Fluid Machines.

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Lecture-5.

Concept of Specific Speed.

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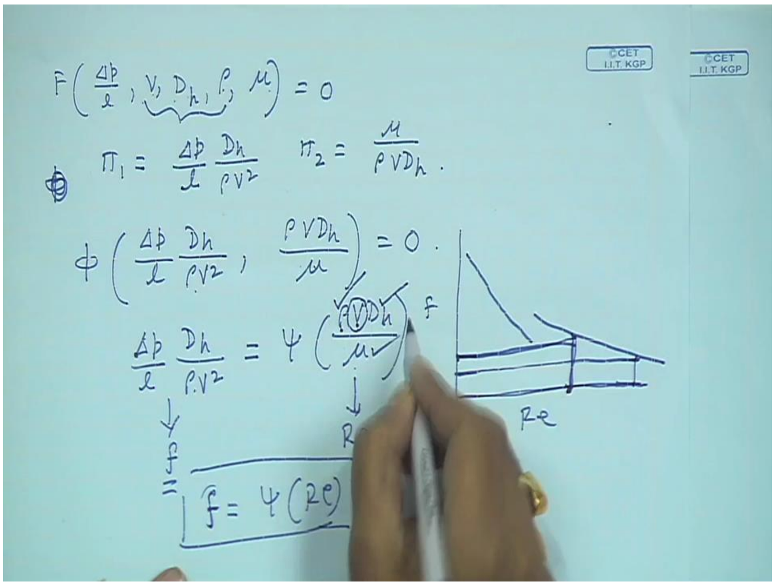

Good morning and I welcome you all to this session of fluid machines. In the last class we have seen that a pipe flow problem, how the nondimensional terms which represent the physical similarity of different pipe flow problems, pipe flow problems in practice and that is done in the laboratory prevails.

And how it is done, that a pipe flow problem which was defined by a functional relationship like this, a function of Delta P by L, V, the characteristic velocity, the characteristic dimension which is the hydraulic diameter of the pipe rho and mu and by the application of Buckingham's pie theorem and by taking this V, DH and rho as the repeating variables, we arrive at relationships that we found that pie 1 is equal to Delta P by L DH by rho V square. And pie 2 is equal to mu by rho V DH.

So this can be written, the same problem described by 1, 2, 3, 4, 5 variables can be written in terms of these 2 variables and in fact this pie 2 is changed by its reciprocal, this does not make any change in the principle of similarity or its concept, rho, this is the conventional term, DH by mu, this is 0. Or simply one can write that Delta P by L, DH by rho V square is a function of rho V DH by mu. And as you know in from our hydraulics knowledge that this is defined as friction factor A and this is defined as Reynolds number rho V DH by mu.

And friction factor becomes a function of Reynolds number for this pipe flow problem. So therefore you see this is reduced to 2 variables, one nondimensional output parameter, one nondimensional variable which contains the output parameter and there is the input parameter. Now if we take others nondimensional set term, that did not V DH rho but V DH mu or rho DH mu, we will arrive at different pie terms but it does not change any sense of similarity.

And by combination of those pie terms, we can come to these 2 pie terms which are the most conventional one. Number of pie terms will remain same but the different pie terms we arrive will be different but they are all interdependent and a combination of these 2 pie terms using different set will automatically give this 2 pie terms. Now the understanding part is that if we, what is the understanding part, that means if we do experiments with pipe flow.

For example we do experiments in laboratory with a small dimension pipe with a different fluid which is not used in practice or different flow rates, we have to ensure one thing that this pie 1 and pie 2, that is combination of this Delta P, DH, rho, V, that is friction factor term and the Reynolds number term will be in the same range as those in practice. So we have to find out that range in practice in terms of nondimensional parameter and we will make the same range and we will vary our dimensional variables according to our ease of availability, space requirement, everything so that this range is varying.

This is number-one, this is the similarity principle, so how we can make the 2 problems similar so that the results from the laboratory can be used to predict that of the performance. And the another beauty is that if you plot this friction factor versus Reynolds number curve, you see this is like this in the turbulent flow region this is like this. If you draw this curve, you will see for a given Reynolds number, we get a value of friction factor, this value, if Reynolds number is fixed.

But it does not know what should be the value of rho, what should be the value of V, whatever may be the variables, if the Reynolds numbers is fixed, F is fixed. If all these things very and make this thing fixed, this is fixed. For example if we want to show the very, for example if Reynolds number is changed to some other value, the friction factor will be changed. So this change in Reynolds number may be effected through change in V to change in DH to change in rho or mu.

That means it is only through the change in Reynolds number, so therefore we can change any parameter in the Reynolds number and can give the influence and get the influence of other parameters. A brand is number is double, so value of F we will know and Reynolds number double can be made by doubling either the velocity, doubling either the characteristic parameter, that is hydraulic diameter, halving the viscosity… So therefore by varying one parameter, we can see the influence of other parameter.

In experiments, because of our ease of investigations or ease of experiments, we vary the flow velocity which is very easy to vary. Having pipe of different diameters will be difficult, we will have to fabricate more number of pipes to have fluids with different viscosity but same density is very difficult. So therefore easy to vary the velocity and varying the velocity we get the variation of rho, influence of rho, DH and mu on the pressure drop in terms of the manifested in terms of the friction factor.

So therefore you now you see, we initially raised 2 questions are very good and answers to the 2 of that physical similarity that we can compare the result from laboratory tests done at altered condition from that the actual ones for its predictions and at the same time we can vary less number of input dimensional variables to conduct the experiments, to predict the influence of other variables on the performance parameter.

Okay. Now with this as the background of the principle of similarity, now I like to tell you how it is being applied to fluid flow situations. Now in the fluid machines, the variables are like these. I will show you here.

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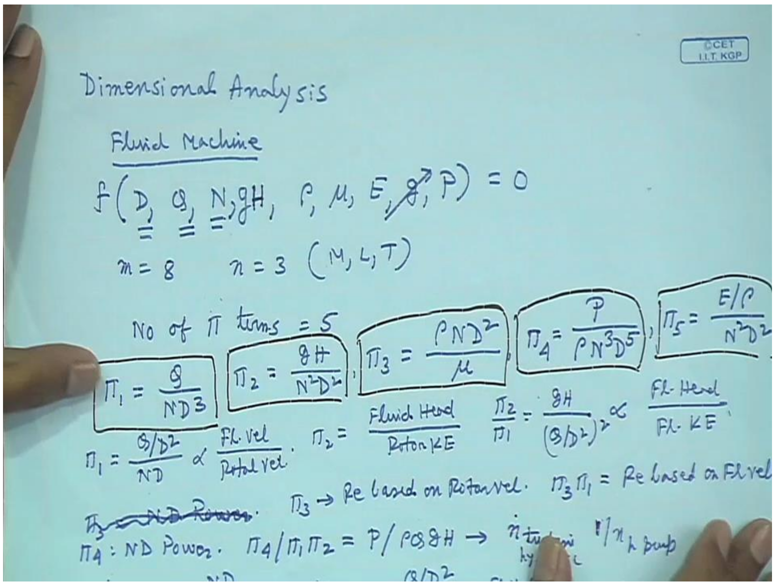

The variables I think you can see. The dimensional alliances if you use for fluid machines, the variables in the fluid machines are its representative rotor diameter D. Representative rotor diameter. The volume flow rate through the fluid, the rotational speed N of the fluid, now initially we take the H, H is the head, this G you do not consider at the present moment or better I do it this way.

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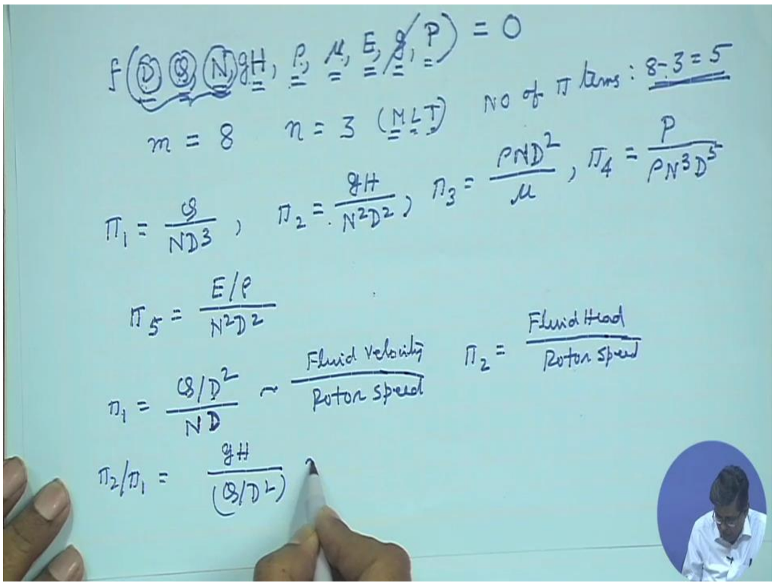

That fluid machine, if we, like that is described other dimensional variables like this, the diameter, the representative rotor diameter, the volume flow rate Q, the rotational speed N, the head available by the fluid or head developed by the fluid depending upon the type of the machine, then density, rho, viscosity mu of the fluid and elastic, coefficient of elasticity. In case of change of density of the fluid and the compressibility effect has to be taken into account, elastic force comes into picture.

So local gravity G and the power, either the shaft power output or the power input to the shaft depending upon whether it is turbine or compressor. So a fluid machines problem can be explicitly, can be expressed implicitly in terms of the function relations. This has to be written from the understanding of the physics of the problem. So a characteristic dimension, flow rate through the machines, rotational speed, head of the fluid, rho and mu, the density and viscosity of the fluid, elastic force of the fluid, G is the local gravity and the power.

Usually the local gravity is taken as an independent dimensional variable where there is a free surface in the physical problem. In absence of a free surface, the G is not considered. In a fluid machine we do not have usually a free surface. So therefore G is not taken into account, whether G is multiplied with H to indicate this is, H is the head and GH is the energy per unit mass, okay, available by the fluid. So if these are the parameters, nondimensional parameters or variables defining the problem, then M as per our nomenclature of Buckingham's pie theorem, total number of dimensional variables is 1, 2, 3, 4, 5, 6, 7, 8. M is 8.

And number of fundamental dimensions remains 3, that is M mass, length and time. No temperature is there. So all the quantities are expressed by the 3 nondimensional terms. So therefore number of pie terms, so therefore number of pie terms will be 8-3 is equal to 5. Now here we can again apply the Buckingham's pie theorem by taking D, Q, N as the repeating variables. If you take D, Q, N as the repeating variables and following the Buckingham's wife there in the same way as I have done for a pipe flow problem, you will find out that different pie term as this.

Pie 1 is Q by N D Q, pie 2 is G H by N square D square, pie 3 is rho N, DN sorry, ND square by mu. Pie 4 is P by rho N Q D 5. And pie 5 is E by rho divided by N square D square. So by routine applications of Buckingham's pie theorem using D, Q, N as variable we can arrive at these 5 pie terms, which is determined by the Buckingham's pie theorem 8-3 is 5 and all these terms are dimensionless terms. And therefore for a fluid machine, these terms, this Q by N D cube GH by N square D square rho N D square mu, P rho N cube D, all nomenclature have already been defined.

E by rho N square D square represents the similarity parameter of fluid machine. That means fluid machines of particular kind to be similar in the operation, whether it is in practice or in laboratory, provided these terms remain same. Now we will see the physical understanding of these terms. Let us 1st start with pie 1, let us 1st start with pie 1. Pie 1 is can be written in this way Q by D square divided by ND and this is proportional to Q by D square is the flow rate divided by D square is proportional to any area.

So, therefore this is proportional to fluid, the numerator can be considered as a representative of the fluid velocity, whereas the denominator ND, D is the representative rotor diameter, N is the rotational speed, so this is rotor speed. So therefore the 1st pie term can be seen as the ratio of fluid velocity to rotor velocity. Similarly the 2nd pie term can be written as GH by N square D square, so GH is the energy, GH is the fluid head, that means fluid energy. It can be written as fluid head, this is proportional to fluid head divided by rotor proportional to fluid head to rotor speed or rotor velocity.

And you can make pie 2 by pie 1, if you want to have a more meaningful term, then this will be GH pie 2, this is pie 2 by pie 2 by pie 1 if you make, then GH by, that is ND will cancel, this will be proportional to pie 2 by pie 1, if you make fluid head, this is whole square, I am sorry, this is whole square, fluid head divided by fluid kinetic energy. So you can see this way, physical interpretation that all this nondimensional term gives the ratio of some same physical entity.

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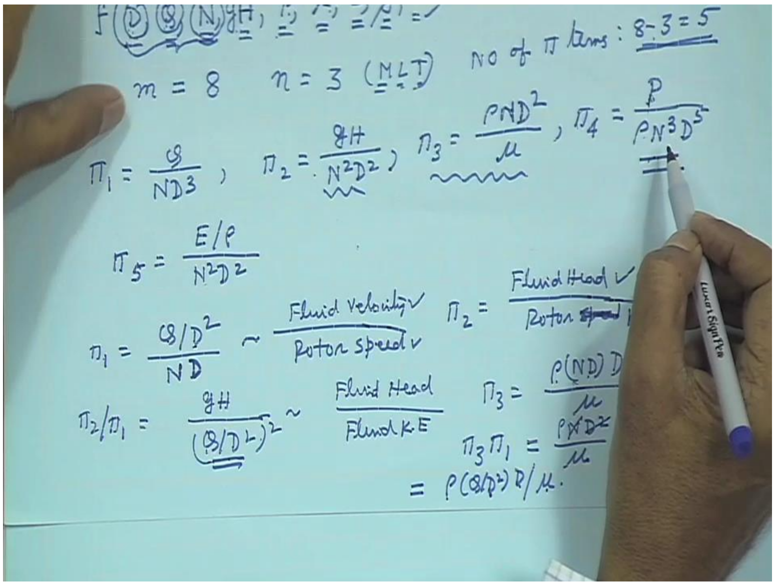

Fluid velocity to rotor velocity, fluid head to, that is not rotor speed, rotor kinetic energy, sorry rotor kinetic energy because this is V square, rotor kinetic energy, I am sorry and if you divided it by pie 2 by pie 1, if you make pie 2 by pie 1, this will be GH divided by Q by D square whole square that is proportional to square of the fluid velocity, so fluid head by fluid kinetic energy. Now if you make similar way, if you see pie 3, pie 3 if you see is equal to rho, now this ND is a sort of rotor velocity into D by mu.

That this is Reynolds number based on rotor velocity. Because this is representative rotor velocity, rho into V into D by mu. And if you make a pie 3 and pie 1 combination, then you get the Reynolds number based on fluid velocity, that means if you make pie 3 into pie 1, if you multiply pie 3 with pie 1, then you will get that N D square will cancel and ultimately it becomes the Reynolds number based on fluid velocity. If you combine the pie 3 and pie 1, so that you get pie 3 pie 1 is the Reynolds number.

What is pie 3 into pie 1, you make it, pie 3 is rho ND square, rho ND square by mu and what is pie 1, Q by ND cube. That is rho ND square by mu, Q by ND cube. This can be made as N N cancel, so D square, it is D, Q by D, so Q by D square into D, you can make it divided by mu. This can be written as rho, Q by D square is the V into D, okay, divided by mu. So therefore rho, fluid velocity times D by mu. So this can simply seen Reynolds number either based on rotor velocity if it is pie 3 or combination pie 3 pie 1 gives that.

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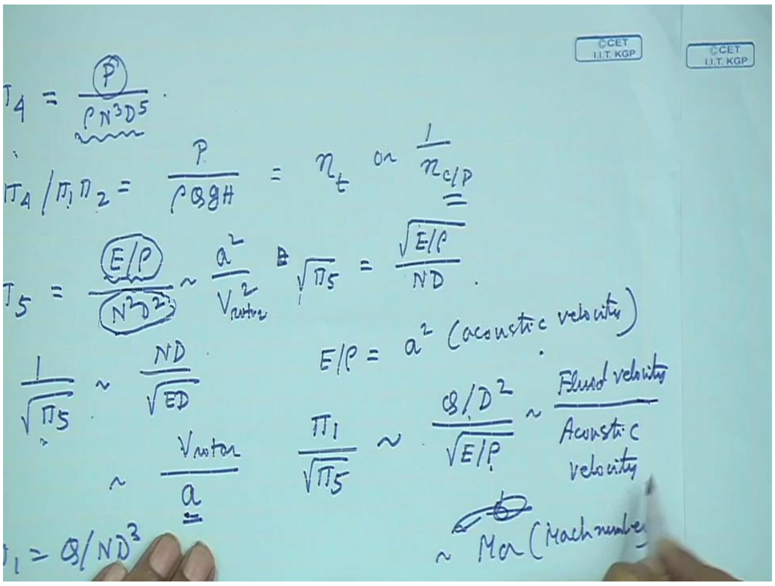

Pie 4 itself is a nondimensional power, pie for gives a nondimensional power P by rho Q GH. So therefore this can be expressed in terms of the efficiency of the machines, how, pie 4 is a nondimensional power, P by, what is that, rho N cube D 5 is a nondimensional power. Okay. But what is its physical significance. If we have to find out the physical significance, you have to make this combination pie 4 by pie 1 pie 2. If you make this pie 4 divided by pie 1 and pie 2, then you get an expression P by, you do it, I am not doing it here, rho Q GH.

That means pie 4, this divided by pie 1 into pie 2, in the denominator pie 1, pie 2. If you substitute the values you get, so this is actually the efficiencies in case of power output turbine or 1 by efficiency in case of compressor or pump. That means this pie 4 combined with pie 1, pie 2 gives the concept of efficiency, otherwise it is a nondimensional power, that means the denominator is power and numerator is also giving a concept of power, physical concept of power, its unit is also power. So nondimensional power.

Now the last term pie 5, as you have seen E by rho, this gives a very important thing. Now if you make a square root of this, so square root of pie 5 is root over E by rho divided by ND and 1 by root over this also is a measure of the velocities. You see why I am making this reciprocal because you see here, this is the square of some velocities, this is also square of some velocities. What is that velocity, you know, the E by rho equal to the square of the acoustic speed, acoustic velocity.

That is the velocity of sound in that medium relative to the fluid. So that is the acoustic speed. And this is the rotor speed, square of the rotor speed, so square of the, this is proportional to square of the acoustic speed and square of the, to make it more conventional I make a square root and take the reciprocal, then it can be written as representative of the rotor speed by the acoustic speed, that is the speed of sound in the fluid medium relative to the fluid flow at that state.

This is V by rotor by A but if you make this pie 1 divided by root over pie 5, this will be still meaningful, that means this becomes Q by D square divided by root over E by rho. If you make it, you know pie 1, pie 1 is Q by ND cube. So if you now pie 1 by root over pie 5, 1 by root 5 is this, so Q by ND cube if you make, it becomes Q by D square E by rho and this becomes is equal to proportional to fluid velocity because this is fluid velocity as we have already seen, Q by D square divided by the acoustic velocity.

That is velocity of sound in the fluid medium at that state relative to the fluid flow and this ratio is known as, that is why I am going to all this operation, this ratio is known as, this ratio is known as this, this ratio is known as Mach number, which is the ratio of representatives fluid velocity divided by the acoustic velocity, that is the velocity of sound in the fluid at that state relative to the fluid. So therefore you see that the pie 5 term gives rise to this Mach number.

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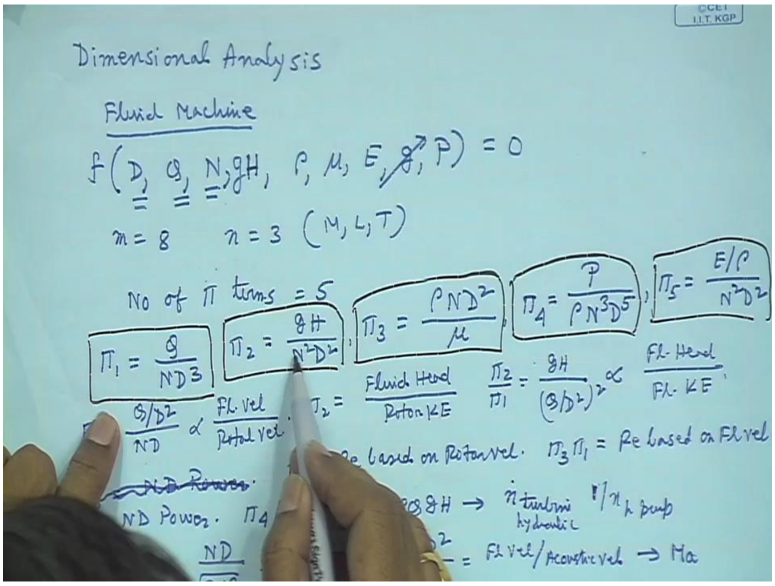

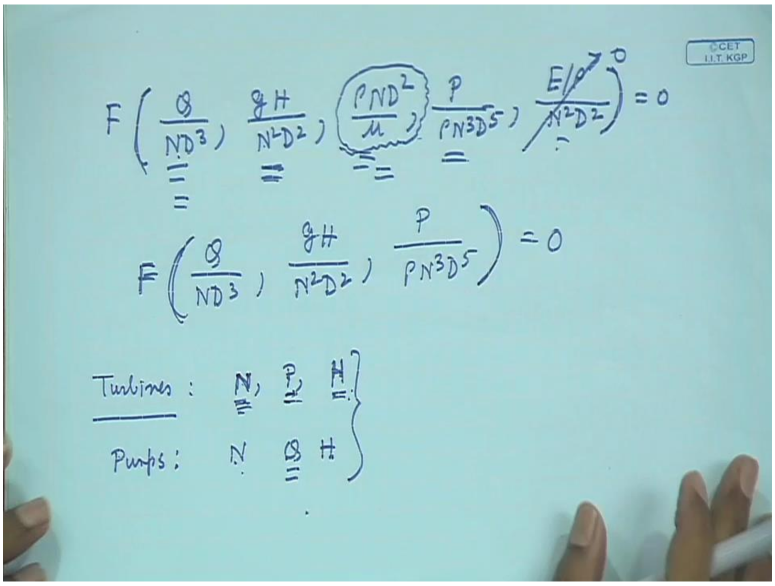

So therefore we get an understanding or apprehension of all these physical concepts of all these pie terms that we get from this dimensional analysis pie 4 and pie 5. Now again, so therefore a fluid machines problem can be expressed by a functional relationships of nondimensional term Q by ND cube, GH by N square D square, rho ND square by mu, P by rho N cube D 5 and E by rho N square D square is equal to, that functionality.

That means instead of 8 number of variables, we can express in 3, 4, 5 number of nondimensional pie terms by the use of Buckingham's pie theorem following the concept of physical similarity. So therefore all these numbers have to be same. Now for fluid machines of a particular kind, now we have to be very careful in this discussion that a fluid, that is similarity principle is sought for a particular type of problem governed by a particular physics.

Now fluids are of different kind, the principles of impulse machines and principles of reaction machines in details are not same. For example a fluid kind is determined whether it is impulse or reaction, whether it is radial flow or axial flow because in radial flow there is a change in the centrifugal, the centrifugal force is acting because of the rotation of the fluid which is not there in an axial flow.

So therefore for a fluid of a particular kind, for example an impulse machine, without any radial flow, tangential flow impulse machine which I will discuss afterwards or for a reaction machine of radial flow type or reaction machine of axial flow type, that means for a fluid of a particular kind, if we have different operating conditions, the problems have to be similar, provided these pie terms remain same for these 2 problems. Now this particular kind, the number of fluids of this particular kind is known as fluids in homologous series.

This kind, this series or the kind represented by this fluid by the different machines which vary in the geometrical dimensions form a homologous series. So that is why we tell that fluid of a homologous series, so therefore we always tell that the for fluids of a homologous series to be similar in the operation, provided these terms are same in both the cases where the investigations are being carried out.

Now if we disregard the compressibility effects, if we deal with incompressible flow, this we can neglect and in many cases the influence of viscosity is not that significant but not always, so in those cases we can get rid of that and we are only with these 3 pie terms and we can tell that the functions of these 3 pie terms, 0 for a special case for handling fluid handling incompressible flow, incompressible fluid, the flow is obviously incompressible in that case and P by rho N cube.

The 3 pie terms only where the influence of viscosity is secondary or has a limited importance or a limited value. But not always in many of the instances there we can only deal with 3 nondimensional parameters. And the fluid machines performance are also expressed in terms of these nondimensional parameters. Now what happens is that, usually in case of turbines, usually in case of turbines, usually in case of turbines we are interested with the operating parameters, rotational speed, power and head.

And in case of pumps, we are interested with the parameters N, Q and H, what is meant by that we are interested. That means turbines are usually specified by these operating parameters, what is the head available by the fluid, what is the power to be developed by the turbine and what is a rotational, what should be the rotational speed of the turbine. In the case of pumps or compressors, instead of power, this comes as a fluid flow.

So therefore our practical interest is that, for example for a turbine, we know that this will be the range of operation N, P and H, that means the rotational speed, power and head to be developed. Then power to be developed, sorry head available. Then what we have to know, which is very important that within this range of operating parameters N, P, H, which machine, that which homologous series or machines of which homologous series will give the highest possible efficiency within this range.

That means which will be most suited for this particular purpose. So therefore we have to know the efficiency of the machines in terms of these parameters, operating, practical operating parameters. In case of turbine N, P and H, that is rotational speed, power and head available. In case of compressors or fans, pumps and also fan, that is N, rotational speed, flow rate and head to be developed.

So therefore it is required that dimensionless parameters should be obtained in terms of these N, P and H and N Q and H instead of this conventional or these parameters which are straightforward obtained by the application of Buckingham's pie theorem.

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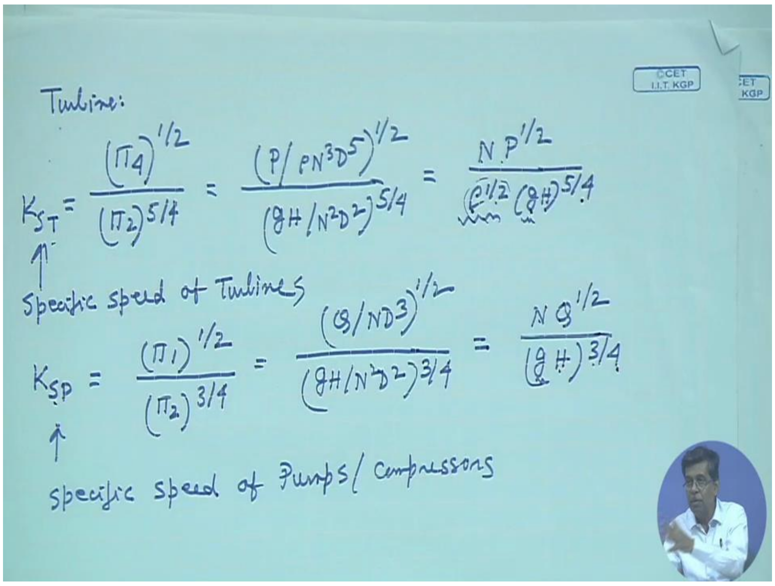

So this can be done in case of turbine 1st, I tell you, in case of turbine, this can be done by making this combination pie 4 to the power half, divided by pie 2. If you see that pie 4 pie 2 which I have did earlier, the term pie 2 pie 4, that means this is our pie 4, this is our pie 2. So if we do that, then this can be written like this. P by rho N cube D 5 is our pie 4 to the power half divided by GH by N square D square to the power 5 by 4. And this becomes is equal to NP to the power half divided by rho to the power half GH to the power 5 by 4.

And this parameter is defined as K ST, that is specific speed of turbine, speed of turbine. That means if we make this combination, the D parameter will be out, that means we want a dimensionless term not in this form but irrespective of the physical dimensions. That means we want a dimensionless term involved in this quantity only, not the physical dimensions. For pumps and compressors this, because these are of over practical interests and we want to find out machines of high efficiency in the particular range of operations given by this parameters in practice.

So therefore we eliminate D from the pie term by this typical combination and get this nondimensional term which is known as specific speed of turbine. Similarly in case of pumps, the specific speed of pump is given by a combination of pie 1 to the power half divided by pie 2 to the power 3 by 4 and pie 1 as you remember, Q by ND cube to the power half divided by pie 2, that is GH by N square D square, that is pie 2 to the power 3 by 4. If you make such physical combination, you get terms nondimensional terms because these are all nondimensional terms free from the diameter, rotor diameter but involving N, Q and H.

So therefore this term is known as, this specific K SP, specific speed of pumps. Pumps, turbines, it is pumps or compressors. So therefore these are the specific speeds. Now before I tell you the physical significance of the specific speed, I tell you one thing that this is the dimensionless specific speed. Usually in practice if we handle incompressible fluid where variation of rho is discarded and in local gravity there is no variation for our practical purposes we define a dimensional specific speed by discarding these terms which is same as that of the nondimensional specific speed in their sense of similarity or in their physical explanation, they are proportional to that.

Because they will be varying by this scale factor rho and G. That means this will be NP to the power half H to the power 5 by 4. In case of compressor, it will be NQ to the power half H to the power 3 by 4. That means if you discard this rho and G, here also G because local gravity is same and if we neglect the influence of rho for incompressible fluid.

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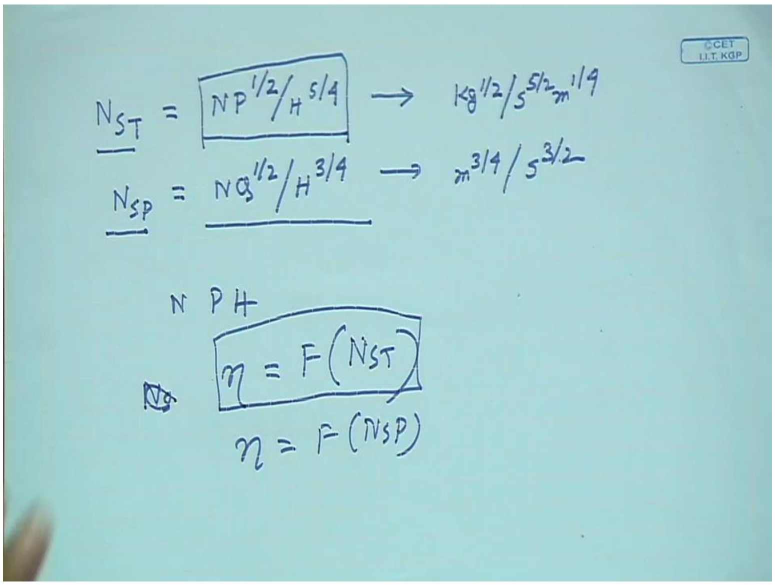

And if the different machines handle the same fluid, then we for our practical purpose can define a dimensional specific speed for turbine as N, so therefore when you discard this rho G, then this becomes dimensional 5 by 4 and N SP, that is dimensional specific speed, in practice, this quantities are known as specific speed, they do not call dimensional nondimensional, specific speed of turbine in practice is given by this, specific speed of pumps or compressors in practice is given by this and they come from the same Genesis of the principal of similarity by combining the pie term this way.

Then by neglecting this G and rho term, therefore for using the same fluid and neglecting the change of density in their operations, we can define the dimensional specific speed or simply this told as specific speed like that. But this has a dimension, if you find out the dimension, this dimension will be in SI unit KG to the power half divided by second to the power 5 by 2, if you do it meter to the power one 4^{th} and this will be meter to the power three fourths divided by second to the power 3 by 2. This is the specific speed.

Now I come to this physical meaning of the specific speed. What is done, up to this we have started from the relations of this basic 5 terms from the principal of similarities we discussed these 3 pie terms or we can retain all the 5 terms depending on the cases where the viscous effect is prominent and the compressible effect is prominent, fluid is handling compressible fluid, so whatever may be the situation, we then discuss that we are concerned about the 3 performance parameters, now I will emphasise on this and to have nondimensional parameters irrespective of D and having only those performance parameters we make such combinations for turbine and define the specific speed of turbine as dimensionless quantity or nondimensional quantity, specifically the pump by this typical convention combination as this.

And if we discard rho and G, there are variations, G is same and rho, there are variations, if they use same kind of fluid and if it is incompressible, this variation is neglected, then in practical purposes, dimensional specific speed. Now the concept is that, for example it turbine, if we have a particular application for turbine where you know the 3 quantity N, P and H. Okay. That you know 3 quantity N, P and H, then what we do, we 1^{st} calculate the specific speed of the turbine, N P to the power half, H to the power 5 by 4.

Then we find out from the given information either in the form of a figure or in the form of a chart that which category of machine or which homologous series machine, whether it is a tangential flow impulse turbine, whether it is axial flow impulse turbine, for example in case of hydraulic turbine, if it is a radial flow impulse, reaction turbine, axial flow reaction turbine, this defines different homologous series as I have told that which type of turbine or which homologous series gives the maximum efficiency under this operating condition range.

That means the specific speed, this we will find out the specific speed, the efficiency as a function of specific speed NST, efficiency as a function of NSP. So we know this, so therefore from a particular type of machine or particular homologous series, we try to find out which homologous series gives the highest efficiency or very high efficiency for this specific speed. We find out the range of specific speed, it is quoted for a given particular value of P, particular value of N… For example turbine is related for this N, this P, this H, means the turbine should give the maximum efficiency, it should work at maximum efficiency, very high efficiency, maximum means at very high efficiency, with this value of N PH.

So what is 1st job, to find out the specific speed with these values of NPH and to see which category of machine gives the highest possible efficiency for that value of specific speed. That type of category of machine should be chosen to suit the purpose, for that purpose we have to know that category of machine is the best fit or best suited for that purpose and that is the 1st point of design of fluid machines. Okay. Similarly for pumps also. So this will be explained again afterwards when we will describe the fluid machines.

So therefore the concept of specific speed brings out a specific parameter representative parameter of a particular series, that is the concept of specific speed, means when we tell that what is the specific speed of a particular machine, it theoretically it may not seem anything because specific speed is NP 2 the power half H to the power 5 by 4 for turbine, NQ to the power half H to the power 3 by4 for pump. So therefore there may be N number of specific speed depending upon the values of N PH and we make that combination.

No, when we tell that this is the specific speed for this group of machines or this homologous, for example impulse machines, Pelton wheel is one impulse machine which will be discussing in the next class that for a Pelton wheel, that hydraulic turbine, this is the specific speed. Means that at this value of this combination, this NPH, this value of specific speed, this turbine gives the maximum efficiency, this turbine gives the maximum efficiency.

And therefore for all the specific speeds where these turbines are giving their maximum efficiency we compare. And depending upon the specific and we know the specific speed from the calculations of the performance parameters required or rated for the practical purposes, we compare these 2 that which specific speed gives the highest efficiency for a particular class, that class we will choose to suit for that particular application. So this is the basic concept of specific speed and that is why the specific speed is known as shape factor of a machine, shape factor of a machine.

For some specific speed, particular series of machine give their maximum efficiency. Okay, so this concept should be made very very clear that the specific speed, this combination is very important and fluid machines are quoted by the specific speed at which their efficiency is maximum and we compare the maximum efficiencies for the given, for the calculated specific speed required for our purpose. And the machine which gives the highest efficiency at that specific speed, we choose that machine category, machines means that category, that homologous series machine.

Okay, that is in fact the concept of specific speed of fluid machines but the specific speed, the genesis of this that originate from the basic principle of similarity, that a fluid machines is governed by 8 dimensional variables, by Buckingham’s pie theorem, following the principle of similarity, we derive 5 nondimensional pie terms governing this physical problem and these are similarity parameters and from there we make a combination to eliminate the and try to find out the nondimensional terms including only N PH or A in case after by, NQ H case of pump and that would define as specific speed which serves this physical purpose which have this physical implication which I have discussed just now. Thank you.