Fluid Machines.
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Lecture-31.
Basic Principle and Energy Transfer in Centrifugal Compressor Part II.

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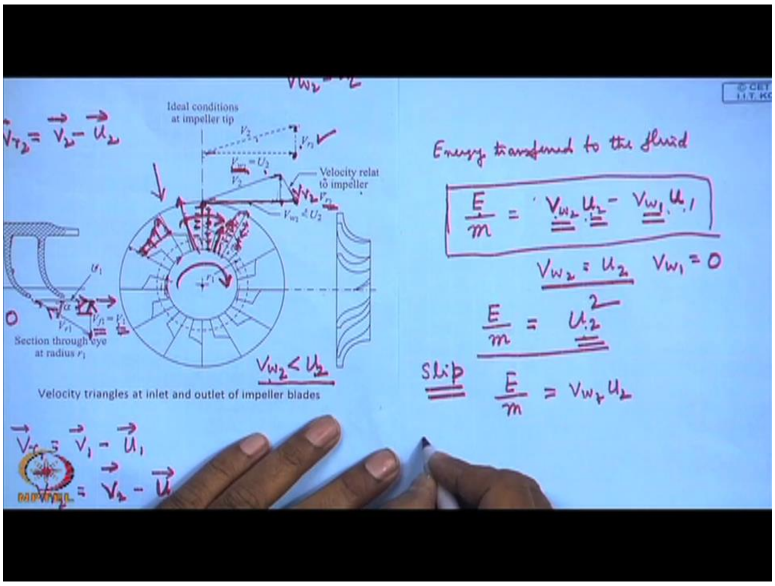

$\vec{V}_{r_2} = \vec{V}_2 - \vec{U}_2$

$V_{w_2} < U_2$

Velocity triangles at inlet and outlet of impeller blades

$\vec{V}_{r_1} = \vec{V}_1 - \vec{U}_1$

$\vec{V}_{r_2} = \vec{V}_2 - \vec{U}$

Energy transferred to the fluid

$$\boxed{\frac{E}{m} = V_{w_2}U_2 - V_{w_1}U_1}$$

$V_{w_2} = U_2 \quad V_{w_1} = 0$

$$\frac{E}{m} = U_2^2$$

Slip $\frac{E}{m} = V_{w_2}U_2$

Good morning and welcome to all of you to this session of this course. I already discussed earlier that there is phenomena known as slip which is very very important. Now I will tell you what is slip. When a fluid flows through a curve vane and the vane rotates, what happens, because of the combined effort of the tangential flow and the radial flow + the curved vanes, there is a difference of pressure in 2 sides of the vane. In the leading edge, the pressure becomes high, the fluid is decelerated and in the trailing edge, the pressure becomes less and the fluid is accelerated.

And there becomes a small recirculated reflow, probably you can number which we discussed in type, in case of centrifugal pump here. Here, let us discuss here. Now if we consider this moving in this direction, in this side for example, there is a positive pressure. Positive means high pressure. And this side, the pressure is low, that means this is because of the movement, this rotation of the blade and the flow imparts the blade in the radial direction. Because of the blade curvature, it moves like that and at the same time fluid flows in the radial direction.

As a combination of that, the fluid element here is decelerated in the leading edge. This is the leading-edge of the blade and fluid here is accelerated. So what happens there is a higher pressure here, there is a lower pressure here, so what happens here, I will show you, this gives rise to a recirculatory flow in this direction. This is the higher pressure region, this is the lower pressure region and what happens in this recirculatory flow, if you see this passage, this happens same way in all the passages, makes a nonuniform distribution of velocity. And that nonuniform is skewed one, that means this makes a distribution like this.

This makes, this was discussed earlier also, this makes a distribution like this, the velocity distribution becomes much skewed. Here the velocity becomes, on this side, between this passage is high and here is this is low. So this way the velocity distribution changes, so as a whole, what happens, because of this, a small recirculatory flow due to the difference in pressure from the leading and trailing edge, there is a, there is a change in the direction of the velocity of the fluid is relative to the blade.

So this results in this way. That means we wanted that the fluid will go in the radial direction, rather it will move in this way. That means the direction of the fluid relative to the blade is changed like this. This is velocity relative to impeller. This is the, this velocity triangle, here you see, this velocity triangle, this, this, this triangle, this is the triangle, this is now the VR2. So this is VR2, and therefore what happens, if this becomes VR2, then this is the changed velocity triangle, it is there similar if you recall what we discussed the case of centrifugal pump. So what happens is that in that case, this is the tangential component of the absolute velocity.

Now if you compare these 2, this is U2, this is U 2, obviously U2 remains same, U2 depends upon the rotational speed and the outer radius of the impeller. Okay, so therefore for the same U2, at the impeller outlet, V2 is reduced and this component is reduced from that of U2. So therefore as a result, what I, what we get VW 2 is less than U2. In this case it was VW2 U2 because the absolute, the relative velocity is radially outward. The other relative velocity is like this.

This is the similar thing what happened in case of centrifugal pump. So this is a slip, but you have to know the fluid mechanical possible of the slip, it is because of the blade curvature and the rotation of the blade, the motion is being imparted to the blade at the same time the fluid flow past the blade of the 2 sides of the blade, leading-edge and the trailing edge which creates, it is a sort of a circulation, circulation combined with a linear flow which is in the radial direction. Which creates this type of difference in pressure and a local recirculatory flow as a combination of this with the radial flow outward, makes a skewed nonuniform distribution of the velocity.

Which finally it results in this type of velocity triangle where we get the relative velocity at the outlet is not radial. As a result of which finally we get VW 2 less than U2. Which means that if we see the energy transfer, so therefore this will be now VW2 is U2, so this will be now actually E by M, if VW 1 is 0 is VW2 U2, since VW 2 equals to U2, we wrote is U 2 square. Now if VW2 is not equal to U2, so it will not be U2 square and if VW 2 is less than U2, then it will be less than U2 square.

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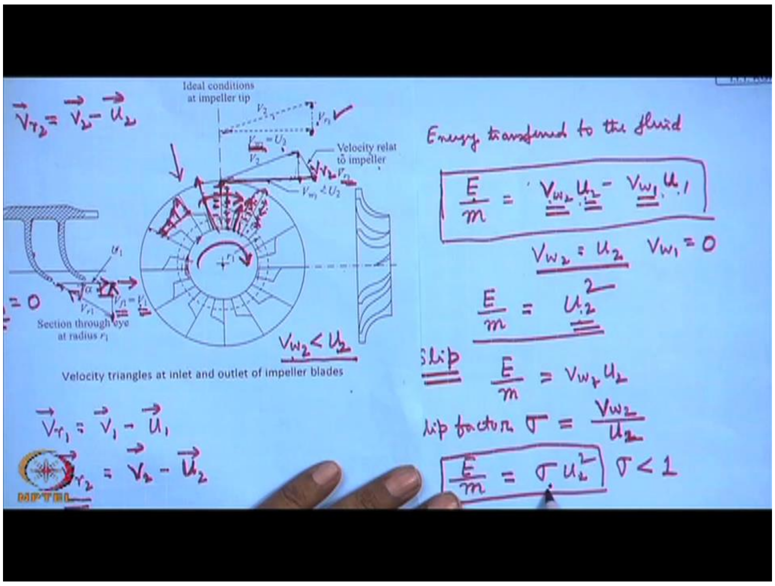

$\vec{V}_{r_2} = \vec{V}_2 - \vec{U}_2$

Energy transferred to the fluid

$$\boxed{\frac{E}{m} = V_{w_2} U_2 - V_{w_1} U_1}$$

$V_{w_2} = U_2 \quad V_{w_1} = 0$

$$\frac{E}{m} = U_2^2$$

Slip $\quad \frac{E}{m} = V_{w_2} U_2$

slip factor $\sigma = \frac{V_{w_2}}{U_2}$

$$\boxed{\frac{E}{m} = \sigma U_2^2} \quad \sigma < 1$$

$\vec{V}_{r_1} = \vec{V}_1 - \vec{U}_1$

$\vec{V}_{r_2} = \vec{V}_2 - \vec{U}_2$

Here a terminology is there defines slip factor. A factor is defined known as slip factor which is symbolised as Sigma, I did it for centrifugal pump also which is here defined as VW2 by the ratio of the outlet whirling component, or the tangential component of the fluid at outlet to the tangential speed of the rotor at the outlet. So if slip factor is defined like this, then in terms of the sleep factor, we can define that the energy per unit mass is then Sigma U2 square. And Sigma is less than 1.

Sigma is less than 1 and therefore we see because of the slip, a less amount of energy is being imparted to the fluid as compared to that if there could have been no slip. So how flip will increase. Now one thing I tell you again and again which I told you while discussing the centrifugal pump that slip is not a consequence of fluid viscosity, slip is a consequence of the motion of the fluid, the rotation of the fluid by the action of the blades. Even if the fluid is inviscid, the slip will occur.

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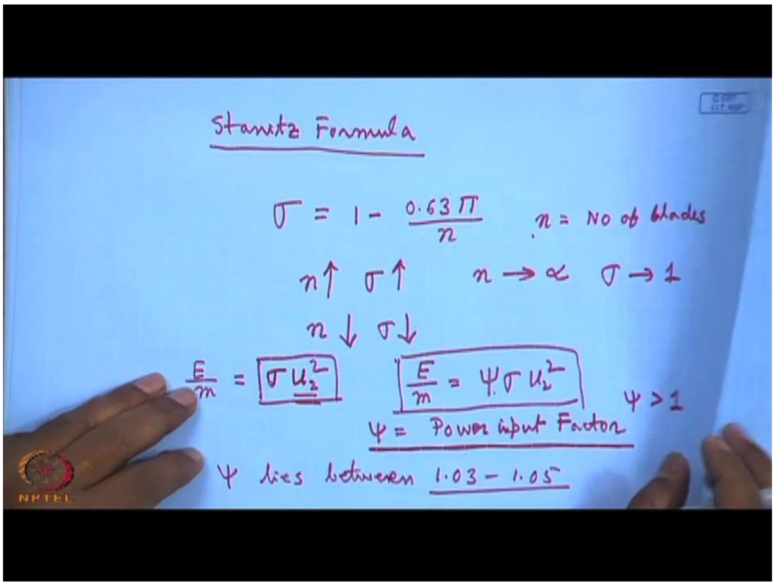

Stanetz Formula

$$\sigma = 1 - \frac{0.63\pi}{n} \qquad n = \text{No of blades}$$

$$n\uparrow \ \sigma\uparrow \qquad n \rightarrow \infty \quad \sigma \rightarrow 1$$

$$n\downarrow \ \sigma\downarrow$$

$$\frac{E}{m} = \boxed{\sigma u_2^2} \qquad \boxed{\frac{E}{m} = \psi \sigma u_2^2} \qquad \psi > 1$$

ψ = Power input Factor

ψ lies between 1.03 – 1.05

So therefore be careful that the slip in and it is not because of the fluid friction to the slip ultimately reduces the energy from what we could have got if there was no slip. There is a formula to find out slip, so this formula is known as standard formula Stanetz formula. Stanetz from a potential flow analysis expressed the slip like this. You can get some idea from here, 0.63 pie by N where N is equal to the number of blade, where N is equal to number of blades or vanes.

You see here very much that is the number of less, so therefore you see the number of blades increases, what happens, Sigma increases, the maximum value is 1. When number of blades tend to infinity, Sigma tends to 1. There is no slip. That means if you make small small streams or more number of blades, infinite number of blade passages, then slip will be totally reduced, almost reduced. On the other hand when number of blades is decreased, slip is decreased, which means slip will be much lower than 1 and there will be much reduction in the energy imparted to the fluid.

Okay. So therefore due to this slip, finally the energy imparted by unit mass is given by Sigma U2 square where U2 is the outlet tangential velocity of the impeller or impeller blade and Sigma is the slip factor. Now if we consider other losses, mainly the mechanical fictions, okay, and windage losses, these types of losses, if you take care of, then you have to give more amount of work as given by that expression. And therefore the energy per unit mass is written by making another term as a product that is psi which is known as power power input factor, psi is known in power input factor.

Okay, power input factor. So this is known as power. That means this takes care of frictional losses, the disc friction, the frictional losses, the frictional losses, the disc friction, windage losses, all these things, I have taken care of by the power input factor which by physical sense will be greater than 1. Definitely that means this will be greater than 1, that means this will be energy requirements of the machine so that out of these, this will be imparted to the fluid which will exhibit the slip phenomena.

And this value of psi usually lies between, lies between between 1.03 to 1.05, like that. So therefore finally we arrive at this expression that energy per unit mass given to the fluid is Sigma Psi, Sigma U2 square.

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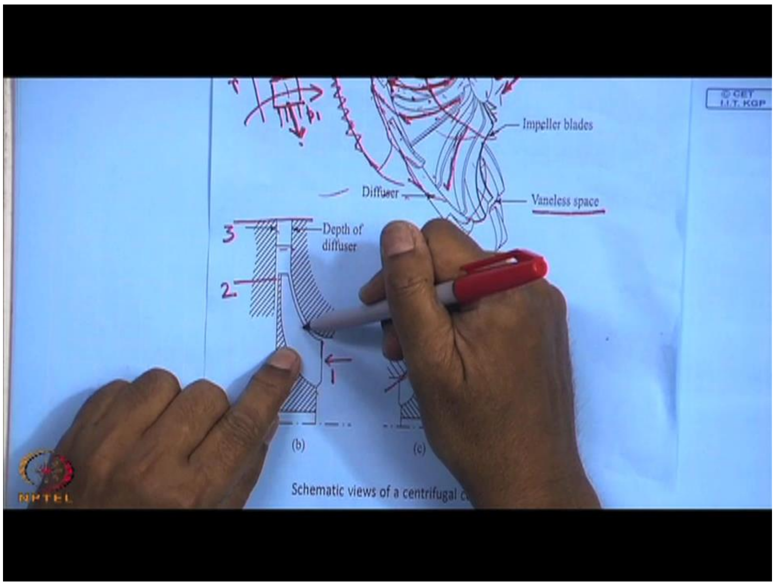

This is the amount of energy given to the fluid. Now let us find out in terms of the pressure rise. Let us do some thermodynamics. Let us consider, no here again I come here, let us consider, this is, this will be better here, we are not going unnecessarily into this complicated figure, let us see that this is the inlet to the impeller section 1, this is the outlet of the impeller section 2, this is the outlet of the diffuser, section 3. Now let us consider this way, that section 1 to 2, when the fluid flows, it gains the energy.

So the energy is being imparted here whose value is just now we have written, psi Sigma U2 square. This is the energy required but actually fluid get this energy, Sigma U2 square. Psi Sigma U2 square is the total energy required part of which is lost in friction. So therefore this energy is being imparted in the fluid only here. So 2 to 3, there is no energy given, so total energy between 2 to 3 remains same, there is no energy given to the fluid, there is no energy interaction. So therefore what happens, the energy level at 2 and 3 remains same but there is the difference in pressure between 2 and 3 because the pressure rise takes place in the diffuser.

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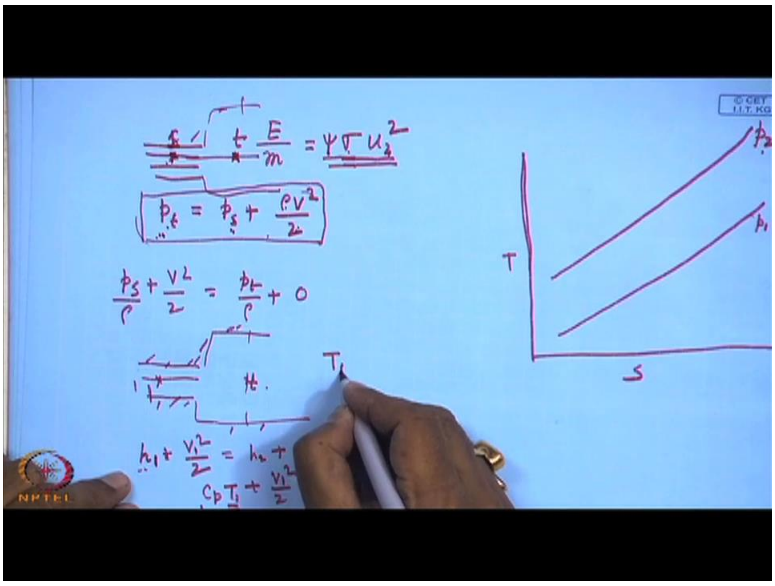

So now if you give this 1, 2, 3, now if we, with this can you see, 1, 2, 3, then we draw a, in thermodynamics, the TS diagram, okay. Let us consider the pressure here is, the total pressure is P1T and at the end, the total pressure is P3T and we show these things like this. P1T and we show this thing in… P2, these are diverging lines, P2T. What is total pressure, total pressure is the static pressure + the stagnation, + the velocity pressure, that means of pressure, velocity head, pressure equivalent of the velocity head or the kinetic energy.

Now I will not show you here, it will be difficult, now 1 is the inlet of the impeller, 2 is the outlet of the impeller. You see one thing that what is this P1 T1, now stagnation pressure, total pressure, I am just calling again, total pressure is the static pressure PS + rho V square by 2. For any fluid stream, if you make the velocity 0 by some way, for example you make the area infinitely large, so therefore is to write the Bernoulli's equation at 2 points along a streamline, then you can get that PT, when the velocity becomes 0, then PT equal to total pressure.

That means the pressure there equal to PS + rho V square by 2. And you can write Bernoulli's equation when there is no energy, no energy at it from outside, okay, and there is no dissipation taking place, no fluid friction, no fluid friction. So therefore under ideal conditions for an inviscid fluid, this is synonymous to an isentropic flow that the fluid is brought to rest, then we can write. For example if this is the point 1, if this is the point T for example where the total pressure is obtained.

So therefore P1, P1 is the letter PS here, let this is S, that PS is the pressure here + PS by rho V square by 2 is PT by rho +0. So which gives rise to this, that with the total pressure is obtained by writing the Bernoulli's equation means that the fluid is decelerated isentropically. Isentropically means without any internal reversibility, without any heat transfer, therefore external reversibility is not there, purely reversible we and without any other energy added or taken out. So therefore this is the concept of the total pressure. So this is the total pressure.

Now let us consider the stagnation temperature, what is stagnation temperature? Similar to that, the concept of stagnation temperature is the temperature which is being gained if a fluid stream is brought to rest but there is no concept of reversible way. That is you can bring to rest even in consideration of the friction because the total energy is conserved.

For example I just gave you the, recalling the stagnation temperature concept, that 1 and T, the similar way if you do that a fluid stream is being retarded from a point 1 to a point here T and if we write this energy question, then steady flow energy equation H1 + kinetic energy V1 square by 2 and if you consider that it is done adiabatically, that makes no energy interaction either in the form of heat or in the form of work takes place, then we can write H2 +0.

So this equation is a steady flow energy equation where we neglect the change in potential energy. So there is no other form of work transfer, either from outside or from the system, outside to the system or system to the outside. Then therefore for this control system, steady flow energy question gives rise to this where friction is not coming into picture. So therefore here, this H2, this H1 is what CP into T1. This is the section 1 for example, + V1 square by 2.

And H2 is CP into T2. So this T2 is corresponding, corresponds to the stagnation temperature of T1. You understand. So therefore a stagnation temperature, if you write the TT, is equal to what? Is equal to TS, this is known as the static temperature + V square by 2 CP in general. That means this is the velocity or kinetic energy equivalent, velocity head or kinetic head equivalent of the temperature, this is kinetic energy equivalent of the pressure. This kinetic energy equivalent of the pressure is realised if this is the being, this is done in a reversible way without any dissipation, absence of friction.

But this is purely energy conversion, so therefore, this is the temperature, this is the velocity equivalent of the kinetic energy equivalent of the temperature added with static temperature gives the total temperature where the fluid velocities reduced to 0 without any restriction by the friction is there or friction is not there. This is because of friction takes care of the pressure but here the friction does not come into picture. I just explain it again for your concept to be clear that H is probably you know U + PV.

So whenever we make this difference, at this balance, that H1 + + V1 square by 2 is H2, so within the H, this U and PV is made, will be made by the friction. That means whether friction is there or friction is not there, that will depend upon this distribution within U and PV, they will add. When friction is there, U will be more, Intel energy will be more because of the friction, whereas, where PV is less, it reverses there. If there is no friction, this U will not be changed because internal molecule energy will remain same because of the temperature and PV will get as proportional to the H, changing H.

So therefore it is the conversion from U and PV that will depend upon the friction but H1 - H2 is V square by 2 or H1 + V1 square by 2 is H2 if the velocity is there any other velocity may be there, this is a steady flow energy equation which is independent of whether there is friction or not. This is a recapitulation of the stagnation, total pressure and the total temperature.

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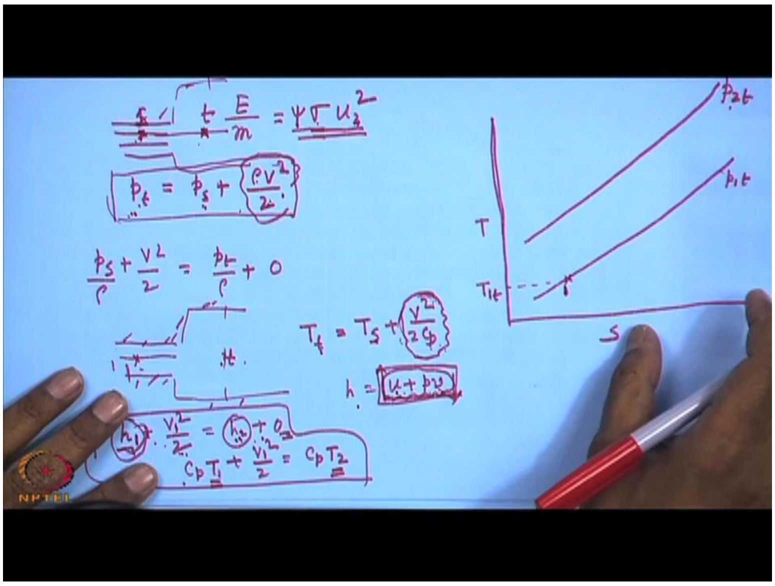

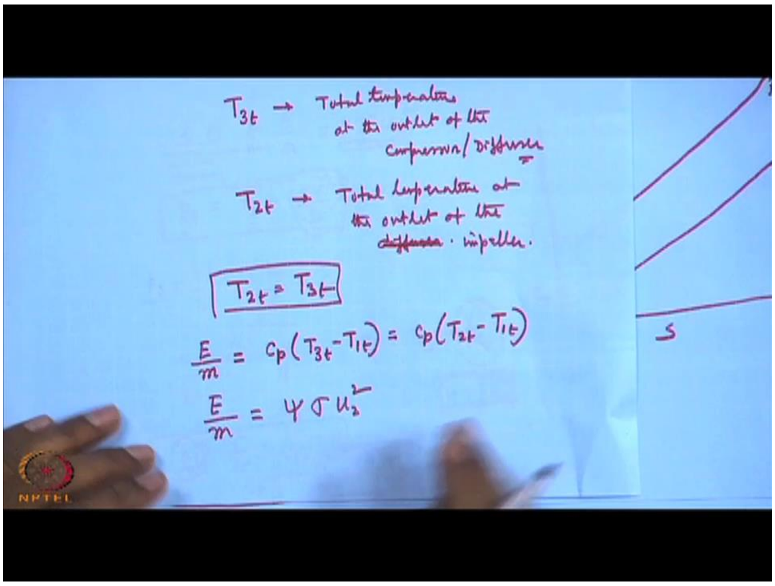

T_{3t} → Total temperature at the outlet of the compressor/Diffuser

T_{2t} → Total temperature at the outlet of the ~~diffuser~~ · impeller.

$$\boxed{T_{2t} = T_{3t}}$$

$$\frac{E}{m} = c_p(T_{3t} - T_{1t}) = c_p(T_{2t} - T_{1t})$$

$$\frac{E}{m} = \psi \sigma U_2^2$$

Now come to the point. If we consider the point 1, that is the inlet to the impeller, at the, at point 1 where we have got the total temperature is, the total temperature, T1T and if we get a total temperature T2 T, now let us before that let us consider this way. Sorry, let us consider this way that, okay. Let us consider this way that if T2, T3 T is the total temperature, total temperature at the outlet of the compressor, at the outlet of the compressor, at the outlet of the compressor at the outlet of the compressor.

That means at the outlet of the, at the outlet of the diffuser. Then T2 T, if we write T2 T is the total temperature or the stagnation temperature, total temperature at the outlet of the diffuser. Since there is no, at the outlet of the impeller, sorry, at the outlet of the impeller, that is outlet of the compressor, that means that the outlet of the diffuser, diffuser, at the outlet of the diffuser. Then we can write T2 T is equal to T3 T because the total temperature remains same because there is no energy added in the diffuser.

Therefore we can write that energy added per unit mass, that work done per unit mass or energy added per unit mass is equal to CP, specifically into T3 T - T1 T or is equal to CP into T2 T - T1 T, clear, that we can write very well this one. CP T3 T - T1 T is equal to CP T2 T - T1 T. Now this becomes equal to what, again E by M we have got is equal to psi Sigma U 2 square. So therefore what we can write now, we can write, we will come to this afterwards, I will do something else here, we will come afterwards.

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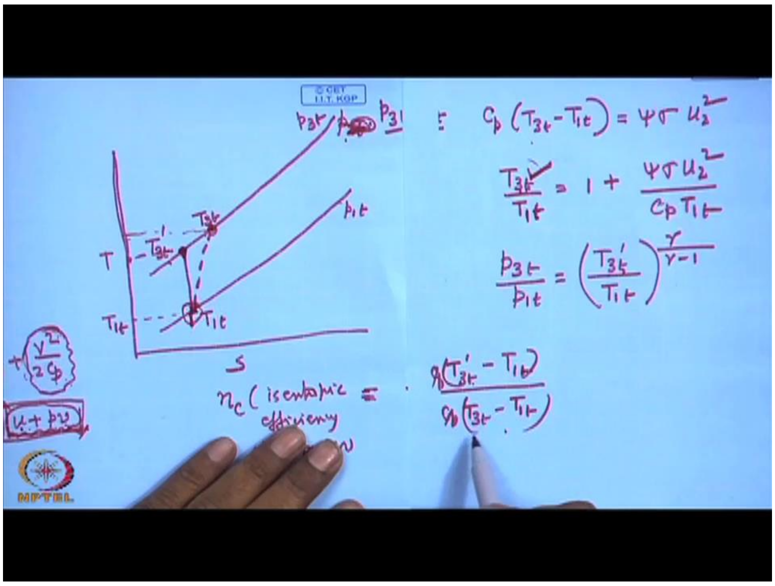

$$c_p(T_{3t} - T_{1t}) = \psi\sigma U_2^2$$

$$\frac{T_{3t}}{T_{1t}} = 1 + \frac{\psi\sigma U_2^2}{c_p T_{1t}}$$

$$\frac{p_{3t}}{p_{1t}} = \left(\frac{T_{3t}'}{T_{1t}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\eta_c \text{ (isentropic efficiency)} = \frac{c_p(T_{3t}' - T_{1t})}{c_p(T_{3t} - T_{1t})}$$

Before that I will do something that we ultimately get, E by M is what, again E by M, E by M is this, then we can write from this, from this we can write CP, let us write here T3 T - T1 T is equal to psi Sigma U2 square. So therefore we can write simply T3 T by one step is equal to this becomes 1+ psi Sigma U2 square by CP T1 T. So a ratio of temperature T3T to T1 T, we get, okay. Now, this is the T3 T, outlet temperature, this is the T1 T. Now you come to this, this is the one, now if we have an isentropic expansion, we get a point here.

This is the T2 T, okay, so therefore we can get this point here. Sorry, this is not P2T, I write it, this is the final pressure of the compressor, the total pressure of the compressor P3 T, sorry, this is sorry, P3T. Now here is the point, this is the isentropic, the actual T3 T is not at this point because the actual process is not isentropic. This point we denote as T3 T dash, that means this temperature.

So therefore, this temperature I know, then I can write the pressure rise, the expression for pressure rise which I am interested to know T3 T dash divided by, this is T1 T, the inlet temperature to the compressor, T1 T to the power gamma by gamma -1. This is the isentropic process relation between pressure and temperature. At actual T3, that is the temperature at the outlet of the diffuser, that is the outlet of the compressor, is different from that of the, from that of the isentropic process.

Actual process is not isentropic. So what will be the actual process, actual process will not be vertical. So there may be 2 options of process maybe like this, process may heal towards left. As you know that adiabatic process, that means the process without any transfer but with friction, or internal irreversibility entropy always increases. What is an isentropic process? Isotopic process is a process which does not have any heat transfer with the surrounding, the system does not have any transfer with the surrounding, at this instance, there is no irreversibility inside.

There is no friction, even know heat transfer within it, temperature is almost uniform at any instant, so that internal dissipation or internal irreversibility is 0. At the same time, there is no heat transfer, external irreversibility is 0, so process is totally reversible and at the same time and at the same time a diabetic, entropy remains constant, that you know from thermodynamics. But if the process is adiabatic, but you cannot avoid the friction, so this is known as adiabatic process with internal area visibility for which we know from the principle of increase of entropy that entropy always increases because of the internally irreversibility.

So therefore the actual process is shown, it is recapitulation of your thermodynamics. So actual process is shown like this, that is T3 T. So therefore this process, this temperature rise here. So therefore this is the actual temperature, here we define a terminology known as isentropic, isentropic efficiency of compressor, isotropic efficiency of compressor which is defined as the ratio of these 2 temperatures in a way that T3 T dash - T1 T divided by T3 T - T1 T.

Actually there is a CP, this cancels out, this ratio is what, physically that it is the actual work that is required divided by the ideal work. Actual work is always more, T3 T is more than that, this is because of the friction and temperature increases because of the increase in entropy, due to internal irreversibility. Actual temperature rise can be written in terms of this, in terms of the isentropic efficiency Eta C, isentropic efficiency of the compressor Eta C.

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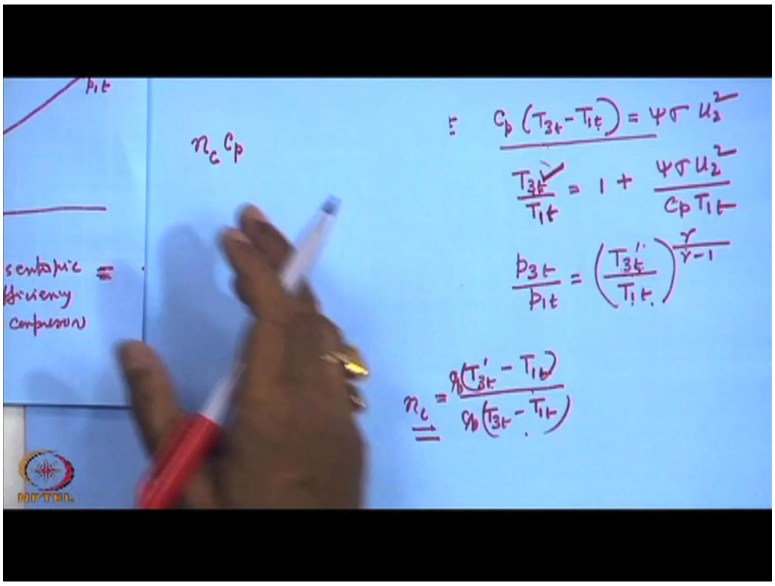

p_{1t}

sentropic =
fficiency
compression

$\eta_c c_p$

$$c_p(T_{3t} - T_{1t}) = \psi \sigma u_2^2$$

$$\frac{T_{3t}}{T_{1t}} = 1 + \frac{\psi \sigma u_2^2}{c_p T_{1t}}$$

$$\frac{p_{3t}}{p_{1t}} = \left(\frac{T_{3t}'}{T_{1t}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\underline{\underline{\eta_c}} = \frac{c_p(T_{3t}' - T_{1t})}{c_p(T_{3t} - T_{1t})}$$

So therefore what we can write, T3 T - T1 T is therefore we can write is Eta C, Eta C into CP into what is this, T3 T dash, this is Eta C is equal to this, so T3 T dash T3 T dash we can write, this is T3 T dash, so 1 Eta C, that means we can write, now CP T3 T - T1 T is this. Here we have T3 T, T3 T dash by T1 T is this. Therefore we can relate this T3 T dash by T1 T in terms, T3 T - T1 T in terms of this, so this Eta C. And we can find out the pressure rise. However today’s time is over. I think I will discuss it tomorrow. Okay, next class. Thank you. Okay.