

Non-traditional abrasive machining process: Ultrasonic, Abrasive jet and abrasive water jet machining

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Lecture – 12 AJM- Numerical Problems

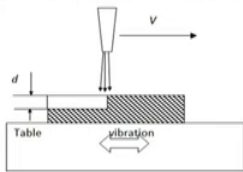
Welcome viewers to the 12th lecture of the online course Non-traditional Abrasive Machining Methods, Ultrasonic Machining Abrasive Jet Machining and Abrasive Water Jet Machining. So, today we start with I mean we continue with some discussion on numerical problems that might be you know solved on the basis of a material removal rates in abrasive jet machining. So, let us start right away with one of the problems. We on the previous day, we had a solved one problem on material removal rate where d by Δd term was being experimentally determined.

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$v = k/d$

Numerical problems – depth of groove

- You are working in a manufacturing concern which cuts grooves in metallic bodies (see fig). The **groove depth (d)** specification is **4 mm with a tolerance of ± 0.03 mm**. Groove depth and velocity of cut are related as
 - $d = k/V$
- At present – you are employing a grooving velocity $V = 10$ m/min to obtain an exact depth of $d = 4.02$ mm. However, due to the running of another machine nearby – a vibration is introduced into the table as shown with displacement where $A = \text{amplitude} = 0.03$ mm, angular velocity = 50π rad/s and t is in seconds. If you still go on cutting the grooves, will they be accepted? why?



So, this problem deals with you are working in a manufacturing concern which cuts grooves in metallic bodies. So, there is a metallic body and you are cutting a groove. It is a brittle metallised material.

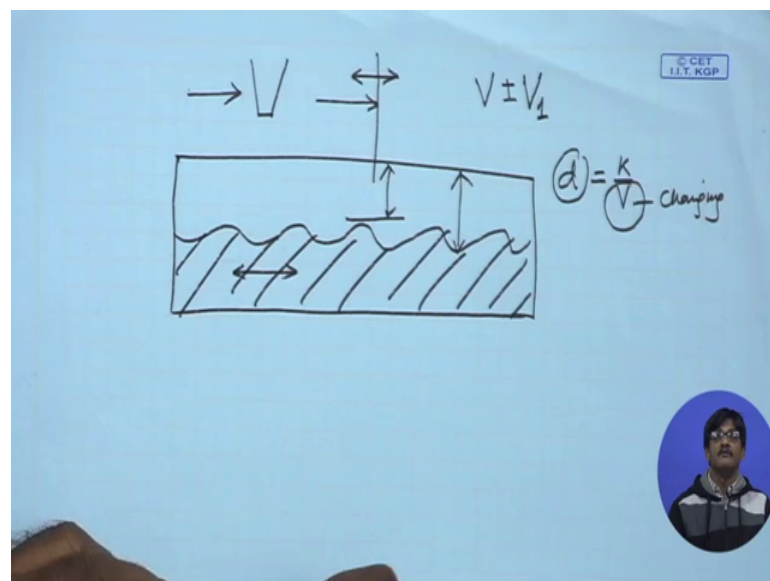
What is stated is that the groove depth d as shown in the figure, the d is specified as 4 millimeter deep with tolerance of 0.03 millimeters plus minus. So, it can have a maximum depth of 4.03 or it can have a minimum depth of 3.97. These are the extreme

values of the groove depth and it is also provided that groove depth and velocity of grooving, they are related as d equal to k by V . v is the velocity as shown in the figure and d is a constant and d is the groove depth. It is being assumed that the groove width which is perpendicular to the plane of the paper that is not affected, neither does it effect this particular relationship.

Also, we are not really concerned about any change that might take place in the groove width due to change in velocity. It is simply being assumed as constant. At present you are employing a grooving velocity of V equal to 10 meters per minute. So, that means I am grooving now with the velocity of 10 meters per minute and I am obtaining an exact depth of groove equal to 4.02 millimeters. So, you are accepted because it is running within the range 3.97 to 4.03, however due to running of another machine nearby, a vibration is introduced into the table as shown with displacement. Just a moment; this expression of displacement has not coming. We can consider this way that it has this particular vibration have an amplitude of 0.03 millimeters and with an angular velocity of 50π radians per second.

So, if you still go on cutting the grooves, will they be accepted? So, what do we have here? The amplitude is given to be 30 microns and the angular velocity is 50π radians per second. So, in that case if there is a vibration introduced, will the cuts be accepted? The grooves will still be accepted.

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Now, what is going to happen here is that if you now look at this particular piece of paper, so if this is you know say top surface of the metal piece and I am having a grooving velocity of this type, but if there is a vibration setting on the table while the grooving element or the nozzle that is still at the constant velocity. Therefore, this velocity will have a reciprocating velocity impressed upon it. So, it will be V plus minus V_1 coming from the vibration.


So, if it is of considerable amount, we will find that velocity will be varying and due to this changing velocity as groove depth is equal to k by V as this is now changing. We will thus have a change in groove depth. Groove depth will be like this. So, we need to find out what is the maximum and minimum groove depth. This is the minimum and this is the maximum whether they have within limits. So, it is as easy as that.

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- **First – let us see the value of k**
- $\frac{4.02}{1000} = 60 \times k / 10 \Rightarrow k = \frac{4.02}{6000} = 0.0067$
-
- Now the highest and lowest velocities are $= V_{\text{cut}} \pm V_{\text{vibr}} = 10/60 \pm A \times \omega$
- $= 0.1713 \text{ m/s} \text{ \& } 0.1619 \text{ m/s}$
- Corresponding depths are

$$d_1 = \frac{0.0067}{0.1713} = 3.91 \text{ mm} \text{ and } d_2 = \frac{0.0067}{0.1619} = 4.13 \text{ mm}$$

- **So – it is not acceptable**



So, let us have a look at the answer. So, what we know is let us find out the value of k . If you know depth of groove is equal to k divided by the velocity, we can say that depth of groove 4.02 millimeter, 4.02 millimeters divided by 1000 to make it meters is equal to 60 into k by 10. Why this 60? This must be 10 meters per minute. If 10 meters per minute is the velocity, 10 by 60 is the meters per second. Therefore, k comes out to be 4.02. This 0 and this 10 cancels out. So, 6000 at the denominator, k is equal to 4.02 by 6000. It will be 0.0067. That is understood. k has been found out.

So, now we try to find out the highest and the lowest velocities. Highest and the lowest velocities will be equal to velocity of grooving which is still 10 meters per minute and also, the vibrational velocity which is introduced first and you know the maximum and minimum values are once it is subtracted and another time it is added. They will define the minimum and the maximum velocities, so that velocity of grooving is 10 by 60 meters per second plus minus $A\omega$. If the viewers kindly recall, $A\omega$ gives us in simple harmonic motion velocity, ok.

The maximum velocity at the mean position, where A is the amplitude and ω is the angular velocity and therefore, once we put in the values of ω and A which are given in the problem, we will obtain that the maximum and the minimum velocities will be coming out as 0.1713 meters per second and 0.1619 meters per second and the corresponding depths can be found out by putting their values in d is equal to k by V , sorry d in d is equal to k by V . So, d_1 comes out to be 3.91 and d_2 comes out to be 4.13 millimeters. Therefore, obviously it is not within the range 3.97 to 4.03 that is not acceptable. So, it is not acceptable.

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- Numerical problem – machine performance index
- In a company carrying out AJM of brittle materials, one machine is working slower than predicted and its performance needs to be checked. The company defines an index of performance = **Volume (cubic meter) of work material removed per kg of abrasive** and starts monitoring it over time for that AJM machine.
- Utilizing this index, find from the table below **whether performance of the machine is improving / deteriorating with time.**

Next, now we take up a problem on performance index estimation; machine performance index in a company carrying out AJM of brittle materials. One machine is working slower than predicted. I should say apparently working slower than predicted and its

performance needs to be checked. So, what is the problem? Somebody has become doubtful about the performance of a particular AJM machine and you know in the comparative market if you are lagging behind, then you might be eliminated or be hunted. So, the company needs to check the performance of the machine. The company defines an index of performance. So, I will check this particular index and I will decide accordingly whether the machine is working all right or not.

The company defines a performance and index of performance equal to volume of work material removed per kg of abrasive volume in cubic meters of work material removed per kg of abrasive. So, it is quite you know easy to understand. I am removing this much of amount material, but I am having to spend this much amount of abrasive for that and this I am defining as my performance index I will check that if it is going down, it is bad. If it is going up, it is good and starts monitoring it over time for that AJM machining utilizing. This index from the table below shows whether the performance of machine is improving or deteriorating with time.

So, what does that mean that the machine must have been checked overtime for a number of occasions and this index must have found out and from that or rather you have to find out this index and decide whether the machine is working, you know whether it is working slower than predicted or not etcetera.

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The table

Sl. No	Date of expt	Abrasive grits			Work pc		MRR m^3/s
		g/min	Jet vel v_j m/s	density $\rho_g \text{ kg/m}^3$	density kg/m^3	Hardness $H, \text{ GPa}$	
1	02-06-09	2	200	3000	3000	4	7×10^{-10}
2	02-06-10	2.5	175	3500			7×10^{-10}
3	02-06-11	1.9	160	3250			5×10^{-10}

So, let us see what is provided that is quite interesting. So, first of all date of experiment. Date of experiments has that in 2009, there was one experiment exactly one year after that. Let us not go into the possibilities of a leap year occurring just in year. So, date of experiment 2609, date of experiment 2610, that means 2010 and date of experiment 26. So, time by this is absolutely no problem here. We are measuring the performance after one year and we have taken three measurements. So, that is easy to understand.

Abrasive grits they are spent you know the abrasive grits have been used up at the rate 2 grams per minute. You know first case 2.5 grams per minute and 1.9 grams per minute whom I say why is there this discrepancy? Why do not we have 2 grams per minute flat that is because for different types of operations that might have been required. The machine had to you know use of abrasives at different rates.

So, you cannot really help that. Just we have to accept different rates of expenditure of abrasives had to be you know had to be there in order to meet with different requirements. Jet velocity was also different; 200, 175 and 160 as we discussed for different requirements that might have been like being there.

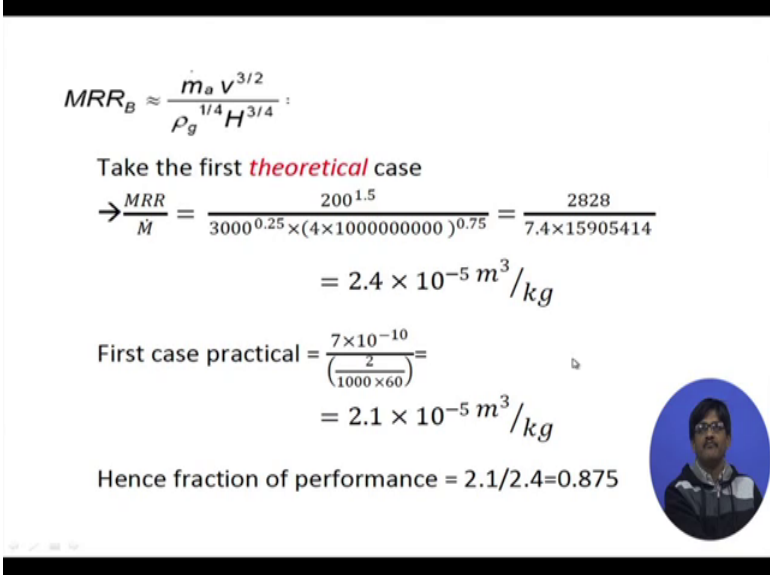
Density of abrasives also was different, 3000, 3500 3250. So, these things were different for the different years and work piece density has been given to be 3000 and hardness to be given as 4 GPA and material removal rates are 7 into 10 to the power minus 10 meter cube per second, 7 into 10 to the power minus 10 once again and 5 into hopefully 10 to the power minus 10.

So, apparently you know of hand a person examining this table might come to the conclusion that material removal rate is coming down, but here we should remember one thing that the main thing you notice is of primary importance that is the input conditions are not the same. We cannot compare these mayor values just like that flat comparison is not possible because the inputs are not the same. So, best is to check up what is the predicted MRR in all these cases separately and compare it with the experimentally obtained values in the last column.

So, let me make it clear. All the abrasive grit details, they are input conditions work piece details, input conditions while MRR at the last column is experimentally obtained. Let me just emphasize on this. These are experimentally obtained values and these are all input conditions. So, best thing would be from the input conditions, find out the

predicted output and compare it with these experimentally obtained values. That is it. That will be best check instead of flat comparison between these values that would be wrong in fact.

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$$MRR_B \approx \frac{m_a v^{3/2}}{\rho_g^{1/4} H^{3/4}}$$

Take the first *theoretical* case

$$\rightarrow \frac{MRR}{\dot{M}} = \frac{200^{1.5}}{3000^{0.25} \times (4 \times 1000000000)^{0.75}} = \frac{2828}{7.4 \times 15905414}$$

$$= 2.4 \times 10^{-5} \text{ m}^3/\text{kg}$$

First case practical = $\frac{7 \times 10^{-10}}{\left(\frac{2}{1000 \times 60}\right)} =$

$$= 2.1 \times 10^{-5} \text{ m}^3/\text{kg}$$

Hence fraction of performance = $2.1/2.4 = 0.875$

Let us have a look how this is done. We know that MRR of a brittle material is proportional to or rather we can also say it is equal to 1.04 into m dot a. M dot a means you know abrasive mass flow rate multiplied by velocity of the gas jet to the power 3 by 2 divided by density of the abrasives to the power on fourth and hardness of the work piece to the power three-fourth. Now, if this is MRR, the performance index would divide the MRR by the mass flow rate of abrasive.

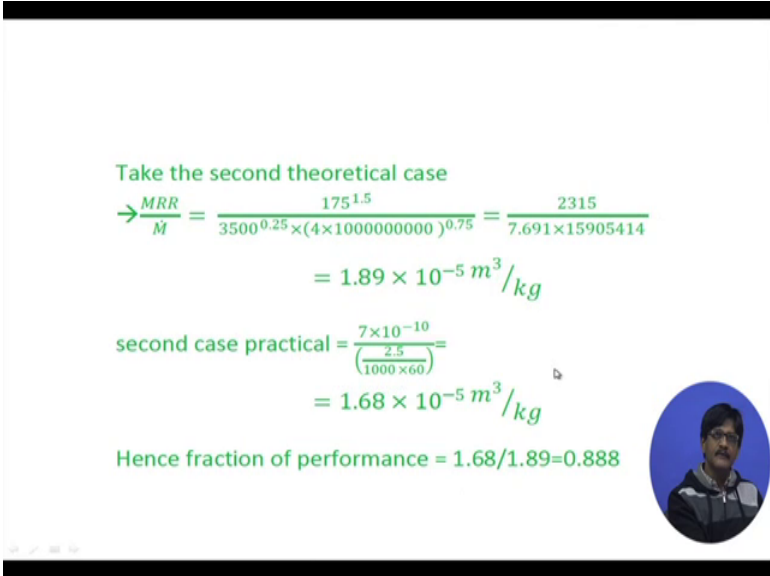
So, simply mass flow rate of abrasive will vanish and it will be v to the power three by two divide V by divided by rho g to the power 1, 4 into H to the power three-fourth and therefore, we have put in the values. Let us see the first theoretically predicted value of the performance index is how much is that 200 to the power 200 is the gadget velocity to the power 1.5 divided by rho g 3000 density of the abrasive grit, 3000 to the power 0.25 multiplied by the hardness of the work piece 4 GPA. So, that converted to you know Pascal's multiplied by to the powers 0.75 and all sorts of number crunching gives us 2.4 into 10 to the power minus 5 meter cube per kg. That is understood 2.4 into 10 to the power minus 5 meter cube per kg of abrasive.

Now, let us see the experimentally obtained value. What is the experimentally obtained value? It is the actual experimentally obtained value MRR divided by the consumption of abrasives. So, that value was 7×10^{-10} divided by 2 into you know divided by the consumption of abrasives. Let us quickly have a look by going back consumption of abrasives. Consumption of abrasive was 2 grams per minute. So, this has to be converted to 2 divided by 1000 into 60. 60 for you know second and 1000 for changing grams to kgs.

So, all this multiplication leads to 2.1. It is 10^{-5} meter cubes per kg. So, fraction of performance which means that the index which has been found out theoretically is higher than the index which has been found out experimentally; I mean the experimental performance is not as good as the theoretical one. That means the predicted one and therefore, it is working at 0.875 fraction of it is predicted you know capacity 0.875.

Let us remember this particular fraction.

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Take the second theoretical case

$$\rightarrow \frac{MRR}{M} = \frac{175^{1.5}}{3500^{0.25} \times (4 \times 1000000000)^{0.75}} = \frac{2315}{7.691 \times 15905414}$$

$$= 1.89 \times 10^{-5} \text{ m}^3/\text{kg}$$

second case practical = $\frac{7 \times 10^{-10}}{\left(\frac{2.5}{1000 \times 60}\right)} =$

$$= 1.68 \times 10^{-5} \text{ m}^3/\text{kg}$$

Hence fraction of performance = $1.68/1.89 = 0.888$

Now, let us take the second case. Second case we have to carefully see whether we have introduced or incorporated the changes which have occurred due to different input conditions. So, let us remember we are taking velocity to be 175 density of the abrasive to be 3500 and work piece hardness remains the same. Let us have a quick look, right. If the velocity has been taken to be 175, the abrasive grits are you know consumption is at

2.5. Is that important? No. That is not important, but the density of the abrasives is 3500. This is important and 2.5 will be occurring when we find out the experimentally obtained value. It should be $7 \text{ into } 10 \text{ to the power minus } 10 \text{ divided by } 2.5 \text{ by } 1000 \text{ by } 60$, ok.

Let us have a look now. So, the green one is a second case. So, in the second velocity is coming correctly 175, 3500 for GPA and all this number crunching gives us finally the value $1.89 \text{ into } 10 \text{ to the power minus } 5$. So, this is the theoretically predicted value of the performance index. When we find out the performance index as per the experimentally obtained values, $7 \text{ into } 10 \text{ to the power minus } 10$ and 2.5 as we have rechecked from the table, this divided by this gives us $1.68 \text{ into } 10 \text{ to the power minus } 5$. So, here you can observe that compared to the previous value, the experimentally obtained value is still less, but if we take the fraction of performance, it is higher than the previous one.


So, surprisingly the machine appears to have improved in its performance in the second year as per the value of the performance index defined. What about the third year that I have left for you to find out? I did find it out and it appeared that it went down again. So, now that the idea is clear to you, I am sure that you can work it out yourselves and come to a solution and a decision that yes whether the machine is performing all right or not, ok.

So, I hope that this is clear and you did understand and enjoyed the problem.

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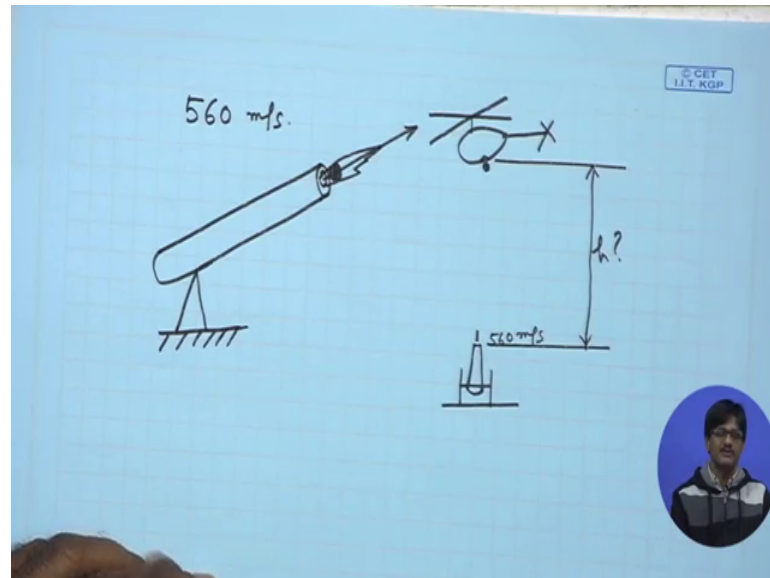
Applications of AJM principle for material removal

Your neighbouring country has purchased an anti-aircraft gun with a muzzle output velocity of 560 m/s and the projectiles are of spherical size and made of solid iron with density 7.6 g/cc. Your army helicopters are being targeted and the defence department asks you to determine a safe flying height at which the indentation damage depth δ can be a maximum of 5 % of the projectile diameter. The helicopter body is covered with TiB_2 plates of hardness 3500 kgf/mm². Here – you can apply the theory of damage caused by abrasives as in AJM.



So, let us move on to next one. This is interesting, but unfortunately it is not coming fully on the screen. What do I do in, let me see I can read it out. In case you have a problem, just one minute your neighbouring country, ok.

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Your neighbouring country has purchased an anti-aircraft gun with the muzzle output velocity of 569 meters per second. I will write it down here. When we come back to this one 560 meters per second is the muzzle velocity. So, let us draw a gun. This is your gun and it looks more like a telescope, but please be a little imaginative and bear with my drawing and understand and imagine that this is gun rather than a telescope signs as both mutations destruction is the less researched.

So, this is a gun and out comes the bullet. So, I think the bullet has been defined as round perfect bullet. I mean perfect shape comes out this way and at this point, it has 560 meters per second. It is quite a reasonably high velocity and the projectiles are of spherical size and made of solid iron with the density of 7.6 grams per cc. They are made of solid iron in reality, perhaps not they must be full of you know explosives and the burst. Anyway for our problem, they are made up of solid iron with the density of 7.6 grams per cc.

Your army helicopters are being targeted and the defense department asks you to determine the flying height at which the indentation damage depth δ can be a maximum of 5 percent of the projectile diameter. My god, what a complex situation; the

helicopter body is covered with TIB two plates of hardness, 3500 kg per millimeter square and here you can apply the theory of damage caused by abrasives as in AJM. Now, it is understood here in lies the link to our problems here.

So, first of all let us draw a picture here you know what is being suggested. So, first of all I make an assumption that in order to get the rise to the maximum height, you have put the gun this way. So, this is the gun. This is coming out 560 meters per second and this goes up and this is your army helicopter. So, the army helicopter it is being assumed that yes in the worst case is that the army helicopter is going to be hit and if it gets hit, there will be a damage and this damage can be tolerable damage is 5 percent of the projectile diameter. It is understood.

So, if that damage is 5 percent of the projectile diameter, what should be this height which will you know restrict this damage to that particular value. This is the problem. What is this height at which we will fly feeling very safe at least? Even though you get of the projectile, you will be sure that whatever you do, you are not more than 5 percent.

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First – apply the theory of kinetic energy of spherical iron projectiles = plastic strain energy of the helicopter body armour


$$\frac{1}{2} \times \rho \times \frac{4}{3} \pi \times \left(\frac{d}{2}\right)^3 V^2 = \frac{1}{2} \times \frac{\pi}{4} \times D^2 \times H \times \delta$$

$$\frac{1}{2} \times \rho \times \frac{4}{3} \pi \times \left(\frac{d}{2}\right)^3 V^2 = \frac{1}{2} \times \frac{\pi}{4} \times 4 \times d \times \delta \times H \times \delta$$

↳

$$\text{But } \delta = 0.05 d = \frac{d}{20}$$

Hence $\rho \times \frac{4}{3} \times \left(\frac{d}{2}\right)^3 V^2 = \frac{1}{400} \times d \times d \times H \times d$



Let us have a look at the calculations. First apply with a theory of kinetic energy of spherical iron projectiles equal to plastic energy strain energy of the helicopter body armour. Remember the helicopter is covered by TIB two plates which have some 3200 kg of per millimeter square or something like that. So, we will come back to those values later on. First of all let us see what has been tried out here.

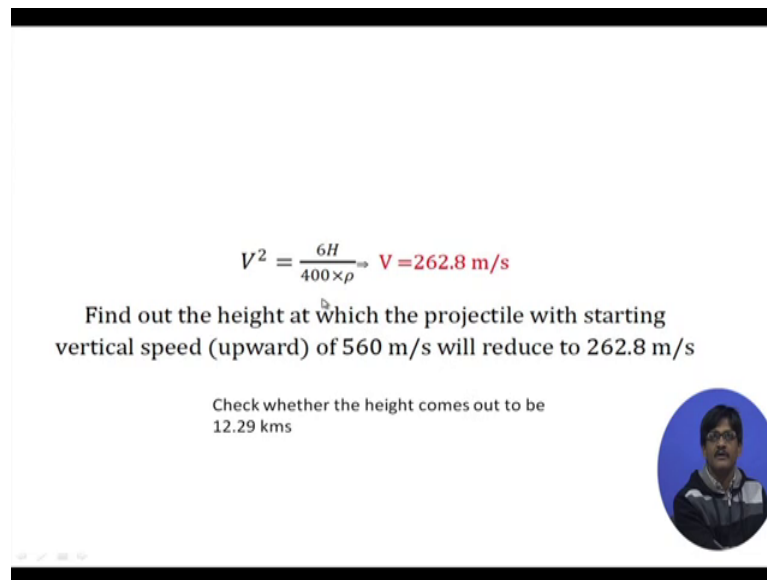
First of all half V square, how is this half V square? This is half, where is that yes this is half. This is density, this is volume $\frac{4}{3} \pi r^3$ multiplied by the velocity square.

So, this is the energy carried out by the iron, what you call spherical iron object missile or bullet at in the point of heating the point at which pointing time. When it heats, it is having this particular kinetic energy and this is equal to half into and you know projected area of plasma indentation multiplied by the hardness of the damage part, damaged armor plate multiplied by the indentation depth. So, this is the full expression and after that I think we have introduced some simplifications like d^2 . We already know d^2 is equal to $\frac{4}{3} \pi r^2$ into δ . This we have done so many times, ok.

So, this we are quite conversant with. We have no issues with this, but now we are introducing the condition which has been provided that is δ can be replaced as 0.05 of the diameter of you know spherical iron projectile. That is good. So, d by 20 is equal to δ . The moment you do that how many δ s do we have? We have 2 δ s here, δ^2 . So, we have δ replaced by d here, d here and also 400 coming from the two 20 at the denominator. So, that is it. We have quite a reasonably simple expression.

So, what do we do? We have d^3 occurring here and d^3 occurring here. That is good. So, we become free of the d s in the world and therefore, after that lots of cancellations will definitely occur and let us see; what are the things which are surviving. Velocity is surviving and the strength of the material. That means, hardness is surviving and that is good.

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$$V^2 = \frac{6H}{400 \times \rho} \Rightarrow V = 262.8 \text{ m/s}$$

Find out the height at which the projectile with starting vertical speed (upward) of 560 m/s will reduce to 262.8 m/s

Check whether the height comes out to be 12.29 kms

So, from here velocity square comes out to be $6H$ by 400 into ρ . Now, what is this ρ ? This ρ must be the density of the iron and therefore, after putting in all these values, velocity comes out to be 262.8 meters per second.

Now, find out the height at which the projectile will start vertical speed of 560 meters per second and will reduce to 262.8 meters per second. In my calculations, it came out to be 12.29 kilometers. This is I think quite common for aeroplanes or whether for helicopters, this is common or not I do not know, but for the sake of you know mathematical calculations, it is no problem. First of all how do we get this particular height from this calculation? That is quite simple.

What you can do is, you can simply apply one of those equations. Remember V is equal to u plus ft , s is equal to ut plus half gt square, V square is equal to u square plus twice fs . One of these will definitely give you start with this velocity. The velocity is gradually eaten up by acceleration due to gravity towards the other direction, I mean towards the center of the earth and therefore, you will alternatively get this particular velocity solved and rather this velocity is provided, so that you will be solving for the height which is coming out to be 12.29 . I am sure you can do these yourselves.

So, we come to the end of 12th lecture and in the 13th lecture, we will take up other aspects of Abrasive Jet Machining.

Thank you.