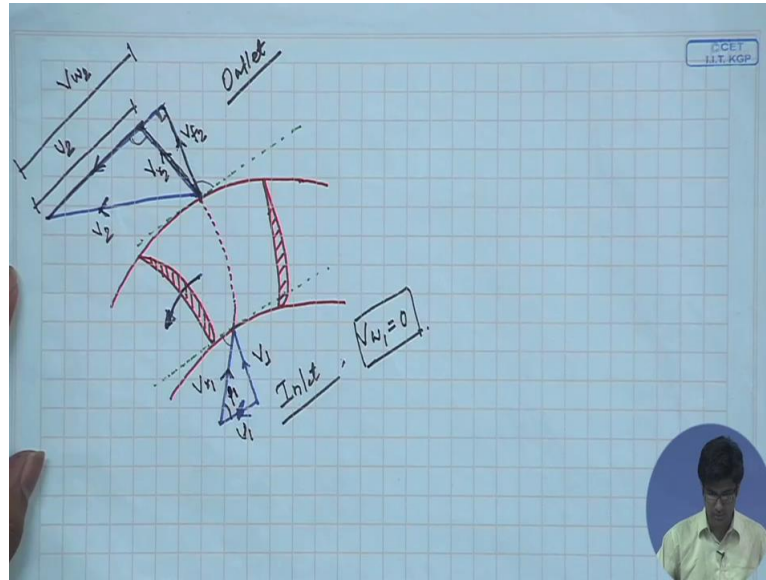
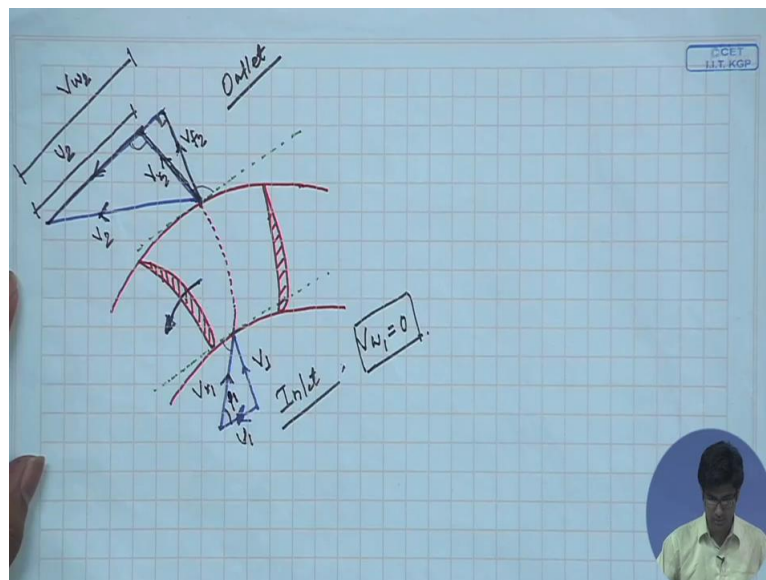


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**Tutorial-7.**

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DCE  
I.I.T. KGP

A single-stage centrifugal pump is to be used to pump water through a vertical distance of  $30\text{ m}$  at the rate of  $0.045\text{ m}^3/\text{s}$ . Suction and delivery pipes will have a combined length of  $36\text{ m}$  and a friction factor of  $0.006$ . Both will be  $150\text{ mm}$  diameter. Losses at valves, etc. are estimated total  $2.4$  times the velocity head in the pipes. The basic design of pump has a specific speed of  $0.074\text{ rev}$ , forward-curved impeller blades with an outlet angle of  $125^\circ$  to the tangent and a width of impeller passages at outlet equal to one-tenth of the diameter. The blades themselves occupy  $5\%$  of the circumference. If a hydraulic efficiency (neglecting whirl slip) of  $75\%$  may be expected, determine a suitable impeller diameter.

Today I will be solving some problems on centrifugal pumps. So let me start with the 1<sup>st</sup> problem. Let me read it out. A single state centrifugal pump is to be used to pump water through it what equal distance of  $30\text{ metre}$  at the rate of  $0.045\text{ metre cube per second}$ . Suction and delivery pipes will have a combined length of  $36\text{ metres}$ , and a friction factor of  $0.006$ ,

both will be 150 millimetre in diameter. Losses at valves, etc. estimated to be 2.5 times the velocity head in the pipes.

The basic design of the pump has a specific speed of 0.074 revolutions, forward curved impeller blades with an outlet angle of 125 degree to the tangent are used and the width of the impeller passages at the outlet equals to the 1/10 of the diameter. The blades themselves occupy 5 percent of the circumference. If the hydraulic efficiency neglecting whirl slip of 70 percent is there. So we have to determine a suitable impeller diameter.

So before moving on to the problem, let me discuss about the velocity diagrams. Since forward type blades are used, so these are the impeller blades. And this is the direction of rotation of the blades and this is the inlet velocity diagram. So this is the relative velocity at inlet, this is the peripheral velocity of the polar blade that the inlet, this is the absolute velocity at the inlet. This is the angle made by the impeller blades at the inlet to the tangent at the bottom of the impeller blade. So this is the inlet velocity diagram.

This on the other hand is the outlet velocity diagram. Here as you can see this, sorry, this is the relative velocity at the outlet, this is the absolute velocity at the outlet, this is the peripheral, this portion is the peripheral velocity at the outlet, peripheral velocity of the impeller blades at the outlet. This is the velocity of flow at the outlet and this whole is the tangential component of velocity of flow at the outlet. If you notice carefully, we have taken the flow at the inlet to be radial in nature.

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C.CET  
I.I.T. KGP

A single-stage centrifugal pump is to be used to pump water through a vertical distance of 30 m at the rate of 0.045 m<sup>3</sup>/s. Suction and delivery pipes will have a combined length of 36 m and a friction factor of 0.006. Both will be 150 mm diameter. Losses at valves, etc. are estimated total 2.4 times the velocity head in the pipes. The basic design of pump has a specific speed of 0.074 rev, forward-curved impeller blades with an outlet angle of 125° to the tangent and a width of impeller passages at outlet equal to one-tenth of the diameter. The blades themselves occupy 5% of the circumference. If a hydraulic efficiency (neglecting whirl slip) of 75% may be expected, determine a suitable impeller diameter.

$H_d = 30 \text{ m}, Q = 0.045 \text{ m}^3/\text{s}, l = 36 \text{ m}, f = 0.006, d = 0.15 \text{ m}$

$h_{wv} = 2.4 \times \left(\frac{v^2}{2g}\right); v = \text{velocity of flow of water through pipes.}$

$= \frac{Q}{\left(\frac{\pi d^2}{4}\right)} = \frac{4Q}{\pi d^2}$

$K_{sp} = 0.074 \text{ rev}$

$\beta_2 = 125^\circ, \eta_H = 0.75, \omega = \frac{D}{10}$

$A = \text{Total area available for flow at impeller outlet}$

$= (0.95 \times \pi D) \times \omega = \left(\frac{0.95 \times \pi D^2}{10}\right)$

$D = ?$

And since this is generally the case, so I had been that we are considering the tangential component to be 0. So next we write down the parameters that are given. So here the, 1<sup>st</sup> the head or the height to which the vertical distance to which the water is pumped up is given as 30 metres which is the head developed. Next the flow rate through the pipe is given as 0.045 metre cube per second. The combined length, the total length of the suction and delivery pipes is given as 36 metres, with a friction factor of 0.006.

Both the diameters equal to 150 millimetre, that is 0.15 metre. Losses involved is given as, that is if you consider this as head loss in the valves, HV, it is 2.4 times the velocity head, that is  $V^2$  by  $2g$ , velocity head in the pipes, where  $V$  is the velocity of flow of water through the pipe. Which is actually given by  $Q$  by  $\pi D^2$  by 4. That is  $4Q$  by  $\pi D^2$  square. Next the specific speed of the pump is given as 0.074 revolutions and it is given that the angle beta-2, that is this angle, this angle being the angle made by the impeller blades at the outlet to the tangent at the tip, that is this, this angle is given as 125 degree.

The hydraulic efficiency is given as 0.75 and it is said that the blades occupy 5 percent of the circumference, that is from this statement we can infer that the total area available for flow at impeller outlet is 0.95 times the circumference, that is  $\pi D$  into  $D$ ,  $D$  which is the impeller diameter which we have to find out into the width. The width was specified here, the width was specified here as the 1/10 of the impeller diameter. So this is the width. So if we substitute the value of  $W$  we get 0.95 into  $\pi$  into  $D$  square by 10.

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Handwritten mathematical derivation on a blue background:

$$V_{w2} = \frac{Q}{A} = \frac{0.045}{\left(\frac{0.95 \times \pi D^2}{10}\right)} = \frac{0.15}{D^2} \text{ m/sec.}$$

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$$u_2 = \pi D N.$$

$$V_{w2} = ? \quad \eta_H = \frac{gH}{V_{w2} V_2} \quad ; \quad V_{w1} = 0, \quad V_{w1} u_1 = 0$$

$N \Rightarrow \text{rev/sec}$

$$= \frac{gH}{V_{w2} (\pi D N)} \Rightarrow V_{w2} = \frac{gH}{\eta_H \times \pi D N} = 4.1635 \times \left(\frac{H}{D N}\right)$$

$$V_2 = \sqrt{V_{w2}^2 + V_{f2}^2}$$

$$H = H_d + h_f + h_v = 30 + \left(\frac{4fL}{d}\right) \times \frac{V^2}{2g} + 2.4 \times \frac{V^2}{2g}$$

$$= 30 + \left[\frac{4fL}{d} + 2.4\right] \times \frac{1}{2g} \times \left(\frac{4Q}{\pi d^2}\right)^2$$

$$= 32.7 \text{ m}$$

So this is the total area available for flow. So the easiest way or the simplest way to solve this problem is to find all these velocity components for the outlet, for the outlet velocity from the outlet velocity diagram and express them in terms of the impeller diameter. After we do so, we can relate by using some trigonometric relations the various velocity components and from them, from that we can find out the impeller diameter. So let us find out the various velocity components.

So 1<sup>st</sup> let us find out the velocity of flow at the outlet which is given by the flow rate and the flow rate divided by the area available at the impeller outlet. That is  $Q$  by  $A$ . Which is,  $Q$  is given as 0.045 metre cube per second, that is 0.045 divided by the total area which was found as  $0.95 \text{ into } \pi D^2 \text{ by } 10$ . So if we calculate this, we ultimately get this as. So next we find the peripheral velocity of the impeller blades at the outlet which is  $U_2$  and it is given by  $\pi D N$  where  $N$ ,  $N$  is the rotational speed in rpm, revolutions per second.

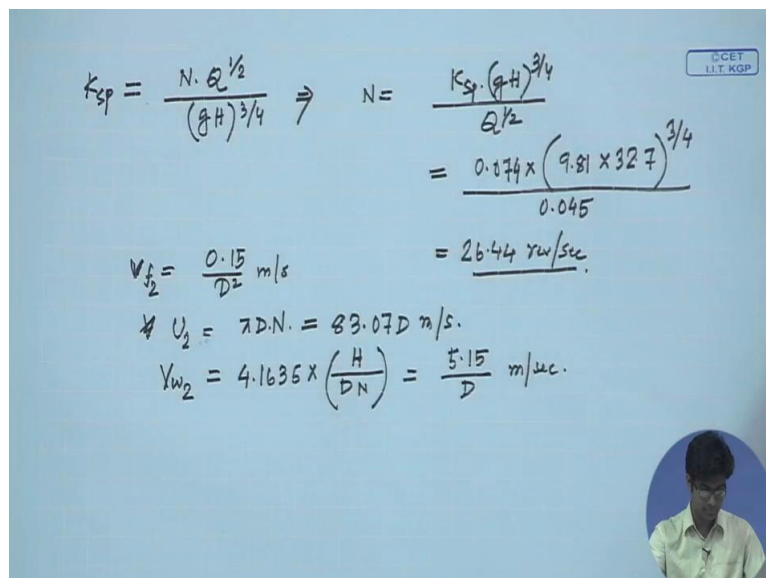
So here we do not know  $N$  right now, so we will leave it like this. Next we have to find out the tangential velocity at outlet. This can be obtained from the definition of hydraulic efficiency which is given as, this is because tangential velocity at the inlet is 0. So the  $V_1$ ,  $W_1$ ,  $U_1$  part of this becomes 0. So ultimately the hydraulic efficiency is  $\frac{GH}{VW_2 U_2}$ . Here we have not found out the head available, so we will keep it like this and we will substitute the value of  $U_2$  as  $\pi DN$ . So ultimately we get by separating all the values, so we can write it like this.

We know the value of hydraulic efficiency as 0.75, so substituting the values we can write 4.1635 into  $H$  by  $DN$ . So this is the value of  $VW_2$  and the absolute velocity at outlet is given by root over of, from this triangle, from this triangle we can say  $V_2$  is equal to  $VW_2^2 + V F_2^2$  square, okay. So right now we have to find out the head available as well as the rotational velocity of the impeller blade at the outlet, impeller blade. So 1<sup>st</sup> let us start with finding out the head available.

So the total head available is equal to or the total head developed is equal to the head, the head that is used to rise the water, that is the height with which the water is raised by using the pump which is  $H_D$  + the head lost due to friction in the pipes + the head lost in the valves. Okay. So this is given in the problem as 30 metres. This we find out by using the Darcy Weisbach formula, that is  $\frac{4fL}{D} \text{ into } \frac{V^2}{2g}$ . This was given in the problem as 2.4 times the velocity head.

So we can rearrange this and write the velocity for flow in the pipes can written as square. So we know all the values here, we know the value of friction factor, the length of the, the total length of the pipes, we know the diameter, and we also know the flow rate through the pipes. So substitute the values, ultimately we get the value of the total head as 32.7 metres. Next we have to find out the rotational speed of the impeller.

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Handwritten equations on a blue background:

$$K_{sp} = \frac{N \cdot Q^{1/2}}{(gH)^{3/4}} \Rightarrow N = \frac{K_{sp} \cdot (gH)^{3/4}}{Q^{1/2}}$$

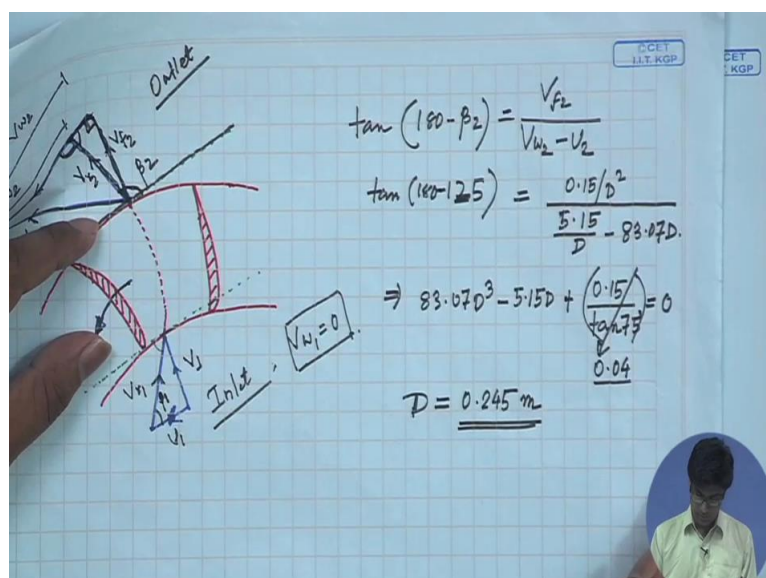
$$= \frac{0.074 \times (9.81 \times 32.7)^{3/4}}{0.045}$$

$$= 26.44 \text{ rev/sec.}$$

$$V_{f2} = \frac{0.15}{D^2} \text{ m/s}$$

$$U_2 = \pi D N = 83.07 D \text{ m/s.}$$

$$V_{w2} = 4.1635 \times \left(\frac{H}{DN}\right) = \frac{5.15}{D} \text{ m/sec.}$$



Velocity triangle diagram and equations on a blue background:

Diagram labels: Inlet, Outlet,  $V_{f1}$ ,  $V_{f2}$ ,  $V_{w1}$ ,  $V_{w2}$ ,  $\beta_1$ ,  $\beta_2$ ,  $V_1$ ,  $V_2$ ,  $V_{w1} = 0$ .

$$\tan(180 - \beta_2) = \frac{V_{f2}}{V_{w2} - U_2}$$

$$\tan(180 - 125) = \frac{0.15/D^2}{\frac{5.15}{D} - 83.07D}$$

$$\Rightarrow 83.07D^3 - 5.15D + \frac{0.15}{\tan 55} = 0$$

$$D = \underline{\underline{0.245 \text{ m}}}$$

For that we use the definition of specific speed for a pump which is given as N into 2 to the power half by GH whole to the power 3 by 4. From this we can write N is equal to K of SP into GH whole to the power 3 by 4 divided by Q to the power half. So KSP is given in the problem as 0.074 into 9.81, the head we just found out as 32.7 and Q is given in the problem

as 0.045 metre cube per second. So from this we get the value of N as 26.44 revolutions per second.

So let us go back to our definition of velocity. We already found out  $V_{f2}$  as  $0.15 \text{ by } D^2$  metre per second, sorry we had  $U_2$  as  $\pi D N$ , so substituting this value of  $D$ , sorry  $N$  here, we get the value of  $U_2$  that is the peripheral velocity of the impeller blade at the outlet as  $83.07 D$  metre per second. The tangential velocity at the outlet is obtained from this relation, that is  $4.1635 \text{ into } H \text{ by } DN$ . So substituting the value of  $H$  and  $N$  here, we get the value of  $V_{w2}$  in terms of  $D$  as  $5.15 \text{ by } D$  metre per second.

Now let us refer to the velocity diagram. As you can see this is a right angled triangle, the right angle being this one. So this angle is  $180^\circ - \beta_2$ , so we can write  $\tan$  of  $180^\circ - \beta_2$  is equal to this by this. This is  $V_{w2} - U_2$ . So all these quantities have been found out in terms of  $D$ , the right, the left side is  $\tan$  of  $180 - \beta_2$  and this, just substituting the expressions we can write as  $0.15 \text{ by } D^2$ ,  $V_{w2}$  we have found out as  $5.15 \text{ by } D$  and this is  $83.07 D$ .

So we can rearrange equation, equation and we can write it as  $83.07 D^3 - 5.15 D + 0.15 \text{ by } \tan$  of  $75$ . So this part comes out to be 0.04 if you calculate. Ultimately solving this equation for the impeller diameter, the only feasible value for impeller diameter comes out to be 0.245, the other values are not feasible. So this is a required value of the impeller diameter.

(Refer Slide Time: 18:05)

A centrifugal pump 1.3 m in diameter delivers  $3.5 \text{ m}^3/\text{min}$  of water at a tip speed of  $10 \text{ m/s}$  and a flow velocity of  $1.6 \text{ m/s}$ . The outlet blade angle is  $30^\circ$  to the tangent at the impeller periphery. Assuming zero whirl at inlet, and zero slip, calculate the torque delivered by the impeller.

Given  $D = 1.3 \text{ m}$ ,  $Q = 3.5 \text{ m}^3/\text{min} = \frac{3.5 \text{ m}^3}{60} = 0.058 \text{ m}^3/\text{sec}$ .

$U_2 = 10 \text{ m/sec}$ ,  $V_{f2} = 1.6 \text{ m/sec}$ ,  $\beta_2 = 30^\circ$ ; Torque = ?

Torque =  $\frac{\text{Power supplied}}{\text{Angular velocity of impeller blade (N)}}$

$U_2 = \frac{\pi DN}{60}$ ;  $N = \text{rpm}$

$= \frac{2\pi r \cdot N}{60}$ ;  $r = \text{Impeller blade radius}$

Now let us move on to the next problem. A centrifugal pump of 1. of impeller diameter 1.3 metres delivers 3.5 metre cube per minute of water at tip, with a tip speed of 10 metre per second and a velocity, flow velocity of 1.6 metre per second. The outlet blade angle is 30 degree to the tangent at the impeller periphery and assuming 0 whirl and 0 slip, we have to calculate the torque delivered by the impeller.

So let us write down the quantities given. The impeller diameter is given as 1.3 metre, the diameter, the flow rate is given as 3.5 metre cube per minute which is also metre cube per second by 60 which comes out as 0.058 metre cube per second. Next the tip speed of the impeller or the peripheral speed at the outlet is given as 10 metre per second, the flow velocity at outlet is given as 1.6 metre per second and the outlet blade angle is given as 30 degree. So we have to find out the torque.

So before moving on, let us draw the velocity diagram for this case. This is the outlet velocity diagram. So this is, this is the relative velocity at outlet, this is the flow velocity at the outlet, this is the absolute velocity at the outlet and this is the tangential velocity at the outlet, this whole is the peripheral velocity at outlet. This angle is the angle made by the impeller blade at the outlet to the tangent at the blade tip  $\beta_2$ , so we have to find out the torque. Now the expression for torque can be written as the power supplied divided by the angular velocity of the impeller blades, that is  $N$ .

Now let us 1<sup>st</sup> find out the denominator. This can be found out by using the expression for peripheral velocity of the impeller blade at the outlet which is  $\pi DN$  by 60, we have to keep in mind that  $N$  here is in rpm. So we can write or this can be written as  $2\pi R$  into  $N$  by 60 where  $R$  is the impeller radius, impeller blade radius. So  $N$  can be written as  $U_2$  by  $R$  into 60 by  $2\pi$ . So this being in rpm, if we convert this into radians per second, we can write it as  $U_2$  by  $R$ , simply  $U_2$  by  $R$ , the other part vanishes if we convert it to radians per second.

(Refer Slide Time: 22:44)

A centrifugal pump 1.3 m in diameter delivers 3.5 m<sup>3</sup>/min of water at a tip speed of 10 m/s and a flow velocity of 1.6 m/s. The outlet blade angle is 30° to the tangent at the impeller periphery. Assuming zero whirl at inlet, and zero slip, calculate the torque delivered by the impeller.

Given  $D = 1.3 \text{ m}$ ,  $Q = 3.5 \text{ m}^3/\text{min} = \frac{3.5 \text{ m}^3/\text{s}}{60} = 0.058 \text{ m}^3/\text{sec}$ .

$U_2 = 10 \text{ m/sec}$ ,  $V_{f2} = 1.6 \text{ m/sec}$ ,  $\beta_2 = 30^\circ$ ; Torque = ?

$N = \left( \frac{10}{0.65} \right) = 15.3846 \text{ rad/sec}$

Torque =  $\frac{\text{Power supplied}}{\text{Angular velocity of impeller blade (N)}}$

$U_2 = \frac{\pi D N}{60}$ ;  $N = \text{rpm}$

$\therefore N = \left( \frac{U_2}{\frac{\pi D}{60}} \right) \text{ rpm} = \frac{2\pi r \cdot N}{60}$ ;  $r = \text{Impeller blade radius}$

$N = \left( \frac{U_2}{r} \right) \text{ rad/sec}$

So this is the value of N. So if we substitute the values, we know the value of U2 and the value of R is 1.3 by 2, that is 0.65. So if we calculate this, it comes out as 15.3846 radians per second.

(Refer Slide Time: 23:22)

A centrifugal pump 1.3 m in diameter delivers 3.5 m<sup>3</sup>/min of water at a tip speed of 10 m/s and a flow velocity of 1.6 m/s. The outlet blade angle is 30° to the tangent at the impeller periphery. Assuming zero whirl at inlet, and zero slip, calculate the torque delivered by the impeller.

Given  $D = 1.3 \text{ m}$ ,  $Q = 3.5 \text{ m}^3/\text{min} = \frac{3.5 \text{ m}^3/\text{s}}{60} = 0.058 \text{ m}^3/\text{sec}$ .

$U_2 = 10 \text{ m/sec}$ ,  $V_{f2} = 1.6 \text{ m/sec}$ ,  $\beta_2 = 30^\circ$ ; Torque = ? ,  $V_{w1} = 0$

$N = \left( \frac{10}{0.65} \right) = 15.3846 \text{ rad/sec}$

Torque =  $\frac{\text{Power supplied}}{\text{Angular velocity of impeller blade (N)}}$

$U_2 = \frac{\pi D N}{60}$ ;  $N = \text{rpm}$

$= \frac{2\pi r \cdot N}{60}$ ;  $r = \text{Impeller blade radius}$



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Power supplied,  $P = \rho Q g H_w$

$H_w = \text{Work input per unit weight of water.}$   
 $= \frac{V_{w2} U_2}{g}$

$V_{w2} = U_2 - V_{f2} \cot \beta_2 = 10 - 1.6 \cot 30^\circ$   
 $= 7.23 \text{ m/sec}$

$H_w = \frac{V_{w2} U_2}{g} = \frac{7.23 \times 10}{9.81} = 7.37 \text{ m}$

$P = 10^3 \times 0.058 \times 9.81 \times 7.37 = 4193.38 \text{ W.}$

$\text{Torque} = \frac{P}{N} = \frac{4193.38}{15.3846} = 274.14 \text{ N-m}$

Next we have to find out the power supplied. This can be written as  $\rho$  into  $Q$  into  $G$  into the head that is supplied as in terms of work which is  $H_w$ , that is we know here the value of  $Q$  and the density of water and  $G$  are already known. So the head supplied in terms of work or the work, that is the work input per unit weight of water used is given by, since we have to keep in mind that here also we have in the problem, it is given that  $0$  whirl at inlet, so here the tangential velocity at inlet is  $0$ .

So keeping this in mind, the work done at the inlet, work input is, what input is actually  $V_{w2} U_2$  by  $G$ . So this work is supplied or actually done to raise the water. So from here we know the value of  $U_2$  and  $V_{w2}$  we have to find out. So if we refer to the velocity diagram,  $V_{w2}$  can be found out as  $V_{w2}$  is equal to  $U_2$ , this whole portion subtract, subtracting this section from this whole section we get the value of  $V_{w2}$ . So we can write this as  $U_2 - \text{of } V_{f2} \cot$  of  $\beta_2$ .

That is this is the  $\beta_2$  angle, so  $V_{f2} \cot \beta_2$  is nothing but this part. So we know all the values here, we substitute the values and  $U_2$  is given as  $10 - V_{f2}$  is given as  $\cot$  of  $\beta_2$  which is  $30$  degree. So we get the value of  $V_{w2}$  as  $7.23$  metre per second. So the value of head input, head used as work input is  $7.23$  into  $U_2$  which is  $10$  by  $9.81$ . This comes out as  $7.37$  metres. So the power supplied is  $1000$  into the discharge or the flow rate into  $9.81$  into  $7.37$ .

So this comes out as  $4193.38$  watts. So the torque is given as, the torque is given as power supplied divided by the angular velocity which is  $4193.38$  and  $N$  was found out as  $15.38$ , this

can be written as 15.3846. So this comes out as 274 .14 newton metre. So with this I end the lecture of today.