

Fluid Machines.
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Lecture-7.

Basic Principles, Analysis of Force and Power Generation Part II.

Good morning and welcome you all to this session on fluid machines. Last class we derived that the total power delivered to the rotor of a Pelton wheel or given to the Pelton wheel by the waterjet is given by this one which we $\rho Q \dot{}$ is the volume flow rate into $1 + K \cos \beta - 2 V_1 - U$ into U and nomenclatures are all known. This is K takes care of the friction in the bucket which reduces the relative velocity at the outlet by this proportion from that of the inlet.

If we neglect the friction, then K will be 0, K will be 1, sorry, $\beta - 2$ is the angle of the blade at the outlet as I have already shown you. V_1 is the absolute velocity of the waterjet at the inlet and U is the bucket speed at the central plane of the bucket, this is the bucket speed, means the linear speed due to the rotation of the bucket. So this is the power obtained by the rotor. Now the input energy input power, power input, power input to the rotor, input to the rotor up to the wheel, this is known as Pelton wheel is given by what.

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$$P = \rho Q (1 + K \cos \beta_2) (V_1 - U) U$$

$$P_{\text{input to the wheel}} = \rho Q \frac{V_1^2}{2}$$

$$\eta_W (\text{wheel/blade efficiency}) = 2 (1 + K \cos \beta_2) \left(1 - \frac{U}{V_1}\right) \frac{U}{V_1}$$

$$\frac{d \eta_W}{d (U/V_1)} = 2 (1 + K \cos \beta_2) \left(1 - 2 \frac{U}{V_1}\right) = 0$$

$$\frac{U}{V_1} = \frac{1}{2}$$

$$\frac{d^2 \eta_W}{d (U/V_1)^2} < 0$$

Input is the kinetic energy of the jet, that means if we see this figure, we will see that this is the input, input is the kinetic energy of the jet, okay. So if we see this, then we can write that input power is $\rho Q \dot{}$ into V_1 square by 2. So we define in this context an efficiency known as wheel efficiency or blade efficiency, known as wheel or blade efficiency of Pelton

wheel. That means it is the efficiency in converting this kinetic energy into the power developed by the rotor if you divide this, then you get $1 + K \cos^2 \beta$, this will cancel.

And this can be written in this fashion U/V_1 into U/V_1 , V_1^2 . And $\rho Q V_1^2$ by 2, so therefore this 2 will come here, sorry there will be a factor 2, so this $2, 1 + K \cos^2 \beta$, so this will be $1 - U/V_1, V_1^2$, okay. So U/V_1 . So this is the expression for the efficiency wheel or the blade efficiency which is defined in case of Pelton wheel. Okay. Now here we see that for a given design of the blade where β is constant and this factor K takes care of the surface friction, that is the for which the V_2 is less than V_1 , this is because of the liquid viscosity and this change in the velocity depends mostly on the type of the surface and the liquid viscosity.

So therefore this for a given design we can take substantially constant, so therefore we can tell that wheel or blade efficiency depends only on this U/V_1 ratio which is very important. What should be the ratio of U/V_1 now for which this wheel efficiency will be maximum, we have to find out that what should be the ratio. That means if I can find it out, then I can direct my jet at that velocity because this rotor speed depends upon the rotational speed of the Pelton wheel, that is usually fixed.

Then we can fix this velocity in such a way that we get the maximum efficiency. So another way for giving jet velocity, we can change the speed of rotation of the Pelton wheel so that we get the maximum efficiency. So if we consider U/V_1 , this velocity ratio is an independent variable and if we just differentiate with respect to U/V_1 , then what we get, 2 into this part is $K \cos^2 \beta$ and therefore $U/V_1 - 2U/V_1$ equals to 0. And you get U/V_1 is equal to half.

Okay, sorry, I am sorry, it is 1 because U/V_1 will be one half. $U/V_1 - U/V_1$ whole square, this will be $1 - 2U/V_1$ is half, so this is the value. And if you take the 2nd derivative, you see this negative. 2nd derivative $D^2 U/V_1$ is less than 0 because it is -, this will be -2. So therefore it has a maxima, this equation has a maxima at U/V_1 by, U/V_1 is sorry it is U only, no U^2 , U/V_1 is half. So when the ratio of the rotor speed to that of the fluid velocity at inlet is half, then we get the maximum efficiency of the Pelton wheel.

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Impulse Hydraulic Turbine: Laster A Pelton

Water jet V_1

Plane of wheel $-U$

Water jet V_2

$U_1 = U_2 = U$

β_2

$V_{r1} = V_1 - U$

$V_{r2} = V_2 - U$

$V_{w1} = V_1 = V_{r1} + U$

Pelton Turbine Wheel

Buckets

Water jet

Fixed nozzle

A Pelton wheel

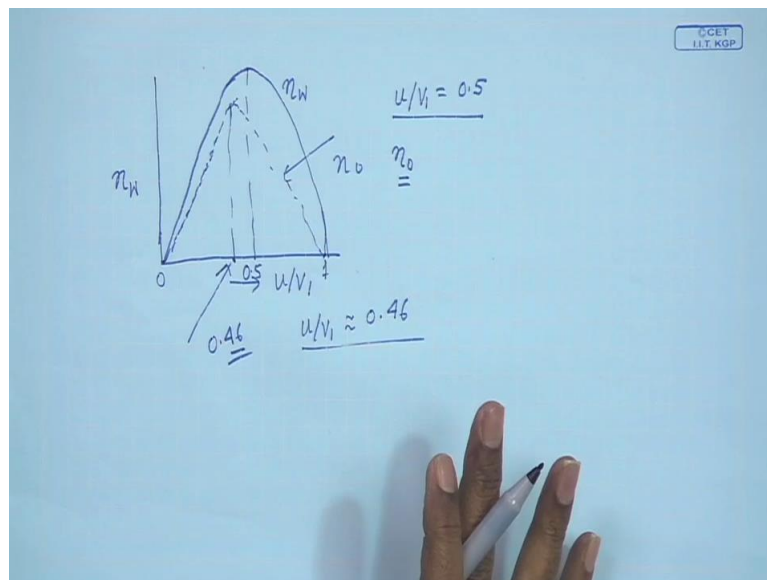
Splitter

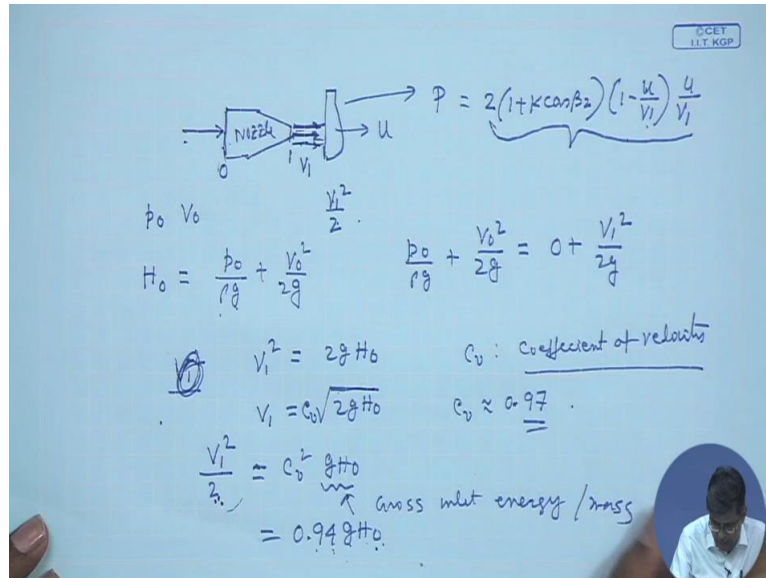
Side view of the bucket

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So therefore this can be shown in a graphical form like this that this is U by V_1 and this is efficiency of the wheel η_w . It is 0 when U by V_1 is 0 because U by V_1 is 0 means there is no motion of the, U is 0, there is no movement of this wheel. Again when U by V_1 is one, then also η_w is 0, that means U by V_1 you see here, when U by V_1 is 1 η_w is 0, when U by V_1 is 1, U is equal to V_1 , V_{R1} is 0. That means no flow will come to the Pelton turbine when V_1 equals to U , that V_{R1} equals to 0.

So therefore at these 2 conditions, the, this will be 0 and the graph will have a maximum at this 0.5, this is η_w wheel. That means when U by V_1 is 0.5, we have the maximum wheel efficiency, wheel will operate on its maximum efficiency condition, that means it will convert the kinetic energy of the water at inlet most efficiently into the mechanical power of the rotor. But the overall efficiency, overall efficiency which is related to the power output at the shaft, at the final shaft at coupling as I have already told you in earlier classes, that is always less than the wheel efficiency or hydraulic efficiency, whatever you call because of the friction in glands, bearings and couplings.

And at the same time when the speed of the machine increases, then the losses, the fictional losses also increases, mechanical efficiency becomes less. Because of these 2 reasons, the overall efficiency is lower than the wheel, will be lower wheel efficiency in case of Pelton wheel. And at the same time it has been found because of these relationships of the speed of the wheel with the mechanical losses, the maximum of overall efficiency occurs at a value of U by V_1 less than 0.5 which can be shown like this.

Its maximum value somewhere like this, which is reported value in practice is 0.46, around 0.46, around 0.46. That means this is, this dotted is the overall efficiency η_o which is less than the maximum value occurs, a value less than U by V_1 is approximately equal to 0.46. Okay. That is the concept behind this efficiency, now one thing should be made clear here that the nozzle is a stator part of the fluid, of the machine, so the energy available at the inlet of the nozzle as I have told earlier that the energy available at the inlet of the machine is same as that of the rotor provided there is no loss in the stator.

Since stator does not take part in any transmission or mechanical energy because this is a fixed one, so no mechanical energy is going out or coming in, it is not given there and if we neglect any heat loss and here the things work in an isothermal state. So therefore the change in the total energy or head from inlet to the stator to the outlet of the stator or inlet to the rotor can take place only because of friction. Friction in this stator, in this case friction in the nozzle for which the, if the efficiency is defined, hydraulic efficiency based on this energy at the inlet, there will be little different from that of the wheel efficiency.

And usually in a Pelton wheel, the hydraulic efficiency is defined in this fashion. That hydraulic efficiency is defined, let us consider this way that if we have a these nozzles and it directs jet and here we have this blade, this this blade is here, here we have this blade, this blade, okay which has got a motion U . Now this is, this is V_1 , so at the blade input energy is V_1 square by 2 per unit mass but here if we consider the input energy, let us consider this plane as the point 0 because here we have told one, so 0, so P_0 is a pressure, V_0 is the velocity, then the total head or energy per unit weight is P_0 by $\rho G + V_0$ square by $2G$, if we neglect the change in the gravity.

Then P_0 by $\rho G + V_0$ square by, P_0 is very less in fact, it is mostly in the form of the pressure. So this is the head at the inlet and this pressure, if it is above the atmospheric pressure, then by applying the Bernoulli's equation one can tell neglecting the friction this V_1 square by, that you can write this way. V_1 square is equal to $2GH$ because we can write the Bernoulli's equation this way that P_0 by $\rho G + V_0$ square by $2G$ is equal to, here the pressure is atmosphere, if the P_0 is the gauge pressure here, then this is $0 + V_1$ square by $2G$ in terms of the head per unit weight. So V_1 square by $2G$ is H_0 , V_1 square is $2G$ is H_0 .

So therefore V_1 is given by root over $2GH_0$. In a very simple analysis what is done by evaluating the velocity in this fashion by writing the Bernoulli's equation without any loss considering, in consideration of no loss, considering the fluid to be inviscid, we just

multiplied with a factor CV which is known as coefficient of velocity which is done in this case, coefficient, this is the culture velocity without going for any detailed analysis for viscous flow, coefficient of velocity which takes care of the friction.

And in fact this is very small, this friction is very small in this case and the value of CV is almost equal to 0.97. So therefore you see that V_1 square by 2 is equal to CV square into GH. That means this is the energy per unit mass at the inlet to the machine, that is inlet to the stator nozzle and this is the energy per unit mass at inlet to the rotor which has been taken as the input energy in defining the wheel efficiency. While the rotor developed power which is equal to $2 \sin \alpha \cos \beta_2 \sin \alpha - U$ by V_1 into U by V_1 .

This is the power developed by the rotor, this is the energy input to the rotor and this energy is output of the stator, that is the nozzle as stator, nozzle as stator which is equal to CV square GH0 which is a gross inlet energy, gross inlet energy per unit mass. And considering CV as 0.97 typical value, this becomes approximately equal to 0.97 square, something like 0.94. That means you see this kinetic energy at the outlet of the stator is 94 percent of the gross, hardly 5 to 6 percent loss is there.

So this way we can analyse that the gross input of energy at the stator inlet, that is the inlet of the machine, nozzle is a part of the machine as stator, then the kinetic energy comes out to be 0.94 GH0 which can be written in terms of V_1 square by 2 and V_1 is the absolute velocity of jet and this is the power which is available by the bucket and which becomes maximum when U by V_1 is 0.5.

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Handwritten equations on a blue background:

$$\eta_h \text{ (hydraulic efficiency)} = \frac{P}{\rho Q (gH_0)}$$

$$\eta_w \text{ (Wheel/Blade efficiency)} = \frac{P}{\rho Q \left(\frac{V_1^2}{2}\right)}$$

$$\frac{V_1^2}{2} = C_v^2 (gH_0) \quad C_v \approx 0.97$$

$$P = 2(1 + K \cos \beta_2) \left(1 - \frac{U}{V_1}\right) \frac{U}{V_1}$$

$$\eta_o = \frac{\text{Mechanical Power at shaft}}{\rho Q (gH_0)}$$

And in this context, a efficiency is defined as hydraulic efficiency which we have already learned earlier, this hydraulic efficiency, the hydraulic efficiency, this hydraulic efficiency of the Pelton wheel is defined as the power developed by the rotor divided by the power, that means this one, this G_H , G_H is the power developed, that is the $\rho Q \dot{V}$ into G_H , that means this is the power at the inlet of the machine, that means at the inlet of the stator stator, at the inlet of the nozzle.

G_H is the per unit mass energy into $\rho Q \dot{V}$, H_0 is the head at the inlet or base of the nozzle. So this is the definition of hydraulic efficiency, this is the definition of wheel or blade efficiency, P by $\rho Q \dot{V} V_1^2$ and V_1^2 G_H is related like this. V_1^2 is $C_V^2 H_0$ which is derived by using the Bernoulli's theorem with the simple modification by multiplying with a factor which takes care of the friction, that is viscous effects of the fluid and C_V value is 0.97.

And the final output from the rotor is equal to again $2 U V_1 \cos \beta$, all these nomenclatures are known to you, U by V_1 . This is, all things which are clear in one place for the Pelton wheel and this is the hydraulic efficiency and this is wheel. And the wheel efficiency becomes maximum when the U by V_1 which is 0.5 but this is not so for hydraulic efficiency because there is a little loss but for overall efficiency, overall efficiency is again, this will be changed. For overall efficiency, we define on the basis of mechanical energy, mechanical power at shaft.

While the input maybe this or input maybe this, that depending upon the purpose. If you define based on this, then this will be the gross overall efficiency. Okay, that is the gross input to the machine at the inlet to the nozzle and this is the final output to the or from the machine that is at the inlet power at shaft. Okay, I think next Class I will discuss about the specific speed of Pelton wheel and the concepts of specific speed and finally the governing of Pelton wheel which is very very important, that I will discuss in the next class. Okay, thank you for today.