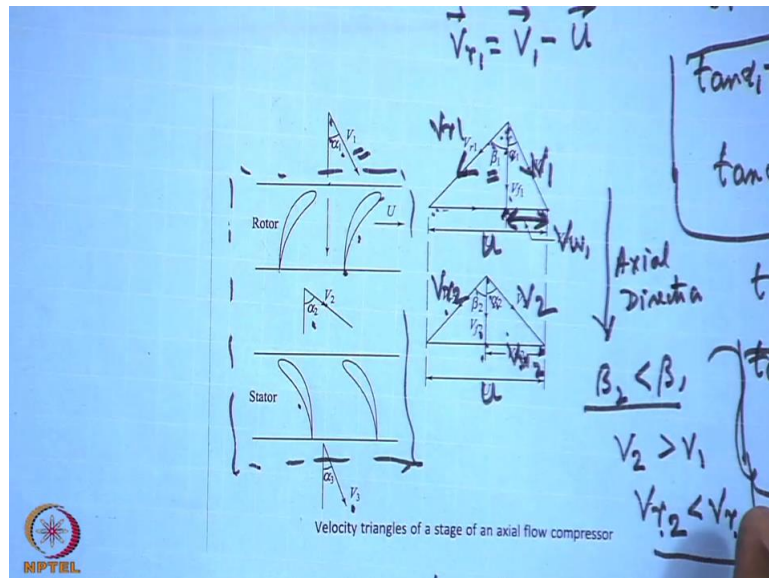


Fluid Machines.
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Lecture-37.

Basic Principles and Energy Transfer in Axial Flow Compressor Part II.

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Degree of Reaction.

$$\Omega = \frac{(\Delta h)_{\text{rotor}}}{(\Delta h)_{\text{rotor}} + (\Delta h)_{\text{stator}}} = \frac{(\Delta T)_{\text{rotor}}}{(\Delta T)_{\text{rotor}} + (\Delta T)_{\text{stator}}}$$

$\frac{(\Delta h)_{\text{stage}}}{(\Delta T)_s}$


$u_1 = u_2 = u$

$$\eta = \frac{(\Delta h)_{\text{Rotor}}}{(\Delta h)_{\text{Rotor}} + (\Delta h)_{\text{Stator}}} = \frac{(\Delta T)_{\text{Rotor}}}{(\Delta T)_{\text{Rotor}} + (\Delta T)_{\text{Stator}}}$$

$(\Delta h)_{\text{stage}} \qquad (\Delta T)_s$

$$\eta = \frac{(\Delta T)_A}{(\Delta T)_A + (\Delta T)_B}$$

$A \rightarrow \text{Rotor}$
 $B \rightarrow \text{Stator}$

$$\frac{W}{m} = c_p \Delta T_{st}$$


Good morning and welcome you all to this session of the course. Now we come to a thing which is known as degree of reaction, which is very important, degree of reaction. What is meant by degree of reaction? Try to understand one thing, again I will repeat the thing which I told you earlier, in the fluid machines class when a difficult to the hydraulic machines, the machines using water. Now one has to understand that any fluid machines has a rotor and a stator, okay whether it is a turbine or it is a compressor or pump.

Okay. So the basic purpose of the stator or the diffuser in a pump or compression is to change the velocity to static pressure. But the question comes whether in a rotor, the static pressure will change or not. 1st of all, try to understand in terms of pressure. Static pressure will change or not. So static pressure will change in the rotor depending upon the rotor construction. If it is a radial flow machine, machine type, rather I will tell machine type, that means the radial flow machine, automatically the pressure changes because of the centrifugal, action of the centrifugal head, that is a centrifugal force.

When the radial location changes, the peripheral speed is changing, so therefore this is manifested in terms of the increase in a static pressure. I explained so many times that a radial pressure gradient is imposed when the fluid has a initial velocity and it changes its radial location. But in axial flow machines, when there is no change in the tangential flow from inlet to outlet, not necessary there will be change in the static pressure. The change in the static pressure will depend upon the rotor design and construction.

That means if we have to change the flow area, if you change the velocity relative to the rotor, in flow, course of flow through the rotor passage, then only there will be change in the

static pressure. And that is a measure of reaction in reaction turbine. That whether there is a change in the static pressure in the rotor itself or not. For example in impulse turbine, if you remember, water turbine, the Pelton turbine, the waterjet is striking the rotor, that is the Pelton wheel at atmospheric pressure.

Throughout the pressure change, this no change in the pressure. This is known as impulse turbine. Whereas in Francis turbine, when it goes through the runner blade, there is a change in the pressure. Similarly in the centrifugal pump, always there will be change in pressure in the impeller, this is because of the radial flow in a rotating tangential flow field. Similar is the case of centrifugal compressor. But in axial compressor, the question comes, whether the flow passage changes in course of flow through the rotor, so that the relative velocity changes or not.

I told you, that if the relative velocity changes in a sense that if VR_2 is less than VR_1 , then we can consider that, we think that there will be change in the static pressure. And when there is a change in the static pressure, there will be a change in the static temperature also. So therefore whether there is a change in the static pressure or not, that will call the machine is reaction type or not. Usually there is a change in static pressure and the static temperature while flowing through the rotor, VR_2 is less than VR_1 .

And therefore the question of reaction comes and the degree of reaction in this context is defined as this way. The in terms of enthalpy, it is defined, the changes in enthalpy, let us consider per unit mass in the rotor divided by the change in the stage $\Delta H_{\text{rotor}} + \Delta H_{\text{stator}}$. This is known as ΔH_{stage} . Since the enthalpy change for an ideal gas is equal to the CP into changing the temperature and since CP is constant here, we are considering the ideal gas, it is independent of the temperature, they can cancel it out. In reality also, the CP does not vary much with the range of temperature.

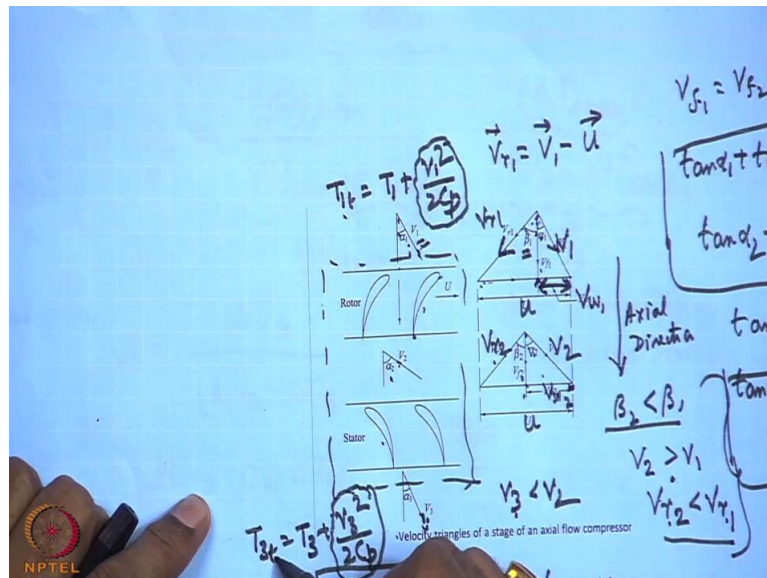
This can be written in terms of ΔT , the Static temperature change in the rotor divided by static temperature change in the stage, that means in rotor + ΔT_{stator} . Now this is the degree of reaction. When there is no change in static temperature or static pressure, degree of reaction is 0. So degree of reaction is 0, for a reaction machine, there will be always a degree of reaction, since ΔT_{rotor} has got a value more than 0.

So this is the measure of the extent by which the total, the fraction by which the total change, that is the total change in the static temperature of the stages taking place to the rotor. Okay.

Now with this definition, let us now see the how we can find out an expression for this. Now let us consider now again, the definition, since we give this S for stage and rotor and stator, these things will not write, so simply we will tell that A stands for stator, for our convenience.

And B stands for rotor, and we write this definition as Delta T, this is A for example, this you write sorry, this you write rotor, this will be better and this you write stator. Then you write Delta A, Delta TA + Delta T B. Okay. Now you see that work done per unit mass is nothing but CP into Delta T, stagnation. Now if we make V1 is V3, now you see this diagram, we told that Alpha 3 is alpha-1. But in our design, V3 is made V1. Now see, that it approaches with some V1 from the earlier stage, then while it passes through the rotor, it gains energy and this V is increased, V2 is greater than V1.

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$$\eta = \frac{(\Delta h)_{\text{rotor}}}{(\Delta h)_{\text{rotor}} + (\Delta h)_{\text{stator}}} = \frac{(\Delta T)_{\text{rotor}}}{(\Delta T)_{\text{rotor}} + (\Delta T)_{\text{stator}}}$$

$$\eta = \frac{(\Delta T)_A}{(\Delta T)_A + (\Delta T)_B}$$

$(\Delta h)_{\text{stage}} \qquad (\Delta T)_s$
 $A \rightarrow \text{rotor}$
 $B \rightarrow \text{stator}$

$$\frac{W}{m} = c_p \Delta T_{st} \qquad \Delta T_{st} = \Delta T_s$$

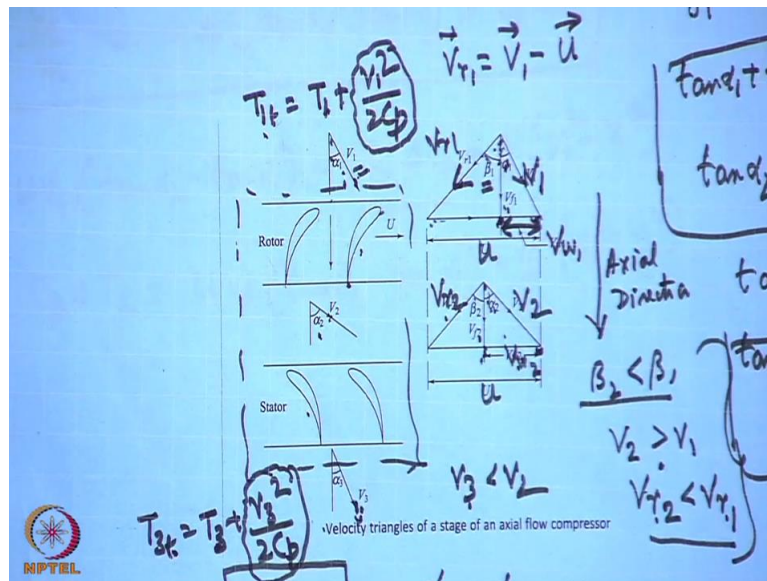
$$= c_p \Delta T_s = u v_f (\tan \alpha_2 - \tan \alpha_1)$$


Then this V_2 is again by the diffusion process, that means this is decelerated to get a rise in static pressure is, that means V_3 is less than V_2 , is less than that. And it is also displaced more towards the axial direction. This deceleration is made in such a way V_3 comes back again to the original V_1 . If we make a design like that, the absolute velocity at the discharge of the or at the outlet of one stage becomes equal to that of its inlet velocity at the stage. You understand. So that we can write V_3 is V_1 .

In that case we can write ΔT , that means, what is T , total T stagnation temperature at 3, for example T_3 . Now I write the total temperature. T . Here you see. Then T_1 , T_1 is T_1 static + V_1 square by $2 C_p$. Similarly here you see, T_3 , the total temperature is $T_3 + V_3$ square by $2 C_p$. We V_1 is V_3 , that means the dynamic equivalent temperature, okay, that when the velocity equivalent temperature are same. That means the difference in the total stagnation temperature has difference in their static temperature.

So that ΔT_{st} , that is the stagnation temperature in ΔT_{st} . And here S is written, that is per stage, that means ΔT_{st} , that means is ΔT_{st} , that is static temperature per stage is equal to the stagnation temperature. That means this can be written as ΔT_s . And this is nothing but the work done formula. That means if you remember that, then what is this formula, $u v_f \tan \alpha_2 - \tan \alpha_1$. So therefore $C_p \Delta T_s$ is given by this. Correct, bracket is there, there will be bracket.

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$\frac{W}{m} = U V_f (\tan \alpha_2 - \tan \alpha_1) = C_p \Delta T_s$
 $\frac{W}{m} = U V_f (\tan \alpha_2 - \tan \alpha_1) = C_p \Delta T_a + \frac{1}{2} (V_2^2 - V_1^2)$
 $C_p \Delta T_a = U V_f (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} (V_2^2 - V_1^2)$
 $= U V_f (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} V_f^2 (\sec^2 \alpha_2 - \sec^2 \alpha_1)$
 $= U V_f (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} V_f^2 (\tan \alpha_2 - \tan \alpha_1)$

sec² - tan² = 1

Now this is correct now how to find out this for Delta TA? So therefore now I find the value for Delta TA. Now rotor, the energy is actually given in the rotor. So W by M is written in terms of the total 1. That means U VF, again I am writing tan Alpha 2 - for your convenience tan alpha-1 and this becomes equal to CP delta TS, static temperature rise per stage. Because the static temperature rise is equal to the stagnation temperature. Now if I write this W by M, that is U VF into tan Alpha 2 - tan alpha-1 in terms of the static temperature change of the stator, I am not permitted.

Because stator static temperature, so entire energy used not to increase the static temperature but at the same time to increase its velocity, V2 square by V1 square. So energy balance in

the rotor gives that rotor energy takes and the air static temperature is increased, this is the increased change due to increase in static temperature, that is the static enthalpy rise + the kinetic energy change. So summation of these 2 from the steady flow energy question is the energy input per unit mass rates. So this is that.

So therefore we can write $CP \Delta T_A$ is equal to $U V_f \tan \alpha_2 - \tan \alpha_1 - \frac{1}{2} V_2^2 - V_1^2$. Now you see, we do V_1 from the velocity triangles. Now from the velocity triangle, if you recall that V_1 is what, if you see this triangle, so this is α_1 , in terms of α_1 , the cosine of α_1 is which one, V_f by V_1 . So V_1 is V_f by cosine α_1 and V_2 is V_f , V_f is same as I have told earlier by cosine α_2 . That means this is V_f by cosine α_1 , this is V_f by cosine α_2 .

So with this thing I can write this equal to $U V_f \tan \alpha_2 - \tan \alpha_1 - \frac{1}{2} V_f^2$. This is V_f^2 , I take V_f common, V_f^2 square into one by cosine, that is sec, sec square $\alpha_2 - \sec \alpha_1$. I think I am correct, sec square α_2 . Now you know again from the trigonometric relationship that sec square $\alpha - \tan^2 \alpha$ is equal to 1. That relationship from the school level you know. So therefore if you use that, this becomes $U V_f \tan \alpha_2 - \tan \alpha_1 - \frac{1}{2} V_f^2$.

Then this is your say $\tan^2 \alpha_2 - \tan^2 \alpha_1$. Now I use this expression for $CP \Delta T_S$, that is stage. And this expression, $CP \Delta T_A$ for the rotor. This stage means $\Delta T_A + \Delta T_S$.

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Handwritten derivation on a blue background:

$$\eta = \frac{(\Delta T)_A}{(\Delta T)_S}$$

$$= \frac{U V_f (\tan \alpha_2 - \tan \alpha_1) - \frac{1}{2} V_f^2 (\tan^2 \alpha_2 - \tan^2 \alpha_1)}{U V_f (\tan \alpha_2 - \tan \alpha_1)}$$

~~$\tan \alpha_2 = \tan \alpha_1$~~

$$= 1 - \frac{V_f}{2U} (\tan \alpha_2 + \tan \alpha_1)$$

$\frac{U}{V_f} = \tan \alpha_1 + \tan \beta_1$

$\frac{U}{V_f} = \tan \alpha_2 + \tan \beta_2$

$$\frac{2U}{V_f} = \tan \alpha_1 + \tan \alpha_2 + \tan \beta_1 + \tan \beta_2$$

$$\eta = \frac{V_f}{2U} (\tan \beta_1 + \tan \beta_2)$$

Then I write, I can write the omega which is equal to Delta TA by Delta TS as CP, CP cancels, so therefore this becomes equal to $U \cdot VF \cdot \tan \alpha_2 - \tan \alpha_1 - \frac{1}{2} VF^2 \tan^2 \alpha_2 - \tan^2 \alpha_1$ divided by the total stage which we derived earlier, $U \cdot VF \cdot \tan \alpha_2 - \tan \alpha_1$. This can be written as, $1 - \frac{1}{2} \frac{VF}{U}$, then VF, VF will cancel, that is VF by 2U and this will be $\tan \alpha_2 + \tan \alpha_1$. Okay.

Now this is the definition. Okay. Now this $\tan \alpha_2 + \tan \alpha_1$ so this is the definition of final definition of the 1. Now if we change it to $\tan \beta_1$, then what we call, that we already have found out this expression if we see this expression at the beginning, which we found out that $\tan \alpha_1, \tan \alpha_2 - \tan \alpha_1$ is equal to $U \cdot VF$, okay, that is what we did. Just a minute, U by VF by U is, what is that one? Very beginning. Which we did if you see that thing, at the beginning, U by VF , $U \cdot VF$ is $\tan \alpha_2 - \tan \alpha_1$ and $\tan \alpha_2 - \tan \beta_1$.

So if $\tan \alpha_2 + \tan \alpha_1$, if you remember this, U by VF , again I am writing $\tan \alpha_1 + \tan \beta_1$ from the geometry, that is from this, this one, we initially write, yes, this one. If you add these 2, you twice U by VF is $\tan \alpha_1 + \tan \alpha_2 + \tan \beta_1 + \tan \beta_2$. So again I am going to write that for your benefit $\tan \alpha_1 \tan \beta_2$. So therefore we write twice U by VF from here is $\tan \alpha_1 + \tan \alpha_2 + \tan \beta_1 + \tan \beta_2$.

Now if $\tan \alpha_1 + \tan \alpha_2$ is substituted or is just eliminated in terms of $\tan \beta_1, \tan \beta_2$ from this, that means $\tan \alpha_1 + \tan \alpha_2$ is written twice U by VF - this. If you write here, then finally you get an expression for the degree of reaction, you get finally an expression, if you do it, it is very simple, there is nothing difficult, you get an expression is equal to VF by $2U$ into $\tan \beta_1 + \tan \beta_2$.

That means simply, you make $\tan \alpha_1 + \tan \alpha_2, 2U$ by $VF - \tan \beta_1 - \tan \beta_2$ and then make it clear, $1 - 1$ will cancel, so you will see a result like that. This is the final expression for the degree of reaction.

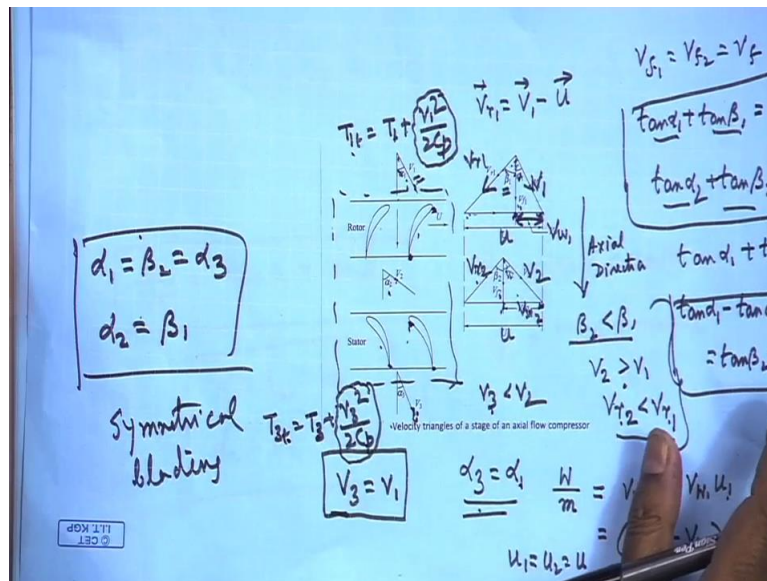
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$$\frac{U}{V_f} = \tan \beta_1 + \tan \beta_2$$
$$\frac{U}{V_f} = \tan \alpha_1 + \tan \beta_1$$
$$\frac{U}{V_f} = \tan \alpha_2 + \tan \beta_2$$
$$\alpha_1 = \beta_2 = \alpha_3$$
$$\alpha_2 = \beta_1$$

Now let us make a case study that where we get a 50 percent degree of reaction. That means 50 percent are half of the total enthalpy static temperature rise of the states takes place in rotor. In that case we get a very good result that U by V_f is equal to $\tan \beta_1 + \tan \beta_2$. Now this is one very very important result. Now if you compare this result with this result U by V_f is, now you compare this with the result earlier that means this one. U by V_f is $\tan \alpha_1$, U by V_f is $\tan \alpha_1 + \tan \beta_1$.

And that just now I wrote, earlier also I wrote, this is simply from the geometry. So with these 3 relationships, you get very beautiful result. When you equate this with this, that means we get $\alpha_1 = \beta_2$. And we equate this with this, then we get $\alpha_2 = \beta_1$. And as we told earlier that $\alpha_1 = \alpha_3$, that $\alpha_1 = \alpha_3$, then that becomes equal to α_3 . So this is one very important result. What does it give?

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This important result gives, if I write here that this results, if I write here alpha-1 is beta-2 is Alpha 3 and Alpha 2 is beta 1, then you see what we get. Alpha-1 is beta-2, beta-2 is the outlet angle of the blade, rotating blade, rotor blade. And that becomes equal to alpha-1, what is alpha-1, alpha-1 is this one, which is Alpha 3. That means outlet angle of the stator blades. That means outlet angle of the rotor blade equals to the outlet angle of the stator blade.

Again beta 1, beta 1 is the inlet angle of the stator blade which becomes is equal to Alpha 2, alpha-2 is the absolute velocity angle at the outlet of the rotor which exactly equals to the inlet angle of the stator blades. That means the inlet angle of the rotor blades equal to the inlet angle of the stator blade. So therefore inlet angle of this rotor blade is equal to the inlet angle of the stator blade, outlet angle of the rotor blade is equal to the outlet angle of the stator blade.

So they are cambered in the opposite direction but their inlet and outlet angles are same. The inlet angle of one blade is equal to that of the other and outlet angle of one rotor blade is equal to that of the stator blade. This type of design of blade is known as symmetrical blading, symmetrical blading and it is easy for construction. So therefore we see for a 50 percent degree of reaction, 0.5, we get a symmetrical blading. That means the inlet angle of the rotor and stator blades are same, similarly the outlet angle of the stator and rotor blades are same. Okay, thank you, today up to this.