

Fluid Machines.
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Lecture-4.
Principles of Similarity in Fluid Machines.

Good morning and welcome you all to this session on fluid machines. In the last class discussed the axial flow and radial flow machines depending on the direction of flow. And next we discussed impulse and reaction machines. Now today we will discuss mainly the principle of similarity applied to fluid machines. But before that we will just see how the efficiencies of fluid machines are defined. As I have already told and you have seen that with machine is a device which converts stored mechanical energy, stored energy in the fluid into mechanical energy and vice versa.

The efficiency accordingly of a fluid machine is defined as the ratio of output energy divided by input energy. Now output energy maybe mechanical energy delivered by the machine or the energy stored in the fluid depending upon whether it is turbine or pump. Similarly the input energy will be mechanical energy, sorry, stored energy in the fluid or the mechanical energy depending upon it is turbine or pump. And in this connection, two efficiencies are defined, one is hydraulic efficiency concerning the energy transfer between the fluid and the rotor and in another one is overall efficiency concerning with the energy transfer between the fluid and the shaft.

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Hydraulic efficiency η_h Overall efficiency η_o

Turbines:

$$\eta_h = \frac{\text{Mechanical Energy Delivered by the Rotor}}{\text{Energy of Fluid at inlet to Rotor/Machine}}$$

$$\eta_o = \frac{\text{Mechanical Energy in output shaft at coupling}}{\text{Energy of Fluid at inlet to Machine}}$$

$$\eta_m = \frac{\eta_o}{\eta_h} = \frac{\text{Mechanical Energy in Shaft}}{\text{M.E. at Rotor}}$$

↑ Mechanical efficiency $\eta_o = \eta_h \times \eta_m$

Now the difference in energy at the rotor and at the output shaft is the energy absorbed by glands, bearings and other couplings due to friction. Now, if we see that in detail, let us write this, there are 2 efficiencies, one is hydraulic efficiency, one is hydraulic efficiency which is given by η_H which deals with the energy transferred between the fluid and the rotor. Another is overall efficiency, overall efficiency, overall efficiency η_O which is concerned with the energy transfer between the fluid and the output shaft.

Now let us define separately for turbines. Hydraulic efficiency is defined as the output energy of the turbines, what is this, this is the mechanical energy delivered by the rotor, that is mechanical energy delivered by the rotor, mechanical energy delivered by the rotor, mechanical energy delivered by the rotor divided by, that is in the denominator is energy, that is the stored energy of fluid at inlet to rotor or the machine, fluid machine. This energy of fluid at inlet is the available energy, stored energy from which the conversion takes place in the rotor and rotor delivers mechanical energy in the form of its rotation.

Now here this energy available to the rotor at the inlet to the rotor and machine is same since the Stator is fixed member, so does not move, so there is no mechanical energy transferred. So therefore if we neglect the heat loss from the machine and the viscous dissipation, the mechanical energy at inlet to the machine and mechanical energy at inlet to the rotor remains same, this is because stator remains fixed.

Now if you think of not mechanical energy, the total energy inclusive of intermolecular energy also which we call as thermal energy, then if we neglect the heat transfer loss from the machine or no heat transfer between the machine and the surrounding, so by the conservation of energy, energy at the inlet to the stator or the inlet to the machine will remain as inlet to the rotor because stator does not perform any mechanical work. So therefore we write energy of fluid at inlet to rotor or machine.

The overall efficiency is now, the denominator remains same, the input energy, energy of fluid, same thing, energy of fluid at inlet to machine or rotor, whatever you write and this thing is the mechanical energy, mechanical energy in output shaft at coupling, in output shaft at coupling. And the difference between the 2 is the friction in glands, bearings and couplings. And if we now divide this overall efficiency by the hydraulic efficiency, overall efficiency if you divide it, this thing is cancelled, energy of fluid at inlet, what we get, we get mechanical energy, mechanical energy at shaft, in shaft, in output shaft.

I am not writing the entire sentence and here the mechanical energy at rotor delivered by the rotor, I am not writing at rotor. That means this becomes overall efficiency divided by the hydraulic efficiency, mechanical energy in shaft output shaft divided by the mechanical energy delivered by the rotor. That means mechanical energy at rotor. And this is always less than this because of the mechanical friction in glands, bearings and couplings. So therefore this ratio is always less than 1 this is defined as mechanical efficiency.

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The image shows handwritten notes on a whiteboard. At the top right, there is a small logo that says 'JCEET U.T. KGP'. The main text is under the heading 'Pumps / Compressors'. It defines three efficiency ratios:

$$\eta_h = \frac{\text{Mechanical Energy Gained by the Fluid at final discharge}}{\text{Mechanical Energy supplied to the Rotor}}$$

$$\eta_o = \frac{\text{Mechanical Energy gained by the Fluid at discharge}}{\text{Mechanical Energy supplied to the shaft}}$$

$$\eta_o / \eta_h = \underline{\underline{\eta_m}} = \frac{\text{M.E to Rotor}}{\text{M.E to Shaft}}$$

This is eta M, mechanical efficiency. And hence we can write the overall efficiency is hydraulic efficiency into mechanical efficiency. Okay. Now next is, similar is the case of pumps and compressors. In case of pumps and compressors, in case of pumps and compressors, we can define similar way the hydraulic efficiency, here output is the mechanical energy gained by the fluid, that is mechanical energy, mechanical energy gained by the fluid, gained by the fluid at final discharge, means discharge from the machines, at final discharge.

And the denominator is mechanical energy supplied by the rotor. Mechanical energy supplied to the rotor or by the rotor, supplied to the rotor. Out of which how much energy is gained by the fluid at final discharge, that is the hydraulic efficiency. That takes care of the losses in the fluid flow, similar is the case of turbine. So this is the definition of hydraulic efficiency of pumps and compressors. Now here mechanical energy gained means this mechanical energy at the final discharge relative to its mechanical energy at inlet because fluid at inlet has some mechanical energy, that is why mechanical energy gained by the fluid at final discharge.

And in the similar way, following the similar line of thought as we discussed in case of turbines, the overall efficiency in this case the output will remain same, mechanical energy gained by the fluid, by the fluid at discharge, final discharge or simply at discharge, now here mechanical energy supply to the rotor, here input will be mechanical energy supplied to the shaft. That means here the final, initial input point is shaft, like the turbine, the final output point is shaft.

So therefore overall efficiency deals with this energy supply to the shaft and the energy gained that final discharge and in the similar way, hydraulic, overall efficiency divided by hydraulic efficiency is the mechanical efficiency which is the ratio of what, ratio of, if you divide this by this is the mechanical energy supplied to the rotor divided by mechanical energy supply to shaft. That means mechanical energy to rotor and I am writing in short, mechanical energy to shaft, input shaft.

And this is always less than 1 because energy supplied to shaft is not going to the rotor because of the frictions in those elements glands, bearings and couplings, so rotor receives less energy. So this is the mechanical efficiency. So this is way usually the efficiencies of the fluid machines are defined. Now we come to a very very important chapter or very very important section is the principle of similarity applied to fluid machines. So before I discuss this principle of similarity applied to fluid machines, I like to have a recapitulation or a brief review of the concept of principle of similarity in physical processes.

Okay. To brush up your knowledge that already you have gathered in your basic fluid mechanics course. As you know that most of the engineering problems or the problems related to the practical applications, their solutions are determined from experiments. This is because of the complex nature of the practical problems or engineering problems in practice, where besides the analytical method, even the most updated CFD tool cannot take care into consideration of all the aspects and features that happen in practice are associated with the problem.

So therefore we always depend on experiments, even if you do theory, these are being calibrated from the experiments. Theories are done, the CFD analysis are made to guide the experiments in which direction the experiments will go and to predict the qualitative trend. But to have a quantitative final solution of those engineering problems, we always do the experiments and their solutions are obtained mostly from these experimental studies.

Now because of economic advantage, saving of time and these of investigations, it is not possible in almost all instances to do the experiments in laboratory under the identical conditions in relation to the operating parameters and the geometry of the system that prevail in practice, it is not possible. So therefore we have to do the experiments under altered conditions that happen in practice. For example, if you have to do an experiment to find out the pressure distributions over an aircraft wheel, we have to do the studies but where we cannot have the aerofoil of the dimensions that an actual aircraft does have.

We cannot have the fluid velocities, the condition of pressure and temperature similar to that happen in practice. Similarly for example if we think of simulating or studying the flow through pipes which will generate data to predict the performance of the pipe networking practice, we cannot do the experiment with such big dimensions of pipe, big diameters of pipe, length of pipe and sometimes we cannot have that flow rate, we cannot generate that flow rate in the laboratory.

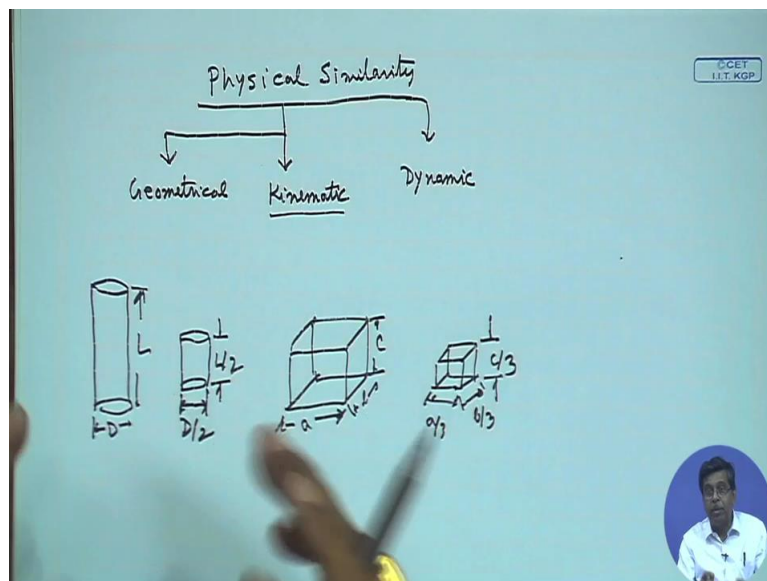
For example to find out the drag in a ship or submarine by our, in our, we do experiments, if you want to do experiments in our laboratory, we cannot have a ship or submarine of its actual dimension and we cannot create a ship in our laboratory. So therefore laboratory tests are always performed under altered conditions that happen in actual practice. But at the same time the results from these tests have to be used to predict the performance of the actual system under actual operating conditions. So now in relation to these, there are 2 very pertinent questions come up.

One is that, which ensures us that the results from these altered conditions, tests at altered conditions in the laboratory will be, can predict the performance parameters of the actual system which are operating under different conditions in practice. How we can compare these, what has to be satisfied to make comparisons or to use these results for predictions of parameters in practice, Number-one.

And number 2 question is that if a physical, if a physical process depends upon a number of independent controlling parameters, then to find out the influence of each and every parameter, we have to do a number of experiments by varying each and every parameter by keeping other parameters fixed. So which involves huge cost, huge time, okay. Can we do this way to save time and cost that vary some less number of variables and can predict the performance, parameters, the influence of the variable, other variables on the performance parameters of the system.

Now these 2 questions are very important and a positive clue in answering these 2 questions lies in the principle of physical similarity. That means this can be done, that means the results from the laboratory at altered conditions can be used to predict the performance of the same process in practice at different conditions and at the same time we can reduce the number of experiments by varying only few independent controlling parameters so that the influence of other controlling parameters can be determined from that, provided the physical similarity exists between the 2 problems.

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So therefore I come to this physical similarity. Now physical similarity, the 1st line of understanding is that, physical similarities has to be sought between 2 problems defined by the same physics. First of all we have to understand that the physical similarity, always, if we say that the 2 problems are in 100 percent or identical similarity, have physical similarity but the problems that may be defined by the same physics. The 1st requirement of physical similarity, that physical similarity has to be sought between the problems governed by the same physics.

For example the flow governed by gravity force and inertia force, for example a surface wave in a sea and another process that flow of fluid through a pipe which is governed by the inertia force, viscous force and pressure force cannot be made similar under any identical condition. We can make similar situations of pipe flow problems at different operating conditions of the surface, problems relating to surface wave at a different operating conditions. So the physics

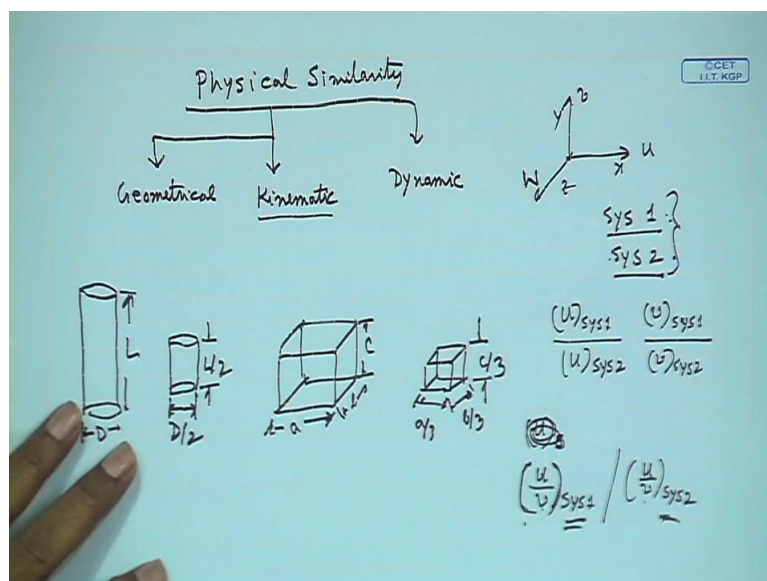
of the problem has to be same. And in that case, the physical similarity to be maintained or to be obtained, we have 3 types of similarities have to be ensured.

One is the geometrical similarity, 1 geometrical, another is kinematic similarity and another is dynamic similarity. Geometrical, kinematic and dynamic similarities. Now geometrical similarity is the similarity of dimensions which tells that the system dimensions in actual case and the laboratory case should be such that the ratio of the corresponding dimensions should be same and it is maintaining the same shape. And this we know since our childhood, an example I am giving.

Let us consider the 2 cylinders whose length is $1L$ and the diameter D , if we have got another cylinder whose length is L by 2 and the diameter is D by 2 , they are geometrically similar. Similarly if we have a parallelepiped if we have a parallelepiped of this A , this is B and this is C and if we have a small parallelepiped where this A , B , C are reduced by the same proportions, say this is A by 3 , this is B by 3 and this is C by 3 , than they are geometrically similar. Now what is kinematic similarity, kinematic similarity is the similarity of motion that requires that the ratio of the velocity, and regular component of velocity at the different system should be same at corresponding point.

That means the ratio of the velocities of a particular velocity, that means particular velocity component of both the system and the in the laboratory, both the system, system in the laboratory and that in practice should be at the same ratio at the corresponding points.

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Let us define this way, let us have U, V and W, 3 components of velocity, XYZ. If we have a system one, if we have a system 2. And we sought the kinematic similarity between these systems, then U for system 1 divided by U for system 2, this ratio has to be same at corresponding points of the system 1 and system 2. Similarly V of system 1 divided by V of system 2 will be same at corresponding points, similar is the case of W. And accordingly one can write this also, U, sorry, this is U by V of system 1 divided by U by V of system 2 at the corresponding points has to be also same for both the systems.

So therefore you see the ratios of the corresponding velocities will be same at corresponding points between the 2 systems and accordingly the ratios of any 2 components of velocity system 1 and system 2 will be same at the corresponding points of the 2 systems. This ensures the kinematic similarity. Similarly the dynamic similarity will ensure the similarity of forces.

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Dynamic Similarity: Similarity of Forces.

I.F., V.F., G.F., S.F., EF, P.F, ...

I.F. ~ mass x acceleration
 $\sim \rho l^3 \times \frac{v}{l/v}$
 $I.F. \sim \rho l^2 v^2$

V.F. ~ Shear stress x Area.
 $\sim \mu \left(\frac{dv}{dy}\right) \times \text{Area}$
 $\sim \mu \frac{v}{l} \times l^2$
 $\sim \mu v l$

$\frac{I.F.}{V.F.} \sim \frac{\rho l^2 v^2}{\mu v l} \sim \frac{\rho l v}{\mu}$
 ↑ Reynolds number

So dynamic similarity is the similarity of forces, dynamic similarity is the similarity of forces, similarity of forces, dynamic similarity is the similarity of forces, similarity of forces. Now problems, physical problems are governed, all physical problems or physical processes are governed by the different forces. Let us consider few such forces. Inertia force, I am writing in short, I am telling inertia force, viscous force, gravity force, surface tension force, elastic force and so on.

A particle problem, a particular process may not be governed by all the forces, some processes may be governed by inertia force, viscous force, pressure force, another force is the

pressure force, inertia force, viscous force, pressure force, some may be governed by inertia force, gravity force, some may be governed by inertia force, viscous force, gravity force. So to ensure the dynamic similarity, the ratio of the forces at the corresponding points between the system and, between the 2 systems, system 1 and system 2 between which the similarity is sought for has to be same.

So to make, to make this, we have to find out the parameters that define the inertia force and viscous force. Now here we know the ratios of velocity we get in terms of velocities but when you say that the ratio of inertia force by viscous force has to be same for both the system, then we have to know that ratio of inertia to viscous force is given by which parameter, which variables of the fluid flow. Usually we deal with the velocity, pressure, density, viscosity, the fluid property, or the characteristic geometrical dimensions of the problem.

So therefore it is customary to express these ratios of forces in terms of those variables concerned with the fluid flow problems. So this is done by scaling the forces. So how to done, now inertia force, if we scale inertia force is equal to mass times acceleration, acceleration. Now what is mass, mass is, can be scaled as ρL^3 . And acceleration is the rate of change of velocity, so if we take the characteristic velocity as V , so acceleration is V divided by the time and time also we can write in terms of L by V , in terms of the characteristic length and the characteristic velocity.

So that this becomes is equal to $\rho L^3 \frac{V}{L}$, this $L^2 V$. So therefore we see we can scale inertia force in terms of characteristic length and characteristic velocity as $\rho L^2 V$. ρ is the density of the fluid, L is the characteristic length and V is the characteristic velocity. Similarly if we see the viscous force, viscous force is proportional, can be written as shear stress times the area, surface area on which the shear stress is acting.

Now this shear stress from the Newton's law of viscosity can be written in terms of the μ into velocity gradient, for a very simple case it is shear rate but in one-dimensional flow, where U is a function of Y only and there is U component of velocity only, then $\mu \frac{dU}{dY}$ into the area. So therefore the scaling is μ the velocity gradient. This will be only the velocity gradient. So μ into velocity gradient, okay so therefore what you write, velocity gradient is V is the characteristic dimension and what is Y , Y is the length dimension L and what is area, area is L^2 .

So this can be expressed in terms of area and the velocity characteristic, velocity and length as μVL . So therefore inertia force by viscous force by scaling is proportional to $\rho L^2 V^2$ by μVL which equals to ρLV by... This is the L we are writing, the same L why I am writing the big L , ρLV by μ . So therefore we see if we have to give the inertia force by viscous force, this ratio same for both the systems between which the physical similarity is sought, we have to keep the combination of these variables same in both the cases.

And the combination of these variable ρ , L , V , ρ is the density of the fluid, L is the characteristic dimension, geometrical dimensions, V is the characteristic velocity and μ is the viscosity is known as Reynolds number. So in a similar way we can derive various dimensionless numbers.

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$G.F. \sim \text{mass} \times g$
 $\sim \rho L^3 g$
 $\frac{I.F.}{G.F.} = \frac{\rho L^2 V^2}{\rho L^3 g} = \frac{V^2}{g L}$
 $\frac{V}{\sqrt{g L}} \quad \sqrt{g L} / V \rightarrow \text{Froude number}$

Another example I tell you that the problems or process which is governed by both inertia force and gravity force. Now gravity force is scaled as the mass into acceleration due to gravity, mass into G and mass is ρL^3 . So therefore the gravity force or inertia force by gravity force will be $\rho L^2 V^2$ divided by $\rho L^3 G$. So that becomes equal to V^2 by GL and therefore for the problems which are governed by the inertia force than gravity force, their ratio has to be same at all corresponding points between the 2 systems to maintain that V^2 by GL has to be kept constant.

And this ratio, the square root of this, V by root over GL or sometimes root over GL by V is known as Froude number. There is no such convention, sometime this is defined, sometimes

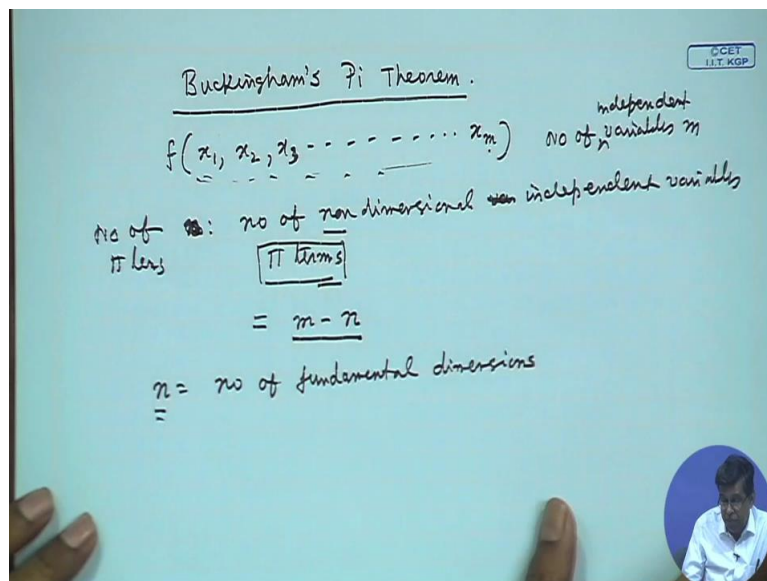
reciprocal of this is also defined as the Froude number. So therefore this is known as Froude number but Reynolds number as a classical convention, it has to be this, inertia force by viscous force, not the viscous force by inertia force. So inertia force by gravity force scale like that.

So this way by scaling the different force we can find out the similarity parameters which have to be made same for both the systems to make the dynamic similarity that the ratio of the forces at corresponding points will be same. Now it is not always possible to do this that the maintaining the similarity, we have to always thought, we have to go for scaling of all forces. So therefore to make it simple, one can find out the nondimensional parameters which keep the principle of similarity, make the 2 systems under physical similar condition by one theorem known as Buckingham's pie theorem.

So again I tell you we have seen that to maintain the similarity between the 2 systems systems, we have to make the ratio of the corresponding geometrical dimensions same, the ratio of the corresponding velocities at corresponding points have to be same, ratio of the different forces at corresponding points between the systems have to be same. All these are nondimensional number and they are usually expressed in terms of the dimensional variables involved in fluid flow like pressure, velocity, density, the property pressure, velocity and the property of the fluid density, viscosity and the geometrical dimensions.

For kinematic similarity ratio of velocities and the geometrical similarity ratio of physical dimension, it is straightforward, apparent. But for ratio of forces, this has to be done by scaling the forces and sometimes it becomes difficult. So a easy method of finding out those nondimensional number which keep all the similarity has been given by Buckingham and known as Buckingham pie theorem.

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Buckingham's pi theorem. Now according to this theorem, Buckingham's pi theorem one can tell that a problem is defined by a number of variables, number of physical dimensional variables, let $X_1, X_2, X_3, \dots, X_m$ number of, M number of physical variables M . Number of variables, dimensionally variables M . Then Buckingham first from his intuitive thinking formed that due to the dimensional homogeneity, the number of independent nondimensional terms governing this problem or the process will be less than the number of dimensional counterparts, that is M .

M is the number of dimensional variables controlling the process and the number of nondimensional variables controlling the process and all these nondimensional variables will come by some of the dimensional terms will be less than that of the dimensional counterparts M . And this was given by Buckingham and he told that if N is the number of nondimensional, number of nondimensional variables, independent variables, rather independent variables, you might name number of here also, independent variables controlling the problem. Number of independent variables, number of independent variables, this is known as pi terms.

Nondimensional independent, if M is the number of pi terms, number of pi terms, then he told that N is the number of pi terms, sorry, here N I will not write, I will write the number of pi terms, pi terms is the number of nondimensional independent variables, this will be equal to $M - N$ where N is the number of fundamental units, fundamental dimensions, number of fundamental dimensions in which the variables are expressed.

Rather it can be told this way, if a problem is defined by M number of independent dimensional variables, then because of dimensional homogeneity, there will be a less number of nondimensional variables controlling the process which is less than the number of the counterpart dimensional variables M. And if these M variables can be expressed in terms of N fundamental dimensions like mass, length, time, temperature, like this, then the number of these nondimensional independent variables pie will be M - N where N is the number of fundamental dimension in which these M variables can be expressed.

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$f(x_1, x_2, x_3, \dots, x_n, \dots, x_m) = 0$
 $m =$ no of variables
 $n =$ no of fundamental dimensions
 $\text{no of } \pi \text{ terms} = m - n$
 Repeating variables $x_1, x_2, x_3, \dots, x_n$
 $\pi_1 = x_1^a x_2^b x_3^c \dots x_n^n x_{n+1}$
 $\pi_2 = x_1^a x_2^b x_3^c \dots x_n^n x_{n+2}$
 \vdots
 $\pi_{m-n} = x_1^a x_2^b x_3^c \dots x_n^n x_m$

Now this can be worked out, now how to find out this number of pie terms. This to find out, we have to do this. Let us consider a problem is 1st defined by its number of M number of dimensional physical variables. Now this line of definition of the problem, a problem is defined by this, means here it has to be maximum implicit implicit functional relationship. Now 1st line of this Buckingham's pie theorem is that we have to write the problem, this mathematical statement of the problem by an implicit functional relation where these are the independent dimensional variables governing this process.

And what are the dimensional, pertinent dimensional independent variables governing the process, that you have to know from the physics of the problem or physics of the process. So after that if we write this thing, if this is the M, M is the number of variables, M is the number of variables. Now we see if these variables are expressed by N number of fundamental dimensions, N is the number of fundamental dimensions, then by Buckingham's pie theorem, the number of pie terms, so pie terms, number of pie terms will be M - N.

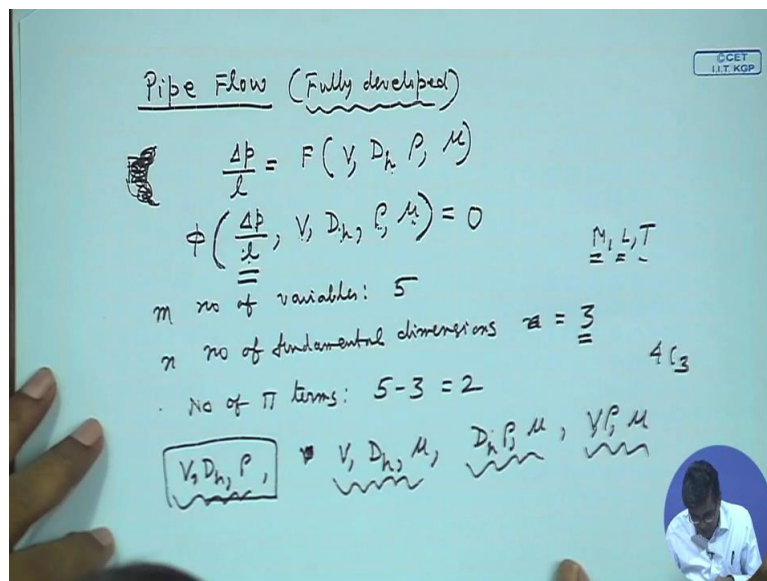
Now how to do it, how to find out this $M - N$ pie terms. Now you first select N number of, N quantities, N number of quantities at the repeating variable. Let X_1, X_2 up to X_N , that means repeating variables with, these are known as reflecting variables. We choose any arbitrarily, any N number of physical dimensional variables, any arbitrarily N number. But there should be a caution. In this repeating variable, N number of repeating variable where N is the number of fundamental dimensions, no variables should be an output parameter of the process, that is number-one caution.

Number 2 caution is that all the variables chosen taken together must have all the fundamental dimensions. We should not choose the variables in such a way that in none of the variable, one of the fundamental dimensions is missing. That means inclusive of all variables, all the fundamental dimensions should be there. And another caution is that no 2, no 2 variables should have the same quantity. That means no 2 variables should be either geometrical dimensions or velocity, so these 2 same physical entities should not be taken.

So if you take so, and you can find out from this, the pie term. How, you make each pie term as raise all the repeating variables raised to the power some unknown X_N to the power N number of indices and taking the rest X_{N+1} . The rest of the $M - N$ variables. So therefore pie 2 will be formed as X , these ABC sets will be different for different pie terms. These sets will be different for different pie terms and it will include X . Like that we will be having $M - N$ items, pie $M - N$ will be $X_1 A, X_2 B, X_3 C$ to $X_N N$ into $X_{N+1}, N + 2 X_M$.

So therefore $N - M, N + 1 M$, okay. So we say, this way we can form the pie terms. Now all AB, A to N sets are different for different pie terms. What we will do now for each pie term, we now replace the dimensional formula of each variable and indices are there. This is nondimensional quantity. That means indices of all fundamental dimensions will be 0. So if we equate each N fundamental dimension indices to 0, we get N number of equations and there are N variables, we solve for A to N from all the pie terms.

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So therefore we get the explicit, the explicit term for this pie term. Okay, so this way the pie terms can be found out. Let us have an example, let us consider the principle of similarity in pipe flow. Let us consider an example of pipe flow problem. A pipe flow problem. A pipe flow, the fully developed flow in a pipe, fully developed flow in a pipe. Now in a fully developed, this is a recapitulation, that is why I am going little fast, fully developed flow we can express that the problem as that pressure drop, rather we write this way, you will be under, you will be able to understand it better.

That in a fully developed pipe flow, pressure drop per unit length of the pipe is usually a function of velocity of flow, some characteristic velocity of flow, some characteristic dimensions of the pipe which we call as hydraulic diameter and the density of the fluid as its property and the viscosity. And this can be expressed as this, implicit functional relationships of Delta P by, this is the output parameter. V, DH, rho, mu, 0. A pipe flow problem is governed by the pressure force and viscous force.

And in fully developed flow, inertia force is 0. So therefore Delta P by L, V, DH, rho, mu defines as in dimensional variables a pipe flow problem. So a pipe flow problem can be defined implicitly by this functional relationship. Now how to find out the, apply the pie theorem, so here the number of variables, number of variables, number of variables are 1, 2, 3, 4, 5. The number of fundamental dimensions, number of fundamental dimensions, number of fundamental dimensions, that is N, that is number of variables M, N the number of fundamental dimensions is equal to, these are expressed as mass, length and time.

All these parameters are expressed in terms of 3 fundamental dimensions. So number of pie terms therefore, number of pie terms therefore equal to 5-3 is equal to 2, so therefore 2 pie terms. Now we see that there are number of, N is 3, so we can have 3 repeating variables. This is the output variable, this should not be there and there are how many choices, there are 4 dimensional variable choices $4C3$, that means we can have many sets of repeating variables.

One set is V, DH, rho, another set is V, DH, this is comma, mu, another set is DH, rho, mu, another set is V, rho, mu. So incidentally all these sets include all the fundamental dimensions. We can take these 3 as repeating variables, we can take these 3, we can take these 3, we can take these 3. Let us take these 3 are the repeating variables V, DH and rho. And if we take this as the repeating variable V, DH and rho.

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Handwritten mathematical derivation on a blue background:

$$\pi_1 = V^a D_h^b \rho^c \left(\frac{\Delta p}{L}\right)$$

$$\pi_2 = V^a D_h^b \rho^c \mu$$

$$M^0 L^0 T^0 = (L T^{-1})^a (L)^b (M L^{-3})^c M L^{-2} T^{-2}$$

$\frac{M L^{-1} T^{-2}}{L}$
 $\frac{M L^2 T^{-2}}{L}$

$$\boxed{a = -2 \quad b = 1 \quad c = -1} \quad - \pi_1 = \frac{\Delta p}{L} \frac{D_h}{\rho V^2}$$

$$M^0 L^0 T^0 = (L T^{-1})^a (L)^b (M L^{-3})^c (M L^{-1} T^{-1})^d$$

$$a = -1 \quad b = -1 \quad c = -1 \quad \pi_2 = \frac{\mu}{\rho V D_h}$$

Then what we have, we have pie 1 is equal to, now we can write here, we have pie 1 equals to V DH rho we have taken V to the power A, DH to the power B, rho to the power C, V, DH, rho, we have taken, so left parameter is Delta P by L and mu. So first we take Delta P by L and this 2nd pie term we take as V A to the power A, DH B to the power, rho to the power C mu. So therefore if we now equate this, pie 1 is what M to the power 0, L to the power 0, T to the power 0, V is L, T to the power - 1 A, L to the power B, M L to the power -3 to the power C.

And what is Delta P by L? M Delta P, Delta P is the pressure were M L to the, M L square, pressure is M L to the power - 1 T to the power -2 and that divided by L. That means M L to

the power -3 T to the power -2, okay. So therefore Delta P is M L to the power -1 , divided by L, M L to the power -2, sorry, M , L to the power -2. What is Delta P, Delta P is the pressure, that is M L to the power -1, T to the power... Okay. Then divided by L. So therefore if we now equate the L, M and T indices, we will get here A is equal to -2, if you do it, B is equal to 1 and C is equal to -1.

If we equate L, $L A + B - 3C$ is equal to 0, similarly for T, $- A - 2$ is equal to 0, N is equal to -2, this way you get this. And if you do the similar procedure for this thing that pie 2, $M_0 L_0 T_0$ is the similar procedure that L T to the power -1 to the power A, L to the power B, M L to the power -3C and mu is the M L to the power -1 T to the power -1, then you get this. In the 2nd situation, A is equal to -1, B is equal to -1 and C is equal to -1.

And if you put this in the pie term, the value of A, B and C, you get pie 1 as Delta P by L, that is the pie 1 as you get DH by rho V square. And pie 2 if you put this, you get pie 2 is equal to mu by rho, V, DH. Okay. So this you get pie, if you put the value of A, B, C here and this you get as pie 2, if you put ABC value we are. So pie 1 will be Delta P by L DH rho by V square and pie 2 will be mu by rho V DH. Thank you.