ by 2, because it is velocity cube. And velocity square is equal to p therefore, after you know simplification, let us quickly now reversed back. So, we have going to have π by 8, we have going to have d 0 square c_d η square R into $1 + R$ whole square into $2 p$ to the power 3 by 2 divided by ρ is here once and ρ is here once and 3 by 2. So, it is 1 by ρ to the power half.

So, let us look at the expression now yeah, almost everything is in place π by η has been predicted c_d into d 0 square into ρw this ρw where this ρw will ultimately produced ρ 1 by ρw to the power half and η by $1 + R$ whole square into R into $2 p w$ etcetera. So, everything is matching. So, we find that the power of the abrasive phase can be written to be exactly proportional to the power of the liquid phase water. Water only, how can we say that? Let us now go back to the expression of water phase power. Did we derive it? Power carried by the jet.

See the power carried by the jet is having the very same expression, c_d into π by 4 into d square into p to the power 3 by 2 which we are getting and ρ to the power 1 by ρ to the power half. So, the power of the abrasive phase is simply this power multiplied by a constant. And hence we come to the conclusion that the power of the abrasive phase of the jet is a proportional to the power of the water jet and therefore, mass material removal rate can be expressed, material removal rate can be expressed to be proportional to the power of the abrasive phase itself. Material removal rate is proportional to the power carried by the abrasive phase.

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- If the mass flow rate of abrasive is 1 kg/min, determine the abrasive water jet velocity assuming that momentum is conserved during mixing process
- Abrasive jet velocity = $(1/(1+(1/3.817))) \times 900$ (this comes from momentum balance)
- Ans 713.16 m/s
- Can you get the same outlet velocity by using water from the tap at the ground floor of Burj Khalifa (tallest building in the world, 828 meters), with tank being on the topmost floor and no losses?




So, since we have a little time I let us have a taste of the typical problems that we might be facing here, and watch this if the mass flow rate of abrasive is 1 kg per minute. What is this? If mass flow rate of abrasive is 1 kg per minute, determine the abrasive water jet velocity assuming that momentum is conserved during the mixing process. So, let see what it says all right. Mass flow rate of abrasive is given and from the previous problem we can say that we have already found out the mass flow rate of water to be 3.817 kgs per minute. And the velocity of the water jet was found out to be 900. So, let me write it down quickly on this piece of paper.

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$$P_{\text{out}_{abr}} = \frac{1}{2} \times \frac{\pi}{4} \cdot d_o^2 \times C_d \times R \cdot \left(\frac{V}{1+R}\right)^2 \times \left(\frac{2\gamma}{\rho}\right)^{3/2} \times \rho$$

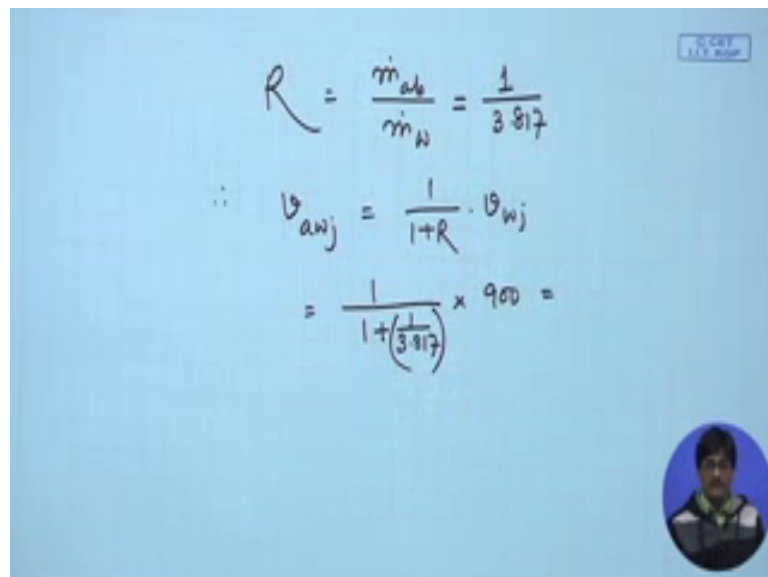
We know

$$V_{wj} = 900 \text{ m/s}$$
$$\dot{m}_{wj} = 3.817 \text{ kg/min}$$
$$\dot{m}_{abr} = 1 \text{ kg/min}$$


This is what we start with, we know that velocity of water jet is equal to 900 meters per second. And water jet mass flow rate, mass flow rate of water jet was equal to 3.817, 3.817 kgs per minute ok.

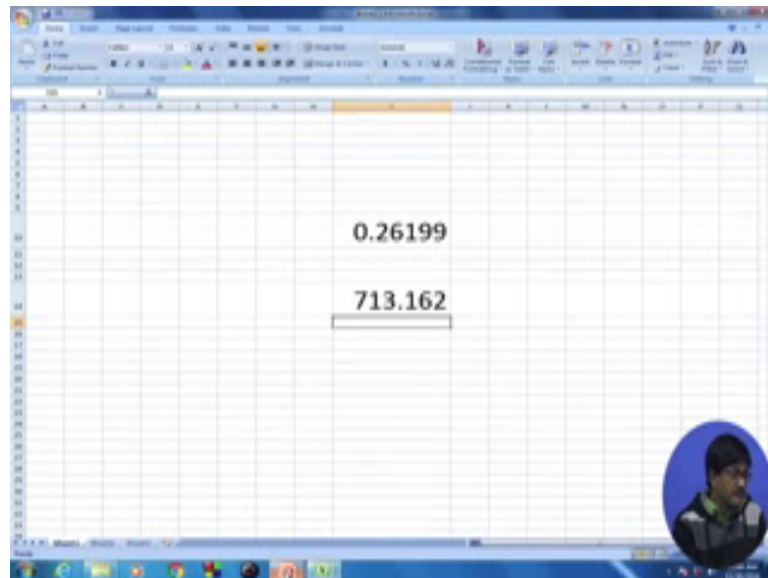
And the mass flow rate of abrasive, abrasive was being stated to be 1 kg per minute. In that case what will be the final velocity? Do we have to change velocity to meters per minute in that case? We could, but I think the units will be cancelling out anyway because you know the R value has no unit. The value has no unit. Because here what we are going to say is that find out the R value.

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$$R = \frac{\dot{m}_{ab}}{\dot{m}_w} = \frac{1}{3.817}$$
$$\therefore v_{awj} = \frac{1}{1+R} \cdot v_{wj}$$
$$= \frac{1}{1 + \left(\frac{1}{3.817}\right)} \times 900 =$$

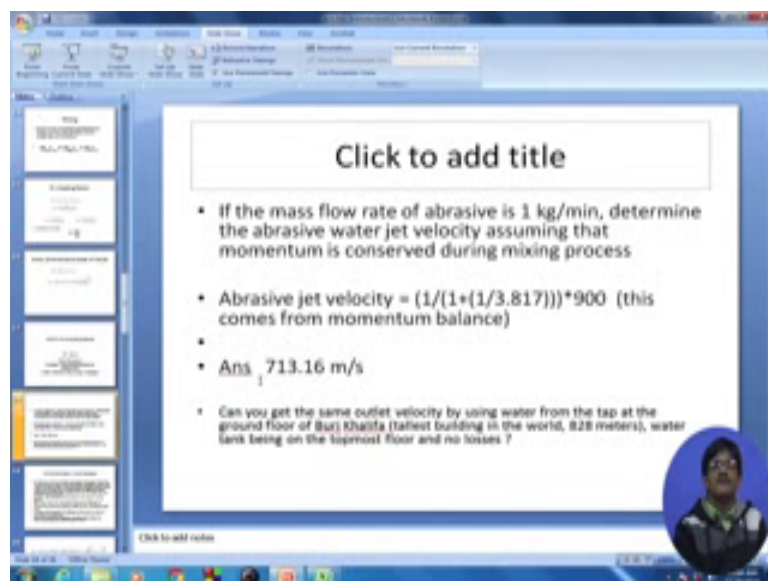
It is equal to mass flow rate of abrasive, by mass flow rate of water and that is equal to 1 divided by 3.817 and therefore, velocity of the abrasive phase sorry, velocity of the abrasive water jet must be equal to 1 by 1 plus R into velocity of water jet. So, this is equal to 1 by 1 plus that is it yeah.

(Refer Slide Time: 26:52)



So, this is equal to sorry, very small in font right equal to 1 by 3.817, this is the value of sorry what is that? Equal to 1 by 3.817. How is that possible? Remind me am I making some mistake, no that is it 0.26199. So, this is coming out to be the value of R. And hence let us do the next part of it equal to, why do not we have large font for all of them. Let us do it. Anyway I cannot increase the font let me try that is good equal to 1 divided by 1 plus was this one, I 10, I 10 multiplied by 900. 713 it is too small how do I increase it let see right. This is equal to 713 by a 0.162 and let us written to the yes, 713.16.

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So, this way we can find out the final abrasive water jet velocity from the initial velocity. So, with this let us draw an end to the 17th lecture. In the 18th lecture we will take harder problems, harder numerical problems so that we become conversant with the working of the machine and its process capability.

Thank you very much.