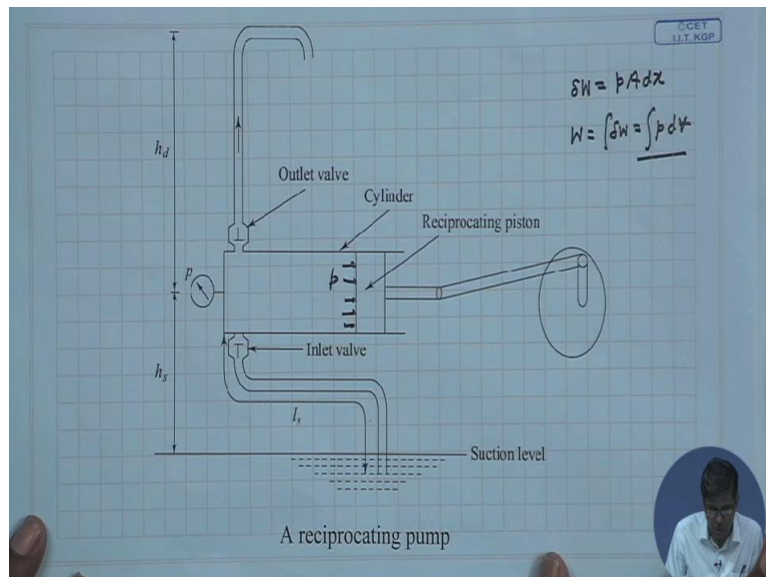


Fluid Machines.
Professor Sankar Kumar Som.
Department Of Mechanical Engineering.
Indian Institute Of Technology Kharagpur.
Lecture-27.
Reciprocating Pump Part I.

Good morning and welcome you all to this session of the course on fluid machines. Now today we will be discussing reciprocating pumps. At the beginning of this course we discussed that there is basically 2 types of fluid machines, one is Rotodynamic machines, another positive displacement machine. And these 2 categories are differentiated on the basis of the principle of the operation.

In a Rotodynamic machines, energy transfer between the machine rotor and the fluid takes place by the hydrodynamic action between a continuous, in a continuous stream of fluid flowing through the rotor blade passages, while in a reciprocating machines, the energy transfer takes place by changing the volume of a certain mass of fluid content within the machine by the physical displacement of one of its boundaries. Now reciprocating pumps where the fluid pressure is increased, the energy is manifested in terms of rising static pressure is made by the principle of positive displacement.

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Now usually in practice, the reciprocating pumps looks like that. It basically consists of a piston or plunger moving in a cylinder, executive reciprocating motion in a tightly fitting cylinder, this is cylinder, this is the piston or plunger. And this movement is being executed

by a connecting rod and a crank which is connected to the crankshaft, final output shaft, input shaft here where the energy is given.

Now you see the operation is like that, when this piston moves this side, outward of the cylinder, then there is a fall in pressure of the liquid and due to this the liquid from the sump, that is lower reservoir comes into the cylinder and there is an inlet valve which automatically opens because of the reduction in pressure. That means this opening is automatically done by changing the fluid pressure during the suction stroke.

Now at the end of the suction stroke when the piston which is at outer dead centre, this is known as dead centre at the 2 extreme points, inner and outer. Then what happens, the piston again while coming inward changes the pressure in the fluid and during that operation, that means this stroke from outward to inward direction, the moment of the piston, this valve is closed at this valve gets opened, that is the outlet valve because of the high pressure created here by the movement of the piston which pushes the liquid and creates a high pressure.

And then during this stroke towards the inner dead centre position or inward, the fluid at high pressure is being discharged through the pipe. Now this is the suction lift which is known as HS, which is the height of the Centre of the pump's cylinder from the lower head reservoir and this height, the height from the centre of the pump cylinder to the discharge point, the overhead reservoir or the discharge point vertically above is known as the is HD, that is the delivery link, this is HD.

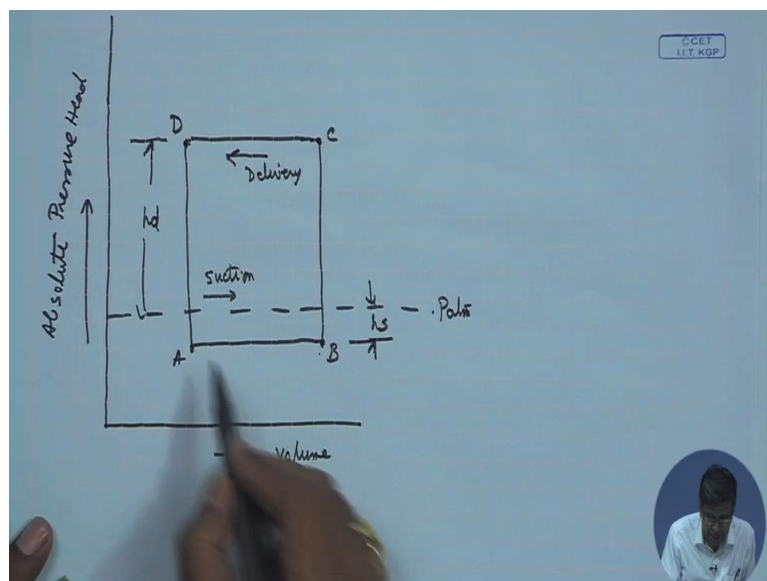
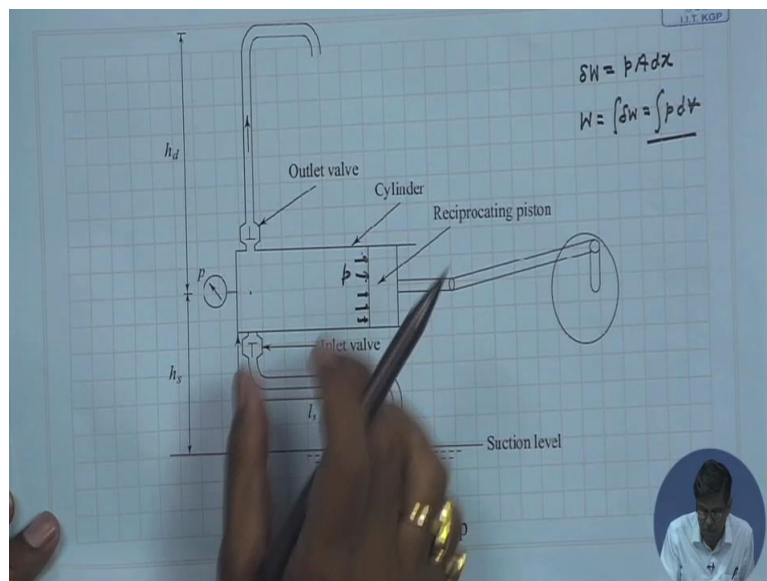
So I pump delivers a head is equal to $HS + H$, which is the static lift as we have already seen in centrifugal pump. Now here you see this, as the piston moves here, piston exerts pressure, now at any instant if you consider the piston pressure is P which is exerted on the liquid is continuous pressure, then we can write that the small amount of work that is being transferred to the fluid or done by the piston with that pressure P . If we consider at that instant piston momentarily in equilibrium under that pressure which is again acting on the piston on this side.

This is the pressure acting on the piston, the piston exerts the pressure on the liquid, pressure acting on the piston, and if we consider a cross-sectional area of the cylinder A and for a infinite small displacement of the piston DX . That means the work which equals to integral $P \cdot A \cdot DX$ is nothing but integral, this $A \cdot DX$ can be written as elemental volume. That means it is integral $P \cdot DV$, it is almost similar to this $P \cdot DV$ work in thermodynamics in consideration of

the piston, local and instantaneous thermodynamic equilibrium, the piston is not at all in thermodynamic equilibrium, it has got acceleration, deceleration and the pressure is changing, so it is continuous pressure by which you can express the work done as and integral of PDV.

That means it is the change in the pressure in the liquid, in the cylinder and the liquid volume in the cylinder because of the piston motion is responsible for the energy transfer and can be expressed like that.

(Refer Slide Time: 6:20)



So with this in mind if we now try to draw a pressure volume diagram for this cylinder, that means this side and this side if we write the absolute pressure, absolute pressure, then we get

a diagram like that. Let me 1st draw, then I will explain you, let me 1st draw then, let me 1st draw then I will explain you the ideal pressure volume diagram, like a cycle diagram in a thermodynamics, for a thermodynamics cycle looks like a rectangle ABCD. What is this, let this is the atmospheric pressure line and this is our suction lift, HS, that is the vertical height of the pump cylinder from the lower reservoir.

Now at the, in the ideal case, the pressure if we neglect the velocity and acceleration of the fluid in the piston and in the pipeline, then we expect in an ideal case that the suction pressure should be sufficient enough to lift this water by height HS. So therefore suction pressure head will be equal to that of the HS, okay. So suction pressure head will be equal to that of the HS and in an ideal case, that will be constant throughout the process as the piston moves in this direction.

It is similar to if you recollect in thermodynamics is the suction process of an air standard cycle like auto cycle or diesel cycle. So it is a constant pressure suction. So this is absolute pressure head rather, when I express this in terms of HS, this is absolute pressure head, absolute pressure head, so therefore this is HS, below the atmospheric pressure A to B is the suction. Now at the end of the suction, the piston pushes the liquid.

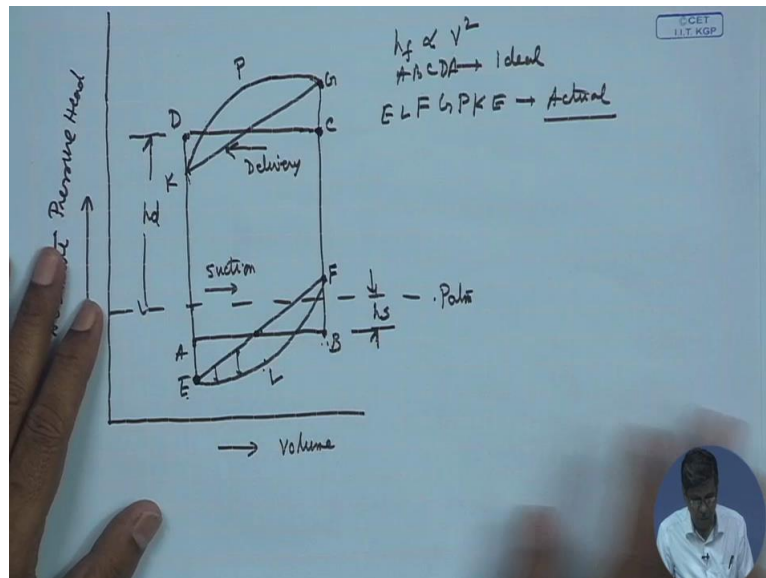
Now if we consider the liquid to be ideally incompressible, fully incompressible, the property of incompressible flow is that it gets pressurised instantaneously without changing in volume. Why, this is because that is the piston pushes it, a pressure wave is generated which is actually transmitted upstream to pressurise the entire fluid. In an 100 percent incompressible fluid, an ideally incompressible fluid, this travelling of the pressure head in the fluid medium is almost infinite, is very very high.

So therefore it is instantaneously it is sensed upstream which means any pushing here resulting in a rise in pressure will be sensed by the entire fluid in this cylinder and that is the property of the incompressible fluid. The fluid is perfectly incompressible, fully incompressible, then we get the, that means the modulus of elasticity is infinitely high so that this is immediately sensed. So this is considered to be an instantaneous pressure rise without any change of volume and that is why this C is there.

So therefore this is the direction of suction, this is suction and again at the delivery, this is the delivery line, delivery direction sorry delivery direction. So therefore pressure is raised to delivery pressure, that is equal to HD, that is the height, that is the delivery lift HD. So this is

HD, so therefore one can say that this is HD and the total is HS + HD is the total lift of the pump. And similarly at the end of the delivery stroke when the piston comes, the inner that position, the innermost position of the cylinder, then immediately at the start of this stroke or reciprocating motion outwards, that means towards the outward that centre position, the pressure falls instantaneously to the suction.

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So this is an ideal diagram rectangle in case of PV. Now this diagram is changed because of the fact that when piston executes the reciprocating motion, it has acceleration and deceleration. Piston executes a motion through this connecting rod and crank mechanism executives and accelerating decelerating motion. In an ideal case where the connecting rod is much longer than the crank, we can consider this as a simple harmonic motion.

So it executives simple harmonic motion where there is both acceleration and deceleration, in that case what happens, the piston pressure within the cylinder varies in suction and in delivery stroke. Now in suction stroke what happens, in the beginning of the suction stroke, the fluid has an acceleration. Fluid has got 0 velocity here at the inlet dead centre position. When it starts from the inner dead centre position, the velocity increases and not only velocity, acceleration also decreases.

Velocity increases but acceleration decreases but there is a sharp acceleration, so acceleration starts at the beginning. So because of this acceleration, the suction head required will be more than that of the static lift in consideration of the acceleration head. That is known an acceleration head. That means an extra force has to be given an extra energy has to be created

to draw the fluid with acceleration. And this acceleration in the pipeline will be more than that in the cylinder because of the difference in cross-sectional area.

No liquid will be moving with the same velocity and acceleration of that of the piston in the cylinder. And at any instant of time, the liquid velocity in the cylinder, velocity and acceleration in the cylinder and the liquid velocity and acceleration in the pipeline will be related to their cross-sectional area. So the liquid velocity and acceleration in the pipeline will be more than that in the cylinder because of lower cross-sectional area of the pipeline than that of the cylinder.

So therefore finally the liquid velocity and acceleration, both in piston and the pipe, in cylinder and the pipeline depends upon the piston velocity. Initially provides an acceleration, so more head will be required to draw this. And acceleration ultimately reduces, as I told you it starts at the beginning but ultimately there is an increase in velocity, the acceleration reduces and in the middle of the stroke, if we consider this to be executing a simple harmonic motion velocity will be maximum and the acceleration will be 0.

And then acceleration switches off its direction, that means it goes on the retardation, that means velocity from a maximum value goes on decreasing and comes to 0 when again acceleration becomes -, maximum with a - sign, that means deceleration. So the rest part is deceleration where a higher pressure is needed in the cylinder during the suction stroke. So to take care of initial, 1st part acceleration and subsequently the deceleration, the head requirement at the in the suction stroke can be modified like this.

Can be modified like this and I give this nomenclature as EF. That means in consideration of a simple harmonic motion, this will be a straight line where E is the minimum pressure in the cylinder which is lower than this static head HS, the suction head or suction lift HS, this is the point where to take care of an acceleration, initial acceleration, the additional head is required. At the same time because of deceleration, here the velocity is maximum and acceleration is 0, the pressure needed is less so that ultimately it ends here.

So finally EF is the modification of this ideal suction stroke AB in consideration of piston acceleration and deceleration. Now along with that there are viscous losses in the pipeline and also in the cylinder. So these viscous losses, that means the loss of energy to fluid friction, this has to be also surmounted by the head in the suction side. So these losses are

proportional to velocity. As you know the viscous losses are always proportional to square of the velocity.

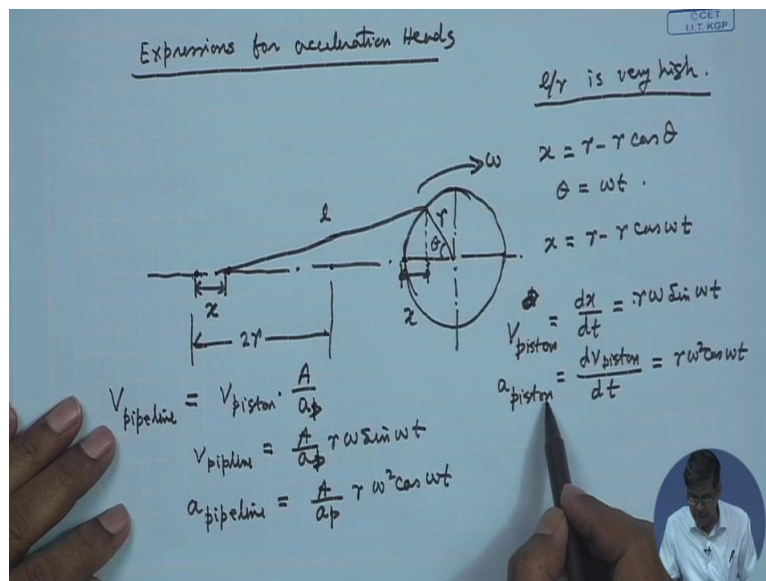
So at this point, at the dead centre, that is the inner dead centre position, at the start of the suction stroke, this is 0, again at the end this is 0. Here the velocity is maximum. So the loss is maximum here. So if you take care of this parabolic law of the losses in its variation with the flow velocity which is 0, 0 here at the 2 dead centre and being maximum at the middle, we can modify this curve like this ELF.

So therefore you see the ideal suction stroke AB is finally modified as ELF in the suction stroke in consideration of the inertia of the piston that means its acceleration and deceleration and in consideration of a simple harmonic motion executed by the piston we can write, we can draw this like this. And this is the frictional loss, additional frictional loss. Similarly this will be modified in this diagram also like this, let us give a nomenclature KG and with the frictional loss, this will be also the final diagram.

Again I explain this this is a constant delivery head at the beginning, again the acceleration of the piston which takes care of that and it produces more head, that means this head is above this lift, this delivery side lift. Again the losses will be maximum by the velocity is here maximum. Velocity is 0 as these 2 ends and again it is a deceleration and comes here, so the velocity, that is the discharge head is at lower than this, this is because to take care of the... Fluid is decelerated there and this part of the fluid is in acceleration.

So therefore in the delivery stroke also, initial acceleration part gave greater head, that greater head has to be generated at discharge end, similarly to take care of the deceleration, the head generated will be little less. So therefore the head generated in the cylinder of the pump will be like this, KPG. Therefore the final diagram from an ideal one which is rectangle ABCD is changed to ELF, from ABCD is the ideal one, ideal and ELFGPKE, this is ABCDA we should write to completed, this is actual in consideration of the inertia of the piston and the liquid and at the same time the frictional losses.

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Now after this let us consider the expressions for acceleration head. Let us consider the expressions of acceleration head. Now let us consider the expressions of acceleration head. Expressions for acceleration heads. How to calculate these acceleration heads? Let me draw a picture 1st. Let, okay, let this is the crank of radius R and let this is a connecting rod which is sufficiently long L. Let this is given by X and similarly this is X. Now we derive an expression of acceleration in consideration of the crank rotation.

Now when L by R is very high, we can consider this as an executive, we can assume a simple harmonic motion executives by the piston due to the rotation of this crank with a uniform angular speed omega. Let us consider the rotation. At some time from the inner dead centre position because this is the inner dead centre position when the piston is here, the crank is here, when the crank goes here, that means 180 degree degree rotation, then it goes to the outer dead, that means this is the outer dead.

So the outer dead to inner dead position is 2 R and this is known as one stroke, so one stroke is completed when the crank moves from 0 degree to 180 degree. Okay, so now if we consider at any time T, the crack has rotated an angle theta with this direction axis, then if this is the displacement X, this is also X, that means the piston has moved from its inner dead centre position to a distance X which is given by this. So this X can be written from here as R - R cos Theta. This is R cos Theta and this is R. And Theta is the angle for the constant angular velocity Omega T.

So X is equal to $R - R \cos \Omega T$. Now to find the velocity, sorry we find the velocity of the piston, velocity of the piston is equal to $\frac{DX}{DT}$ which is equal to $\sin \Omega T$, that is $R \Omega \sin \Omega T$. Then the acceleration of the piston, a piston, acceleration of the piston is equal to $\frac{d}{dt}$, derivative of this. And if we make this derivative $\frac{d}{dt}$, now this is the, sorry, I am extremely sorry, this is the acceleration of the piston is okay but I think we will find out the acceleration of, okay, pipeline.

Acceleration of the piston will be definitely $R \Omega^2 \cos \Omega T$. Now we are interested in the velocity in the pipeline. That means either in discharge pipe or in the suction pipe, inlet pipe. So at any instant, velocity in the pipeline and velocity in the piston is related like that, velocity in the piston time the cross-sectional area of the piston and cross-sectional area of the pipe. That means we use A_p as the cross-sectional area of the piston and A is denoting the cross-sectional area in the pipeline.

Maybe inlet pipe, suction pipe delivery pipe. At any instant because of the continuity for a steady flow, steady flow means constant flow rate, volumetric flow rate, we can write this equation. So therefore we can write the V pipeline, V pipeline is equal to what? $A_p v_p$ from here $R \Omega \sin \Omega T$ and correspondingly the A pipeline, A pipeline, that is acceleration, here a is $A_p a_p$ rather. That is the cross-sectional area of the pipeline, rather I give A_p , A_p Rather.

So A by A_p , so the acceleration will be $\frac{d}{dt}$ of this, that means $A_p \frac{d}{dt} (R \Omega^2 \cos \Omega T)$. As I told earlier that the velocity and acceleration at any instant between in the piston and the pipeline of the liquid in the piston in the cylinder and in the pipeline because in the cylinder it is that of the piston, in the cylinder and the pipeline is just varying by this area because of the continuity. So this is the velocity in the pipeline, this is the acceleration in the pipeline.

Now with this in mind, that acceleration in the pipeline is this, we can write that the force now to create this acceleration in the pipeline, we require a force which is equal to this acceleration times the mass of the fluid. If you consider the mass of the fluid in the pipeline as this where L is the length of the pipeline, that into, this is of course A_p , $A_p R \Omega^2 \cos \Omega T$.

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$$V_{\text{pipeline}} = V_{\text{piston}} \cdot \frac{A}{a_p}$$

$$\underline{V_{\text{pipeline}}} = \left(\frac{A}{a_p}\right) r \omega \sin \omega t$$

$$\underline{a_{\text{pipeline}}} = \left(\frac{A}{a_p}\right) r \omega^2 \cos \omega t$$

$$F = \rho a l \left(\frac{A}{a_p}\right) r \omega^2 \cos \omega t$$

Pressure head caused by $F = \frac{F}{a_p \rho g}$
Acceleration head

$$\boxed{h_a = \left(\frac{l}{g}\right) \left(\frac{A}{a_p}\right) r \omega^2 \cos \theta}$$

Expressions for acceleration Heads

l/r is very high.

$x = r - r \cos \theta$
 $\theta = \omega t$
 $x = r - r \cos \omega t$

$\dot{x} = \frac{dx}{dt} = r \omega \sin \omega t$
 $V_{\text{piston}} = \frac{dx}{dt} = r \omega \sin \omega t$
 $a_{\text{piston}} = \frac{dV_{\text{piston}}}{dt} = r \omega^2 \cos \omega t$

$$V_{\text{pipeline}} = V_{\text{piston}} \cdot \frac{A}{a_p}$$

$$\underline{V_{\text{pipeline}}} = \left(\frac{A}{a_p}\right) r \omega \sin \omega t$$

$$\underline{a_{\text{pipeline}}} = \left(\frac{A}{a_p}\right) r \omega^2 \cos \omega t$$

So ultimately it becomes $\frac{A}{a_p}$, $\frac{A}{a_p}$ will cancel, so therefore the force required to do this will be $\rho A l$, if A by a , that means the force required to accelerate the liquid mass will be A by $\frac{A}{a_p} r \omega^2$. Okay, we may not write anything, straight we can write the pressure head caused, so therefore the pressure head caused, this is the force required to accelerate the liquid mass either in the suction or delivery line. So pressure head now caused by that force, that means which is known as acceleration head.

Which is known as acceleration head is nothing but this divided by $\frac{A}{a_p} \rho g$ because force divided by area, this cause the pressure corresponding to acceleration, acceleration pressure divided by ρg . So if you substitute this, this acceleration head H_A rather I write, the

pressure at caused by the acceleration will become L by G , A by a , R omega square, again $\cos \omega T$ I replace by θ .

That means this is force, this divided by A , A means again AP , sorry I forgot. That means the area of the pipeline. So this AP AP cancels, again one AP is coming. So A by AP , L by G , okay this ρ and ρ cancels. R omega square I write \cos . So therefore this is a very important expression relating to the acceleration head because the acceleration head can be found out for the liquid or the water, whatever may be in the piston by using this acceleration.

But usually the piston acceleration is neglected, velocity and acceleration as compared to the pipeline because of the high ratio of this, here AP is much less compared to the area of the cylinder. So therefore we are more interested in the acceleration head in the cylinder, suction delivery lines, delivery pipe. And this is the expression, length L by G , L is the length of either suction delivery pipe and A is the area of the cylinder, AP is the area of the pipeline which may be suction, which may be delivery R omega square $\cos \theta$, where θ is the crank angle. Okay. So today up to this. Thank you.