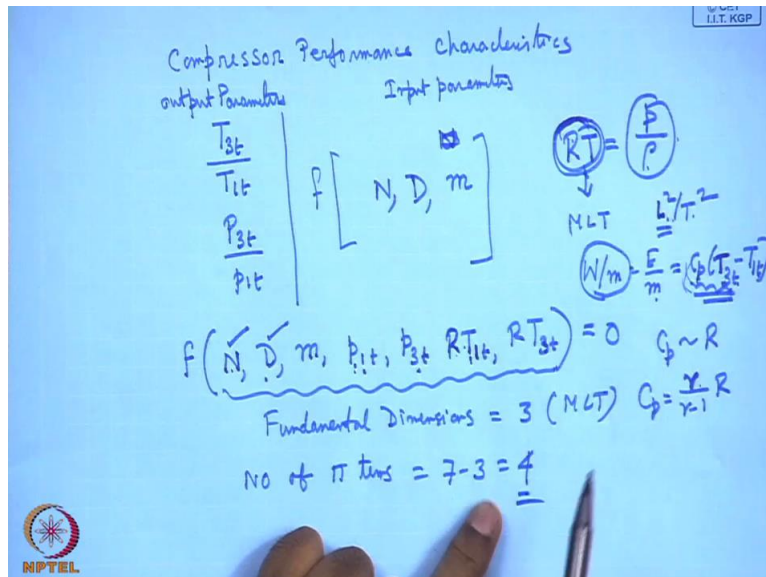
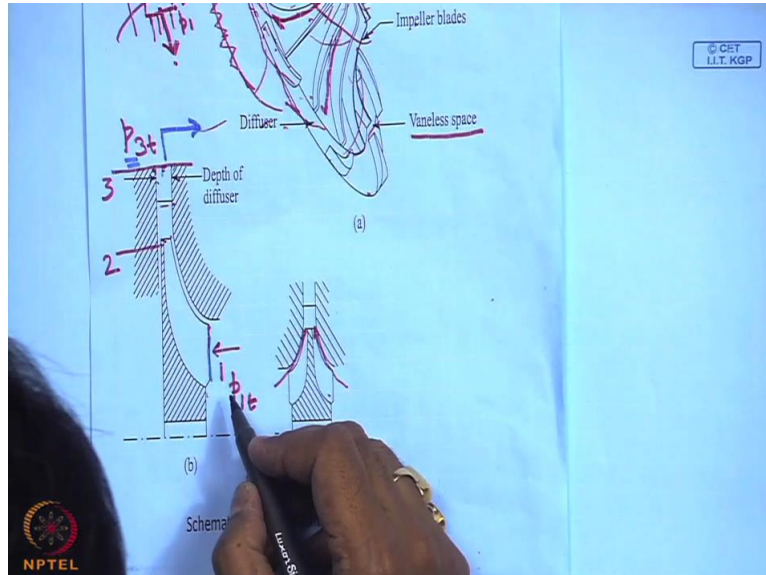


**Fluid Machines.**  
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**Lecture-34.**

**Performance Characteristics of Centrifugal Compressors Part I.**

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Good morning and welcome you all to this session of the course. Today we will discuss the compressor characteristics. Now compressor characteristics or performance characteristics or compressors are usually expressed in terms of the ratio of the stagnation pressure and temperature. Now let us 1<sup>st</sup> have a look at how do you make the nomenclature. This is the

stagnation pressure at inlet to the compressor and this is the outlet from the compressor. We will use the same nomenclature, that is at the outlet of the diffuser.

And similarly the stagnation of total temperature is  $T_1$  and the  $T_3$  is the outlet temperature, total temperature. So with this nomenclature, the compressor performance characteristics, compressor, today we will discuss compressor performance characteristics, performance characteristics. So performance characteristics for a centrifugal compressor is usually expressed in terms of the ratios of the total temperature stagnation temperature, these are output parameters. As a function of input, this is the output parameters and this is input parameters usually this is done, input parameters, this is output, output parameters, parameters. Okay.

As a functions of  $N$ , the rotational speed, sorry here I can write,  $N$  the rotational speed, the size of the compressor  $D$ , the mass flow rate  $M$ . So this speed, then size of the compressor and the mass flow rate, these are the parameters, input parameters and the ratios are expressed in terms of these parameters. The speed, the size and the mass flow rate. So now you see that this can be expressed in a functional relationship like this that a function of, if we think in terms of the functional relationship, we can write this,  $N$ ,  $D$ ,  $M$  separately,  $P_1/T_1$ ,  $P_3/T_3$ , in case of this temperature, we include  $R$ ,  $R T_3/T_1$  and these are the variables which define the performance of a centrifugal compressor.

Now let me explain 1<sup>st</sup>, these are the input parameters as I have told, that this is the rotational speed, this is the size, that is the overall diameter of the impeller, this is the mass flow rate  $M$ , this is the total pressure at the inlet to the compressor, this is the total pressure at the outlet of the compressor, outlet from the diffuser and this is the total temperature at inlet, total temperature at outlet and they are multiplied with  $R$ , there are, because of the 2 things, it is multiplied with  $R$ ,  $T$  has a fundamental dimension, temperature.

But if you multiply which  $R$ ,  $RT$  becomes equal to  $P$  by  $\rho$  and its dimension as a whole can be expressed in terms of  $MLT^{-2}$  because  $P$  by  $\rho$  is  $V^2/L$  square, you can very well know that  $P$  by  $\rho$  is  $L^2/V^2$  square. So therefore the few find out the dimension of literati, it becomes  $V^2/L^2$  square, that means  $P$  by  $\rho$  is  $V^2/L^2$  square, the dimension wise. Okay, dimension wise, this dimension is  $V^2/L^2$  square by  $T^2$  square, sorry,  $L^2/T^2$  square.

So therefore it is L square by T square, so therefore multiplying with R, taking care of R as a whole to reduce the fundamental dimension and at the same time taking care of the physical concept that  $T_1 T$  and  $T_3 T$  are very important parameters describing the centrifugal pump performance. Again another logic is there that you know that work done per unit mass or energy added per unit mass is given by change in CP times, this CP times  $T_3 T$  or  $T_2 T$ , whatever you call.

That means it is the CP times the T and CP is what, CP is proportional to R. That means CP in case of, specifically the constant pressure, it is  $\gamma$  by  $\gamma - 1$  into R.  $\gamma$  is the specific heat ratio. That means it takes care of R. That means T alone has got no function, if you take, multiplied with CP, CP T is the index of the energy, CP, this  $T_3 T - T_1 T$  is the work or energy input. On the other hand, RT if you take together, this reduces the fundamental dimension by 1 and things become little simple.

So therefore is to do this way, you can now explain that the entire, another thing very important that why we have not considered density? Because it is a compressible flow machine, from the beginning I am telling that the density is very important in a compressible flow machine but I am not including density. Density is implicitly included because P is included, RT is included. So their ratio is the density. So therefore density is not included explicitly, it is implicit.

So therefore all the variables described in the centrifugal pump performance is, are there. How many, 1, 2, 3, 4, 5, 6, 7 and fundamental dimensions, fundamental dimensions are how many? Fundamental dimensions are 3, ML is equal to 3. That is ML T, since we have considered the total RT. So therefore by Buckingham's pie theorem, number of pie terms will be  $7 - 3$  is 4, while you are doing this dimensional analysis, Buckingham's, by applying Buckingham's pie theorem, because we want to express this relation in terms of nondimensional variables rather than dimensional variables which will be reducing the numbers from 7 to 4.

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$$F\left(N, D, m, P_{1t}, P_{3t}, R_{1t}, T_{3t}\right) = 0$$
 Repeating variables:  $D, P_{1t}, R_{1t}$   
 Fundamental Dimensions = 3 (MLT)

$$\pi_1 = \frac{ND}{\sqrt{R_{1t}}}$$

$$\pi_2 = \frac{m\sqrt{R_{1t}}}{D^2 P_{1t}}$$

$$\pi_3 = P_{3t}/P_{1t}$$

$$\pi_4 = T_{3t}/T_{1t}$$

$$F\left(\frac{ND}{\sqrt{R_{1t}}}, \frac{m\sqrt{R_{1t}}}{D^2 P_{1t}}, \frac{P_{3t}}{P_{1t}}, \frac{T_{3t}}{T_{1t}}\right) = 0$$

Now by applying the standard procedure of dimensional analysis, what we do, we take 3 repeating variables. Here what are the repeating variables? We take the repeating variables as D we take the repeating variables as P1 T and we take the repeating variable as R T1 T. This D P1 T and R T1 T is taken as the repeating variables. These are taken as, they are taken as, just see that the repeating variables. They are taken as you can see repeating variables. And following the dimensional analysis, with these 3 as the repeating variables, if we combine with M, then you get a pie term, that is your task you can do ND divided by root over R T1T.

I am now writing this, that means this takes the N. Now D is the repeating variable, where you combine with M, the 3 repeating variables D, P1 T, R1T, then the 2<sup>nd</sup> pie term comes as M root over R T1 T divided by D square into P1 T. Okay. Then if you take this P3 T as the one, then automatically when, obviously when the one of the repeating variables has the same dimension, it is a thumb rule, and if you do it, you will get that T3 T by P1 T is the pie 3.

And similarly pie 4, 4<sup>th</sup> pie term which will be found out, with, what is that, T3 T, then with this, you will get since T3 T and T1 T are the same dimension, then automatically get T3 T by T1 T. This is by thumb rule, it will always come and if you follow this dimensional analysis, you will find the same thing. Okay, now therefore the equation can be written as some functional relationship of the nondimensional term. That means ND by root over R T1 T. M root over R T1 T divided by D square P1 T, P3 T by P1 T, T3 T, the ratio of total pressures and the ratio of total temperature is equal to 0.

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$$\Pi_2 = \frac{m \sqrt{R T_{1t}}}{D^2 P_{1t}}$$

$$\Pi_3 = P_{3t} / P_{1t} \quad \Pi_4 = T_{3t} / T_{1t}$$

$$F\left(\frac{ND}{\sqrt{R T_{1t}}}, \frac{m \sqrt{R T_{1t}}}{D^2 P_{1t}}, \frac{P_{3t}}{P_{1t}}, \frac{T_{3t}}{T_{1t}}\right) = 0$$

$$\frac{ND}{\sqrt{R T_{1t}}} \propto \frac{U}{a} = MR \quad \frac{m \sqrt{R T_{1t}}}{D^2 P_{1t}} = \frac{(\rho A V_f \sqrt{R T_{1t}})}{D^2 P_{1t}}$$

$$ND \propto U \quad \sqrt{R T_{1t}} \propto a$$

$$P_{1t} / \rho \sim R T_{1t} \quad \rho A V_f / (D^2 \sqrt{R T_{1t}}) \propto \frac{V_f}{a} = M_f$$

Before proceeding further, I would like to tell you that these 2, these 2 are very clear, they are the ratio of total pressure and total temperature, these 2 have some physical significance. For example this pie 1, ND, what is the physical significance?  $R T_{1t}$ , what is the physical significance? Now ND is proportional to the tip speed of the impeller, rotational speed into the impeller diameter. And root over  $R T_{1t}$ , I told you that this sound speed is given by root over, RT, so it is proportional to the sound speed, acoustic speed in the media relative to the flow.

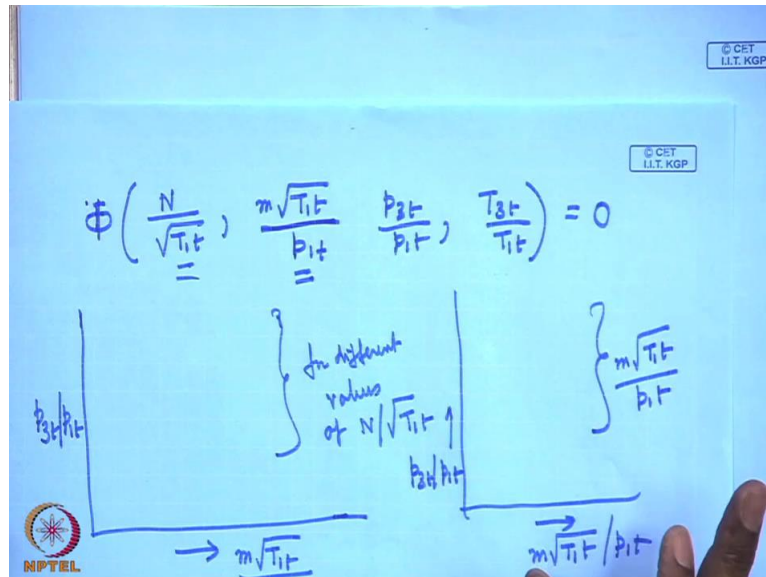
So therefore this is proportional to U by A. And which is known as Mach number based on rotor speed. So therefore this pie term signifies physically some sort of Mach number based on rotor speed. Now this pie 2 if we write, that pie 2, M root over  $R T_{1t}$  divided by D square  $P_{1t}$ . Now this can written, M can be done in terms of flow velocity and the density. Rho, the flow area A and the flow velocity VF root over  $R T_{1t}$  divided by D square  $P_{1t}$ .

Now one can write  $P_{1t}$  by rho is proportional to root over, proportional to  $R T_{1t}$ . P by rho is  $R T_{1t}$ , so though this is total pressure and this is the density, so therefore it is at  $R T_{1t}$ . So therefore this can be written as by cancelling that this is proportional to A VF, then this will be cancelling out, D square root over  $R T_{1t}$ . Because this square root and this is, this is under root, this is not under root, so this is root over T. So therefore this will be proportional to VF by, again A.

That means this is Mach number based on flow velocity, this is known as flow Mach number, this is known as rotor speed Mach number. That means physically this pie term represents a

Mach number based on rotor speed velocity and this pie term represents a Mach number based on flow velocity. This is just for your physical implication. Now one thing that if we express this relationship, try to express for a particular machine, then D is not necessary to be included, D is constant, we can drop D.

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And moreover it is for a particular gas, for example the air, then R also can be omitted, here R is there, here R is there, so therefore for a given machine, with a given gas, the same relationship which is used nondimensionally, this can be written as some other function, for example the function, a function of N by root over T1 T, then this will be M root over T1 T, follow it clearly by P1 T, P3 T by P1 T, T3 T by T1 T. And for a given machine, with a given gas, the relationship can be expressed like that.

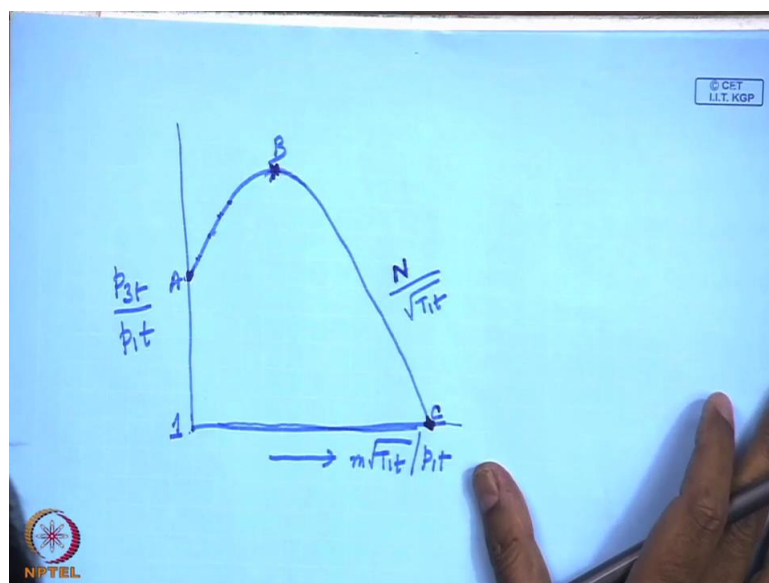
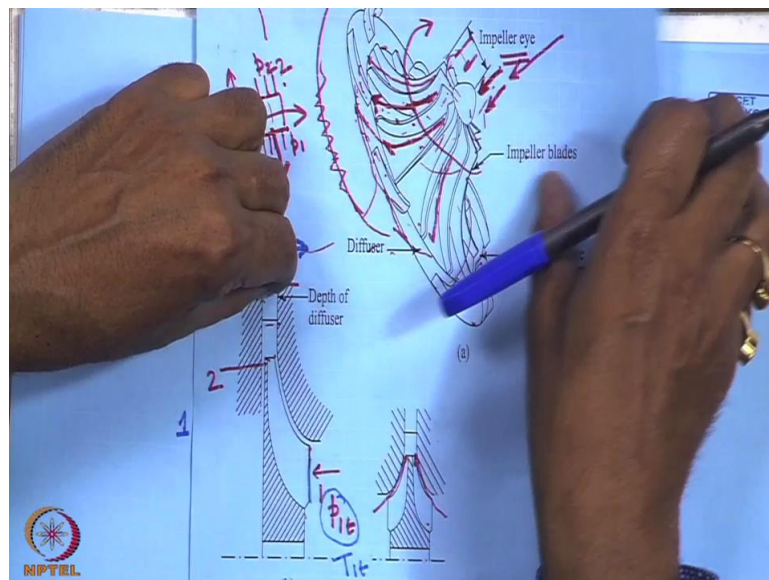
Now we see here these 2 are not truly dimensional less or truly nondimensional because we have dropped the term R and D. But what happens is that even if they are dimensional terms, but we take the help of the nondimensional analysis to reduce the number of terms. So the number of variables are reduced where some of the variables which combines other primary variables may not be nondimensional but does not matter. For a given size and for a given fluid, we can use these as a function relationship of the performance.

So usually what happens is that the performance now is expressed like this that a family of curves is generated as the ratio of the pressure with the, I will not tell this is nondimensional mass flow rate. This is normalised mass flow rate, the word normalised does not mean nondimensional. Different families of curves, I am not drawing the curve at present, different

families of curves for different values, for different values of  $N$  by root over  $T_1 T$  in one family, another family is the different values, the same thing, the same thing is  $M$  root over  $T_1 T$  by  $P_1 T$  and here is the ratio  $P_3 T$  by  $P_1 T$  with the different values of  $M$  root over  $T_1 T$  by  $P_1 T$ .

That means 2 families of curves for both the ratio of the total pressures and total temperatures for different parameter values for each family of the normalised rotational speed and normalise mass flow rate. This is basically the way the performance parameters, performance characteristics of a centrifugal pump is expressed. Now I will show you how does it look in case of a pressure ratio. Now I will show you the very important, a very important curve.

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P3 T, that is the pressure ratio  $P_1/T$  versus the nondimensional mass flow, normalised, sorry normalised mass flow  $P_1/T$ , the curve looks like this, I tell you, this curve looks like this. The curve looks like this, I will explain, let me 1<sup>st</sup> draw the, label the curve. Now I will explain, the 3 points are important points in understanding this. Now what happens, if you make an experiment and draw the points, you will get a curve like that.

Initially it increases with the positive flow, reaches a maximum, then it has a negative slope, continuously decreases and probably at high value of mass flow rate, it touches the officer where the pressure ratio is 1, actually the pressure ratio starts from 1. Okay. Now try to understand physically the fact that when the mass flow rate is 0, there is a pressure ratio. Why, this is because in a centrifugal pump, try to understand when the mass flow rate is 0, mean that you stop the delivery valve here.

Then what happens, the impeller goes on rotating, so therefore centrifugal head or centrifugal, the energy is imparted on the fluid in terms of a pressure rise. So therefore a pressure rise will take place in the impeller because of the centrifugal action which we call as centrifugal head is impressed on the fluid is imposed on the fluid. The fluid may not move in the diffuser, there will be a static fluid, a static field is there, pressure field and that pressure is due to the churning action of the fluid in the impeller which imposes a static pressure rise because of the centrifugal action, this we call a centrifugal head.

That means the centrifugal head because of the rotation of the impeller is imposed on the fluid even if the valve is closed here, so a pressure ratio will be developed. So that is the pressure ratio by the action of the impeller rotation which is shown here at this point A. Now when we slowly open the valve of a, of the delivery line, then what happens, the flow commences and when the flow commences, again you see that the flow takes place through the diffuser vanes, so and also the vane less space.

Now when the diffusion process takes place through vanes less space and the diffuser vanes as a whole which is shown in the diffuser, then what happens, again pressure rise takes place because of the diffusion process, so therefore the rise in pressure takes place as we increase the mass flow rate. Okay. As we increase the mass flow rate, the rise in pressure takes place. This means that diffusion contributes its quota, diffusion contributes its quota to the pressure rise because of the diffusion process. Okay.



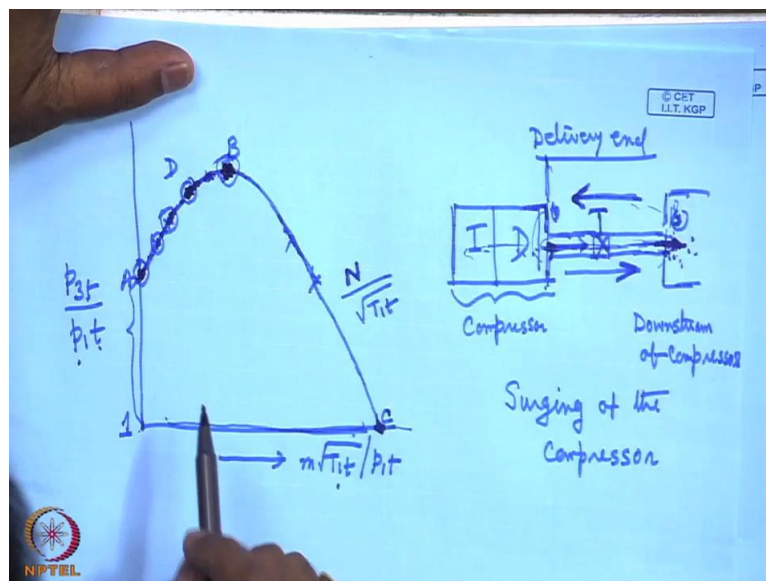
So because of that, the pressure rises and reaches a point of maximum while beyond which if you increase the mass flow rate, it will not be manifested in terms of the pressure rise. This is because of the frictional losses, as I explained earlier in the last class that frictional losses composed of skin friction loss at the same time the losses due to separation. Okay, along with that, the incidence losses are there, so all together the losses increase for which an increase in mass flow rate is not manifested with an increase in pressure ratio rather by a decrease in pressure ratio.

And this point corresponds to the maximum efficiency of the compressor. So below which the compressor efficiency drastically falls, okay, because of the losses. And if we go on increasing the mass flow rate for a given rotational speed, for example, here we give a rotational speed, I tell you this is valid for 1 rotational, normalised rotational speed, one normalised rotational speed, I show this particular curve. For a given rotational speed, there may be a point which may or may not be obtained in practice but there may be a point for a given rotational speed, if I go on opening the valve wider and wider, the mass flow rate may be such that the pressure ratio may be unity.

That means there is no pressure rise, the entire energy given to the compressor to handle a being used to overcome the frictional losses in handling a huge mass flow rate. Okay. So that particular point may not be available for a given speed  $N$ . But it is theoretically envisaged, can be envisaged. So physically it is possible for a given  $N$ , there may be a point which gives a mass flow rate where pressure drop, pressure ratio is unity. That means the entire energy is utilised to overcome the friction.

So therefore A, B, C, 3 points are important and this is a particular curve and this way we can generate a family of curves with different rotational speed and similarly with different normalised mass flow rate. So the characteristic curve is like that which has a positive slope, maximum point corresponds to maximum efficiency, then there is a negative slope. Here is the most important thing now I will discuss is the instability of this part of the characteristic curve.

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Now this part of the characteristic curve is having a positive slope and usually this part is unstable and is very difficult to have this part of the curve in practice. Rather this part, which is associated with the negative slope, that pressure rise and mass flow rate is stable, how I will explain you. Now let us consider a compressor like this. Let us consider a compressor like this, this is impeller, this is diffuser, this is totally the compressor. Try to understand this compressor.

Okay. And what do we do, this is the delivery and we control a delivered, a valve, a delivery valve here and this is further downstream of the compressor, where the compressor is discharging air, this is the downstream, the 2 things you have to understand. Downstream of compressor, compressor, downstream of the compressor and this is the compressor delivery, this is the delivery end, delivery end.

Now consider a case that the compressor is running with a given speed, this valve is opened but partially, at steady state some flow is there is and compressor is discharging steadily. And let the operational point is on the positive part of the curve, let this point is D. Now what happens, by any chance if there is a reduction in flow in the compressor by any disturbance or any closure of the valve, then what happens, decrease in flow rate here if you see is accompanied by a decrease in pressure ratio because this is in the positive, the figure tells like that.

So therefore the delivery pressure will fall immediately. Now what happens, you see again that this part of the curve is such, initial it is very steep, then finally it becomes flat, as it

happens for a curve which has a maximum, then it reaches a maximum because gradient has to be 0 here. So if this point is little bit on the steeper side of the curve, then this pressure falls rapidly. So the delivery end pressure falls rapidly, while due to the reduced mass flow rate, the downstream side, where this compressor is delivering air does not fall that rapidly.

So therefore what happens as a result, this pressure becomes higher than the delivery pressure, that means a pressure gradient for flow is generated in the reverse direction. This is a high-pressure and this is a low-pressure, so therefore, the flow starts from the downstream end of the compressor to the delivery side. That means to the compressor, you understand, because of this, if this does not fall rapidly, whether this, what happens in this part, that there may be a point and usually it happens so that this is reduced more rapidly than that at downstream.

When the flow takes place like this, then what happens, the net flow through the compressor, delivered by the compressor is reduced by the opposing flow, so therefore the flow rate is still reduced and the pressure is still reduced. In turn, it affects in reduction of delivery pressure. Again the reverse flow is increased and this way what happens, this makes the flow in the compressor totally 0, there is no flow. That means the compressor cannot deliver air anymore.

But still with delivery side, there is a pressure, pressure ratio, that I explained because of the impeller action and by that time what happens, since the mass flow delivery totally shutdown, is reduced gradually, gradually to 0, then the delivery side pressure is reduced. When the delivery side pressure is reduced at this condition A, then what happens, this pressure becomes high and it takes up again, repeat the flow in the positive direction. And therefore it starts repeating the cycle.

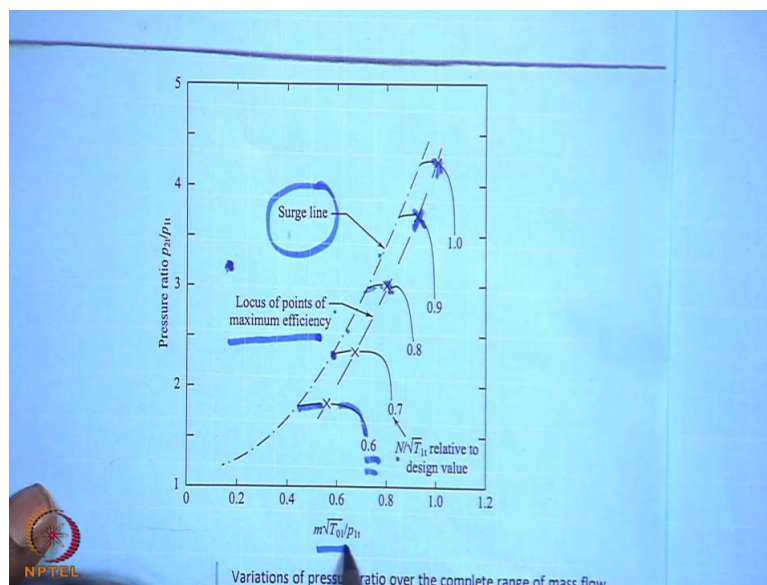
That means it starts flowing in this direction. Again an instability in reducing the flow causes the flow reversal. So therefore what happens, a small disturbance in reducing the flow in this zone makes a repeating cycle, that means the flow reversal takes place, again flow. So flow comes in, again flow goes in this direction, comes this direction, goes this direction. So this type of flow reversal takes place when the operating point is on the steeper side of the positive slope part of the characteristic curve.

And this is known as surging, surging of the compressor, clear, this is very important thing. Now you see this instability, type of a stability known as surging is not there in the negative

slope part of the curve because here what happens, if there is a decrease in mass flow rate, this is associated with increase in pressure. Decrease in mass flow rate, increase in pressure. So no a flow reversal, that means from the downstream to the compressor side can take place.

So therefore this part is unstable and another thing I told you since the slope is steeper initially and then flat, so there is not necessarily that the point has to be immediately, that downstream, upstream of the left side of the B. That means there may be a point here, even there may be a part of this for the slope where the surging will not occur, that means surging may not start when the operating point falls just left of the, maximum efficiency curve.

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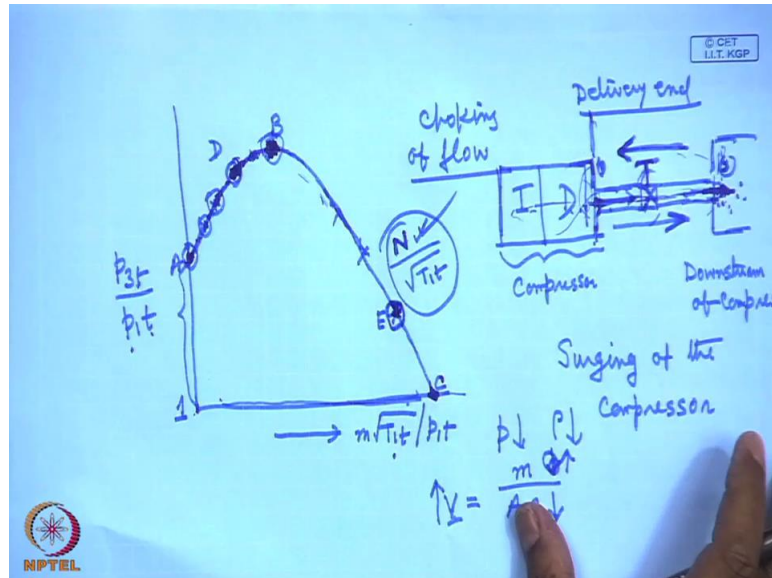


There may be some point away, some distance away from the maximum point where from the surging can start, the onset of surging is there. So this can be well understood by this particular figure. Well, this can be well understood by this particular figure. Now if I draw, show you this figure. You see that as I told earlier that this is the curve, the characteristic curves, now this point, that means if I find out the surge onset of surge point D for all curves of the family for different values of the parameters and if they are joined, this is the surge line.

This is the locus of the starting of the surge point. That means this part of the curve for a given value, 0.6 of this  $N\sqrt{T_1}$  is the stable part, is the stable part, is the stable part, is the staple part. And this cross points at the maximum efficiency, that means this line is locus of points of maximum efficiency and this line is the surge line. So this part of the

curve is characteristic curve is stable. This is  $N \sqrt{T_1}$  and this is with respect to mass.

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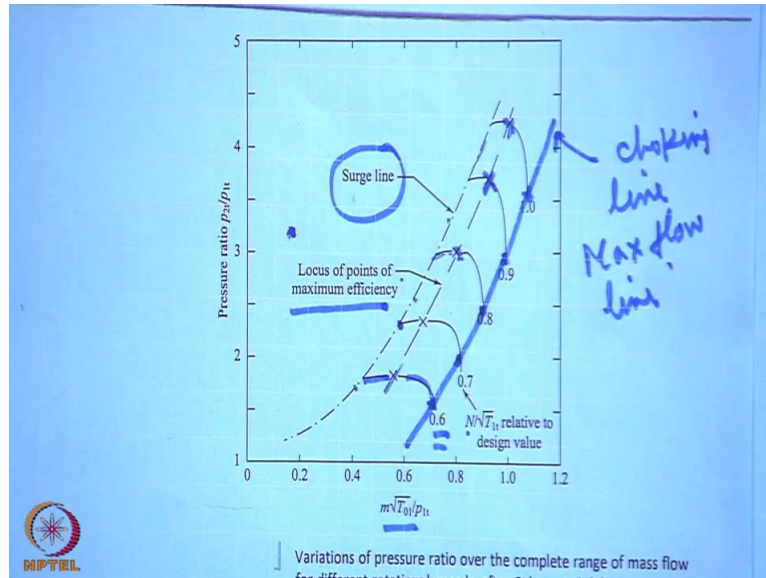
Okay. Now you understand, you have understood this thing. Now next what I would like to tell you that, okay, another important thing is there. On this side of the curve, that is in the negative slope, there is another interesting point E where we may stop. What is that, now you consider when the flow rate is increased, the pressure drop decreases, pressure decreases. For example if you make the valve wide open. So what happens, the flow rate is increased, delivery pressure is decreased and a decrease in delivery pressure, decreases the rho.

Now velocity of flow is proportional to mass flow rate divided by area into rho. So a decrease in the mass flow rate, sorry increase in the mass flow rate and a decrease in the density because of decrease in the pressure. Because you see increase in the mass flow rate is associated with decrease in pressure, the negative part, the negative slope part, this part, makes a huge increase in flow velocity. Then it may so happen, that is also, not always possible depending upon the value of N that a point may come when the sonic velocity maybe attained at some part of the compressor.

So when the sonic velocity is attained, we cannot increase the flow anymore, by any change in the downstream. This will be explained again in detail in your compressible flow class and that is known as choking. That is known as choking. So the maximum flow condition, choking of flow. When the flow at any part becomes sonic, that means the compressor will run, there is absolutely no problem but no further increase in mass flow rate possible. That is

there is a point here, which will, on the characteristic curve which will indicate the limit of the maximum flow rate.

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So therefore I will show you here also, along with the surge, along with the constant, along with the locus of the surge point, onset of surge, the maximum efficiency point, there may be another line, this is the joining of the E point there which is the choking or the maximum flow limit, choking line or maximum flow line. So therefore the stable part of the characteristic line is bounded by the left extreme, by the locus of maximum, locus of the surge line, onset of surge, surge line, in the the extreme right is the choking line, maximum flow line and in between is the maximum efficiency line.

Okay, it is clear, so this is as a whole is the, this is as a whole is the your characteristic curve.

Okay, I think it is all right. Okay, thank you.