**Fluid Machines.**

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**Tutorial-1.**
**Lecture 9.**

Welcome to this session of fluid machines. In today's tutorial class we are going to solve few representative problems related to principle of similarity and dimensional analysis in Rotodynamic machines. So before solving problems, I will quickly go through the important dimensionless parameters that may arrive in Rotodynamic machine.

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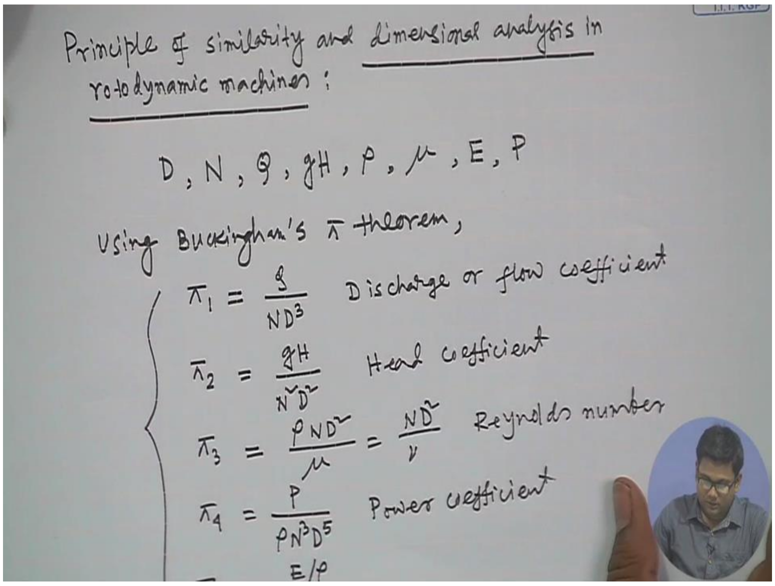


So the topic is principle of similarity and dimensional analysis in Rotodynamic machines. For the physical parameters for dynamic machines which are very much relevant in these problems are the diameter of the rotor, the speed of the rotor, the flow rate Q through the rotor, GH, where G is the acceleration due to gravity and H is the head across the rotor, rho the density of the fluid, mu the viscosity of the fluid, E coefficient of velocity of the fluid, and power transfer. So using Buckingham's pie here one can obtain 5 dimensionless parameters which are very much relevant in this course.

So using Buckingham's pie theorem we obtain the 1st term being the pie 1 is Q divided by ND cube which is known as discharge or flow coefficient. The 2nd pie term which is pie 2 is GH by N square D square, this is known as head coefficient. 3rd pie term is pie 3 rho ND square by mu or ND square by nu where nu is the kinematic viscosity. This is Reynolds number. The 4th pie term is power transferred P divided by rho N cube D to the power 5 which is known as power coefficient. And the last pie term is E by rho divided by N square D square.

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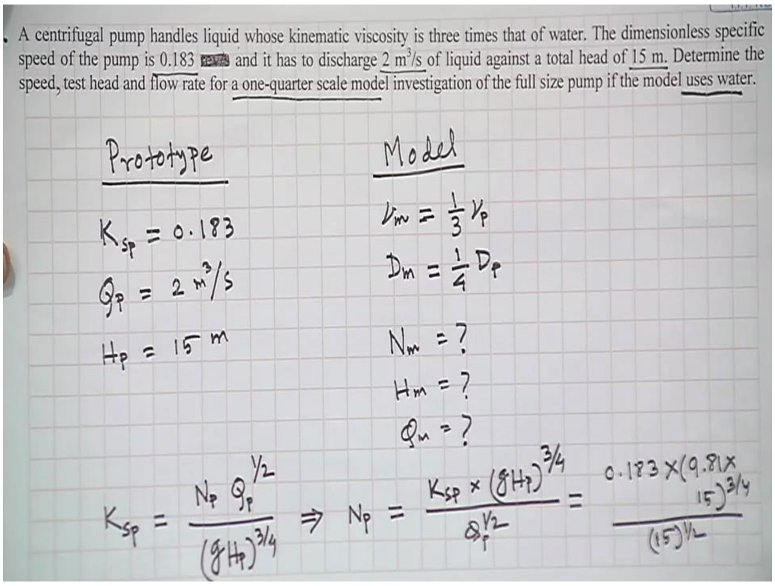


So these 5 pie terms govern the different characteristics of Rotodynamic machines. Now I am going to solve 2 problems related to these principles of similarity. The 1st problem is related to one centrifugal pump. So let me just read the 1st problem. So a centrifugal pump handles liquid whose kinematic viscosity is 3 times that of water. The dimensionless specific speed of the pump is 0.183 and it has to discharge at a rate 2 meter cube seconds of liquid against a total head of 15 meter. Determine the speed test head and flow rate for a one quarter scale model investigation of the full-sized pump, if the model uses water.

So let me just list the important given parameters are associated with this problem. So the prototype, prototype centrifugal pump. For prototype pumps it is given that the specific speed, the dimensionless specific speed is 0.183. So KSP is 0.183, the flow rate through the prototype pump is 2 meter cube per second, so QP is 2 meter cube per second and total head across the pump is 15 metres, so HP is 15 meter.

So important thing to note that is here we are using subscript P to denote the parameters or variables associated with prototype. Similarly the given parameters for model pump, so the kinematic viscosity for the model is one 3rd to that of prototype. Here it is specified that centrifugal pump handles liquid whose kinematic viscosity is 3 times to that of water. Now the model is using water and the prototype pump uses some other fluid having kinematic viscosity 3 times. So model kinematic viscosity will be one 3rd to that of kinematic viscosity prototype.

Another important thing to note here is that we are using a one quarter scale model, so rotor diameter of the model will be one 4th of the diameter of the prototype. So these are given quantities, now we have to determine the speed, test head and flow rate for a one quarter scale model. So speed of the model, test head of the model and flow rate, these 3 quantities are of interest. Now let us 1st start with determination of speed of the prototype pump. Now we know that specific speed of pump is given by the speed of the pump times the flow rate to the power half divided by GH P to the power 3 by 4.

So using this relation we can obtain the speed of the prototype pump as specific speed times GH P to the power 3 by 4 divided by QP to the power half. Now we substitute the given quantities. So 0.183 is the specific, dimensionless specific speed, G is 9.81, HP is given as 15 whole to the power 3 by 4 divided by QP is 15 to the power half.

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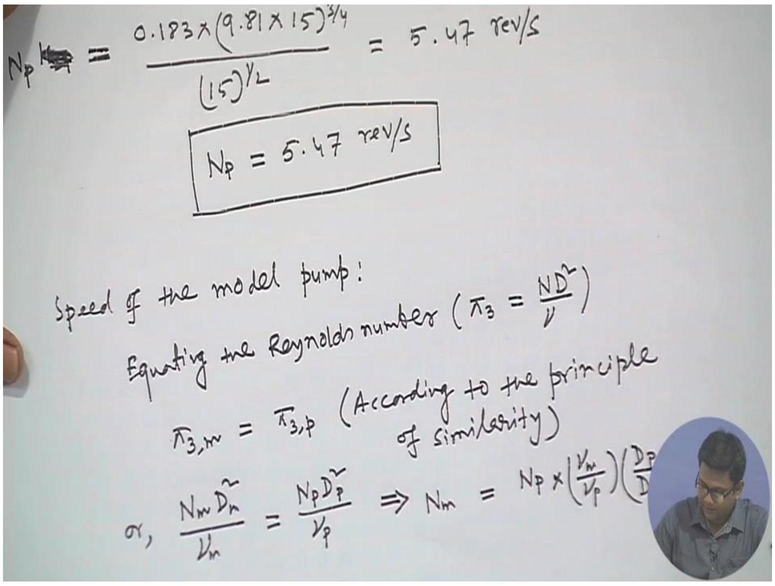


So we can obtain the numerical value of specific speed as, so Q SP is 0.183 times 9.81 into 15 to the power 3 by 4 divided by 15 to the power half. This will be rather this is NP. So specific speed is 0.183, now we are going to determine the speed of the prototype rotor, so N P will be 5.47 revolutions per second. So one desired quantity has been obtained, it is 5.47 revolutions per second. Now we will determine the 2nd quantity of interest which is the speed of the, speed of the model pump.

Now to determine this, we will use the principle of similarity by equating, equating the Reynolds number. Reynolds number is nothing but the pie 3 term which we have previously obtained in here. So this pie 3 term is the Reynolds number. So this is ND square by nu. N D square by nu. So pie 3 of model will be pie 3 of prototype according to the principle of similarity. Now we substitute the expression for pie 3. So NM DM square by nu M, equal to NP DP square by nu D. Using this we can obtain speed of model as NP times nu M by nu P DB by DM square.

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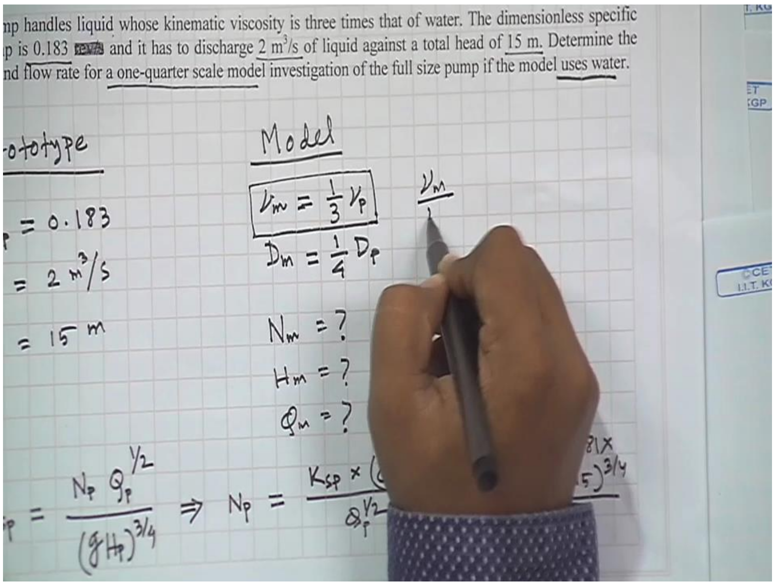


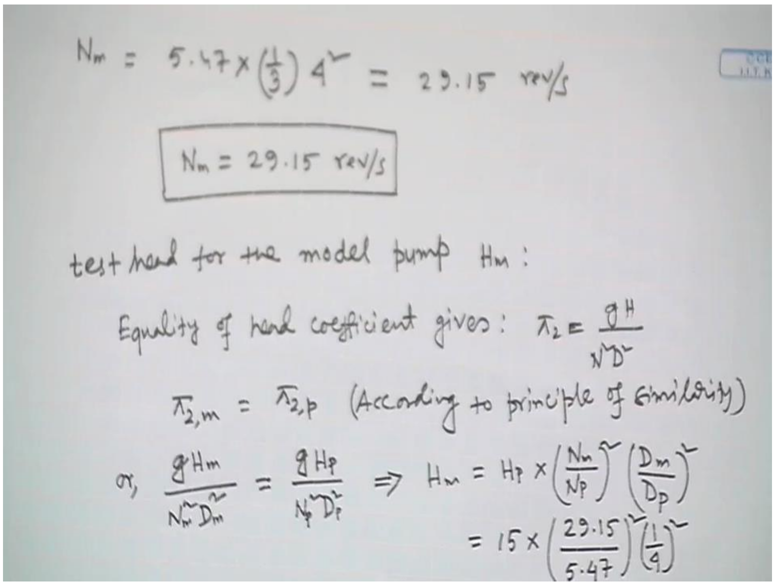


Now we substitute respective numerical values given to obtain NM. So NM will be, NP we have obtained as 5.47 nu M by nu P. So previously you have noted that nu M is nothing but nu P by 3. The kinematic viscosity of model is one 3rd of the kinematic viscosity of prototype. Now we will substitute nu M by nu P as 1 by 3 here. So here nu M by nu P will be one 3rd. So DP by DM as per the prescription of the problem, DM is one 4th of the DP, so diameter of the model pump is one 4th of the diameter of the prototype pump.

So DM by DP, from here we obtain DM by DP equals to one fourth. So here we will substitute DP by DM to be 4. So this gives the NM as 29.15 revolutions per second. So speed of the model centrifugal pump will be 29.15 revolutions per second. Now we will determine the head or the test head across the, test head for the model pump, model pump which is HM. To determine this, we make use of the head coefficient or equality of head coefficient. Equality of head coefficient gives, so head coefficient previously we have defined as pie 2 which is GH by N square D square.

So we will make use of this relation, so pie 2 is GH by N square D square. So pie 2 of model will be pie 2 of prototype, this is according to principle of similarity. Now I will substitute the expression for pie 2. So G HM by N M square DM square equals to GHP by NP square DP square. This will give HM equals HP times NM by NP square and DM by DP square. So HP is previously specified in the problem as 15 meter, so I am going to replace this, so 15 meter.

NM we have obtained as 29.15 revolutions per second, 29.15, NP is also obtained previously as 5.47 revolutions per second. So NP is 5.47 whole square and DM by DP, DM by DP is equal to 1 by 4. So 1 by 4 square. This will give HM as 26.64 meter. So the head for the model pump is 26.64 metres. Now another important quantity of interest is the flow through the model, model pump.

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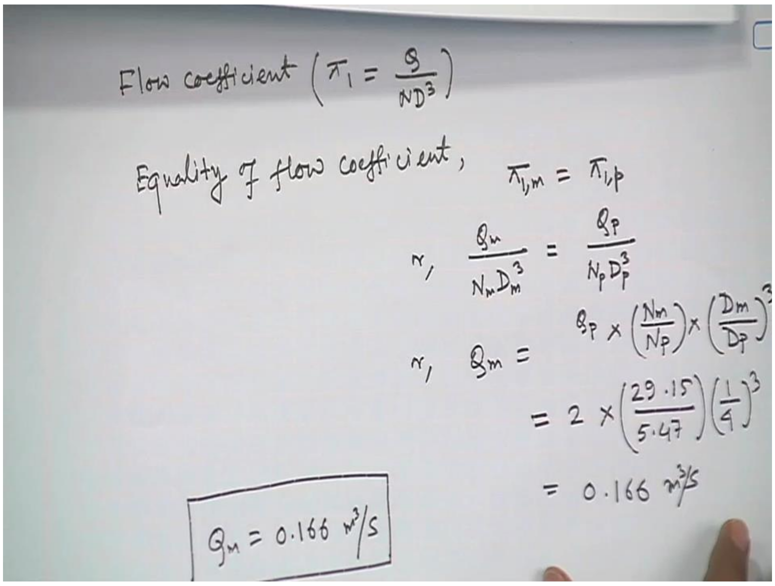


To obtain this flow I am going to use the flow coefficient. So flow coefficient. Flow coefficient was previously obtained as this pie 1 term. So Q by ND cube, so this flow coefficient is pie 1 equals to Q by ND cube. Now equality of flow coefficient, equality of flow coefficient gives pie 1 M equals to pie 1 P. Or Q M by NM DM cube equals QP by NP DP cube. Or QM equals QP times NM by NP into DM by DP Q. Now that QP, the flow through the prototype pump is given as 2 meter cube per second, NM, so NM we have obtained previously, so NM is 29.15, let NP is 5.47, NP is 5.47 and DM by DP is 1 by 4.

So flow through the model pump can be obtained here as 0.166 meter cube per second. So QM is 0.166 meter cube per second. So we have determined all the 3 important quantities that we were intending to obtain, that was NM, HM and QM. Now I am going to solve another problem related to turbine.

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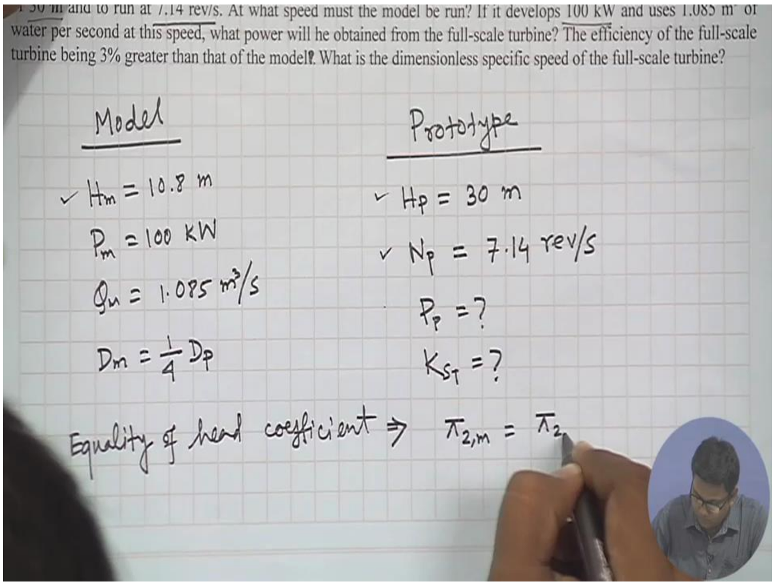


So this is the 2nd problem, problem 2. So I will just read the problem 1st. A quarter scale turbine model is tested under a head of 10.8 metre, the full-scale turbine is required to work under a head of 30 meter and to run at 7.14 revolutions per second. At what speed must the model be run if it develops 100 kilowatt and uses 1.085 meter cube of water per second at this speed. What power will be obtained from the full-scale turbine, the efficiency of the full-scale turbine being 3 percent greater than that of the model? And what is the dimensionless specific speed of the full-scale turbine.

So let us 1st list the important given quantity. So for model, the model is tested under a head of 10.8 metre, so head is 10.8 metre. And power produced by the model is 100 kilowatt where the flow rate through the model is 1.0 85 meter cube per second. And it has been mentioned the quarter scale turbine model. That means diameter of the model will be one 4th of the diameter of the prototype. Now for prototype, prototype turbine, the head is specified as 30 meter and the speed of the prototype turbine is given as 7.14 revolutions per second.

Important quantities of interest are power of the turbine, prototype turbine and the specific speed, dimensionless specific speed. Now as the head for the model and prototype are given and speed of the prototype are given, we can easily obtain the speed of the model by using the equality of head coefficient. So equality of head coefficient, this is, so head coefficient is the pie 2 term which is GH by N square D square. So pie 2 of model will be pie 2 of pump or prototype.

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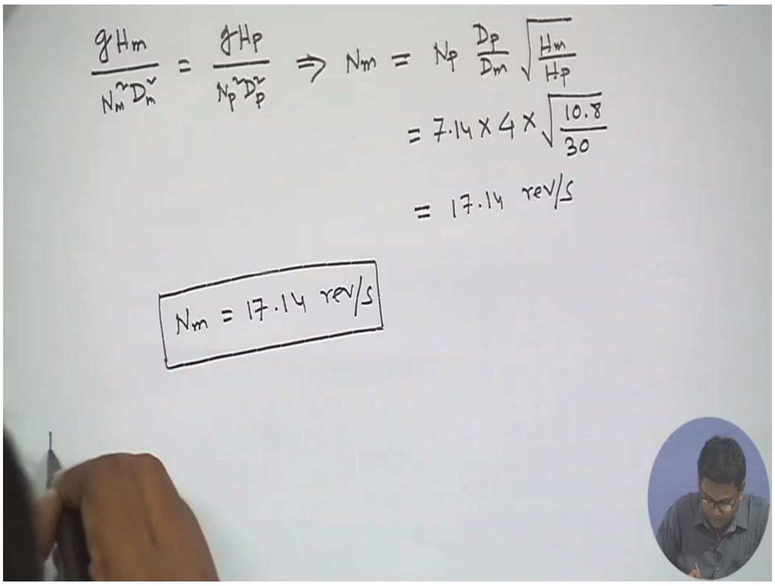


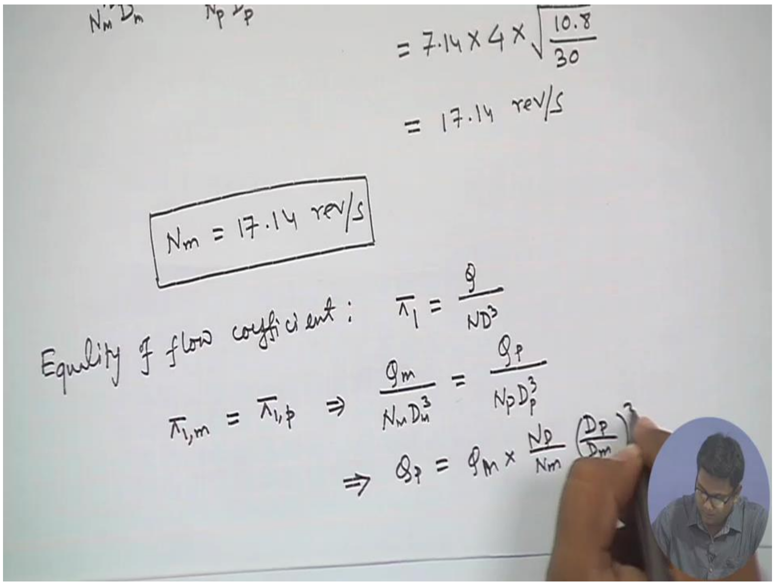


Now substituting the expressions we can obtain that GH of model HM NM square DM square will be GH P by NP square DP square. So NP, HP and HM are known, so only unknown is NM. So NM will be NP times DP by DM root over of HM by HP. Now I will substitute the value of NP as 7.14, so let NP is 7.14. DM by DP is 1 by 4, so DP by DM is 4. HM is given as 10.8 and HP is given as 30. So this will be NM 17.14 revolution per second. So NM is speed of the model turbine is 17.14 revolutions per second.

Now before obtaining the power which is our main interest, the power output from the prototype turbine, I am going to calculate the flow through the prototype turbine by using the equality of flow coefficient, equality of flow coefficient. Flow coefficient was previously obtained as pie 1 equals to Q by ND cube. So equality will give pie 1 M equals to pie 1 P. So let Q M by NM DM cube is equal to QP by NP DP cube. So from here we can obtain Q P as QM , QM times NP by NM DP by DM cube.

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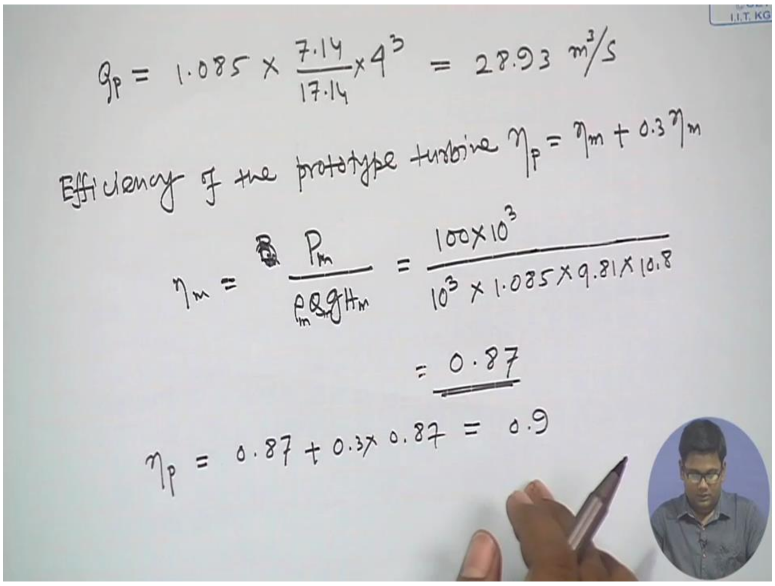


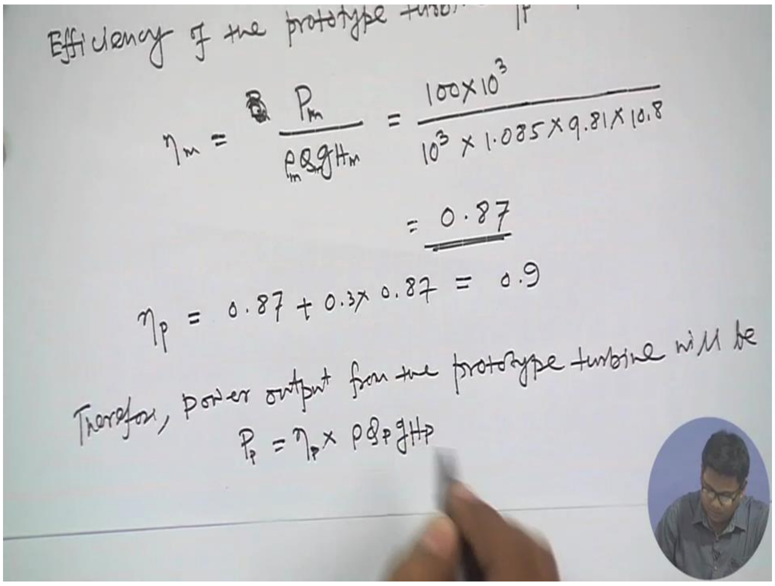


How substituting different quantities we can obtain QP as, QM is given as 1.085 meter cube per second, let NP is given as 7.14, N is obtained as 17.14 and DP by DM is 4, this cube, this will be 28.93 meter cube per second. Now to obtain power of the prototype turbine, one very important thing to note here is that the efficiency of the full-scale turbine is 3 percent greater than the model. So to obtain the power, let us 1st obtain the efficiency of the prototype turbine.

So efficiency, efficiency of the prototype turbine let NP will be NM which is the efficiency of the model turbine + 0.3 times NM. Now NM is power output by the model PM, PM divided by rho Q GH, so rho M QM HM. Power output from the model turbine is 100 kilowatt, density is for water 1000, QM is 1.085 meter cube per second, G is 9.81, HM is 10.8. So this gives efficiency of the model as 8.76.

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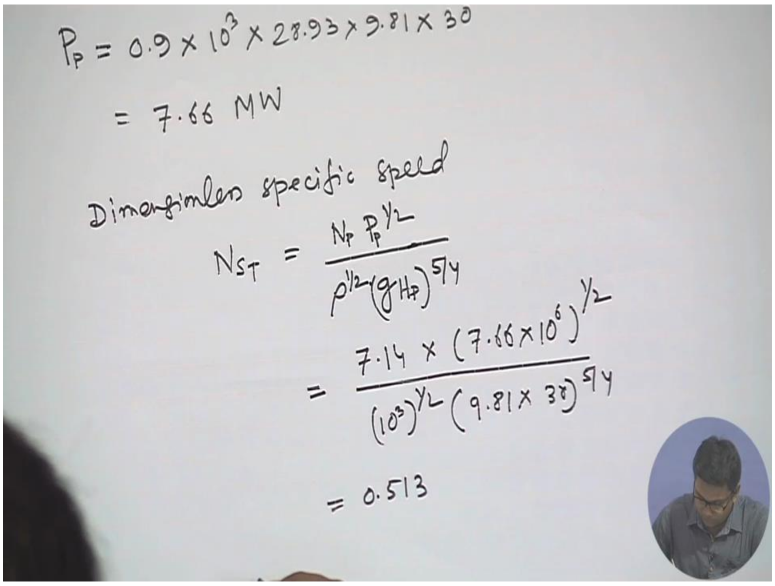


So efficiency of the prototype will be 8.7, 0.87, sorry this is 0.87, 0.87 + 3 times 0.87, which is 0.9. So this is the efficiency of the prototype turbine. Therefore power output from the prototype turbine will be, power output from the prototype turbine will be efficiency of prototype turbine times the rho QP G HP. Now substituting different quantities, power from the prototype turbine now can be obtained as, efficiency is 0.9, rho is 10 cube, QP we have obtained, so 28.93, G is 9.81 and H is given as 30 meter. So this will give 7.66 megawatt.

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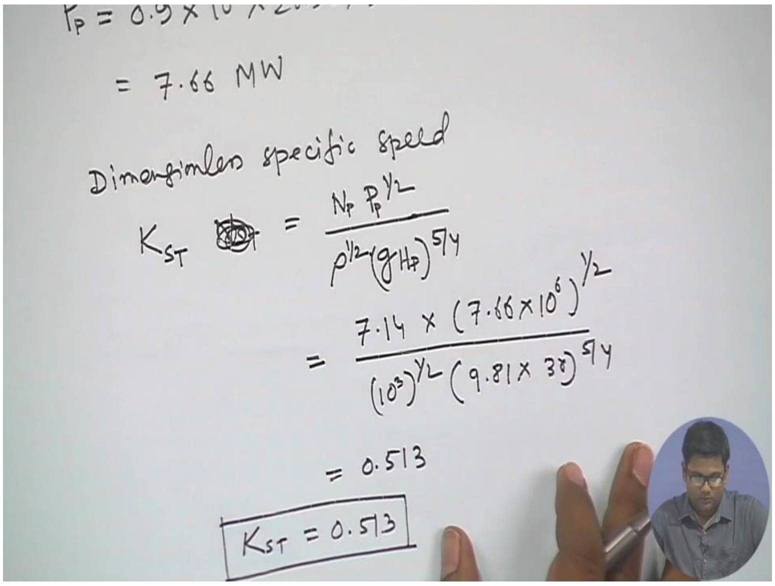


The last quantity of interest is the specific, dimensionless specific speed or NST, which is NP, PP to the power half by rho to the power half GHP to the power 5 by 4. This is the expression for specific speed, dimensionless specific speed of the prototype machine. Now substituting different quantities we can obtain the NP as 7.14, power is 7.66 times 10 to the power 6 whole to the power 1 by 2, rho is 10 cube to the power half, G is 9.81, HP is 30 over 5 by 4. This will be obtained as 0.513.

So the sign for this will be more appropriate will be K SP. So dimensionless specific speed of the turbine will be 0.513. Now important fact to note here is that here determine the power of the prototype turbine, we are not using the equality of the power coefficient. This is because here it has been mentioned that the efficiency of the prototype turbine is not same as the model turbine. So we know that the efficiency, hydraulic efficiency is nothing but, so Eta, hydraulic efficiency we can write in terms of pie terms.

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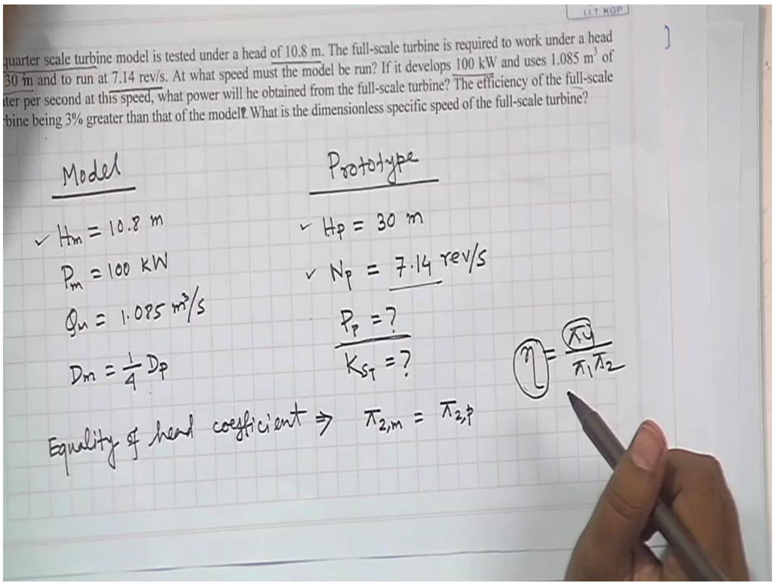


So this is pie 4 by pie 1 pie 2, so hydraulic efficiency, equality of hydraulic efficiency means all the terms will be same for model and prototype but in this case, efficiency of model and prototype are not equal and we have previously used equality of pie 1 and pie 2 in terms of equality of discharge coefficient and head coefficient. So pie 4, which is the coefficient, power coefficient will not be same for the model and the prototype. That is why we have used the efficiency of model turbine and then obtained the power in this way. So with this I am ending today’s lecture, thank you.