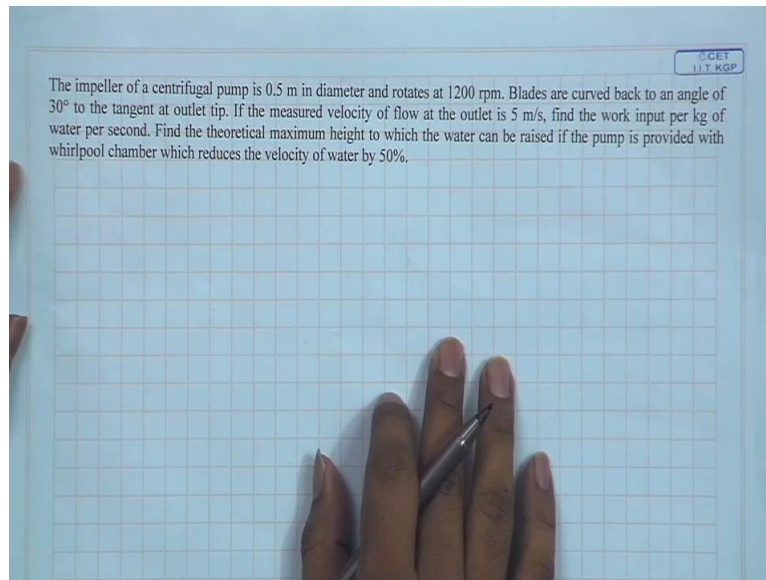


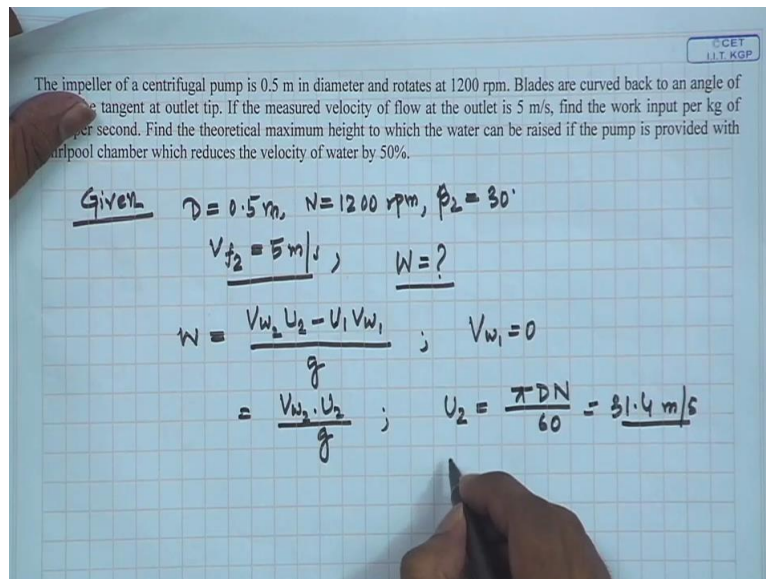
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**Tutorial-6.**

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Welcome to this tutorial class on fluid machines. Today I will be solving 2 problems on centrifugal pumps. The 1<sup>st</sup> will be a basic problem on the concept of centrifugal pump and the 2<sup>nd</sup> problem will be based on cavitation. So let us start with the 1<sup>st</sup> problem. Let me read it out. The impeller of a centrifugal pump is 0.5 metre in diameter and it rotates at 1200 rpm. The blades are curved back to an angle of 30 degree to the tangent at the outlet of tip. If the measured velocity at the outlet is 5 metre per second, then we have to find out the work input per KG of water per second.

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We have to also find out the theoretical maximum height to which the water can be raised if the pump is provided with a whirlpool chamber which reduces at outlet by 50 percent. So before starting the problem, let me discuss about the velocity diagrams. So as you can see, these are the impeller blades and this is the inlet velocity diagram and for the, for a pump in general we assume and in practical also it is seen that the flow actually enters radially. So we will be assuming the flow to enter radially at the inlet, that is we will be considering  $VW_1$  or the tangential velocity at inlet to be 0.

So this is actually the peripheral velocity of impeller blade at the inlet, this is the radial velocity at inlet and this is the absolute velocity at inlet. This angle is the impeller blade angle at the inlet made with the tangent at the bottom of the impeller blade, that is, this is designated by  $\beta_1$ . Next moving onto the outlet velocity diagram, here you can see that this is the relative velocity at outlet, this is the peripheral velocity of the impeller blade at the outlet, this is the, absolute velocity at the outlet of the impeller.

This is the flow velocity, this angle which is made by the impeller blade at the outlet with the tangent on at the tip is the blade angle of the impeller at the outlet and is designated by  $\beta_2$ . And the tangential component of the velocity at the outlet, that is  $VW_2$  is given by this portion, that is  $VW_2$ . So moving back to the problem. So let us write down what the parameters that are given. So we are given the diameter of the impeller for the centrifugal pump as 0.5 metres.

Rotational velocity is given as 1200 rpm, angle beta-2 as we discussed is given as 30 degree. So this angle is given as 30 degree. Next we are given the velocity of flow at the outlet, that is  $V_{f2}$  is given as 5 metre per second. So we have to find out the work input per KG per second, then that is actually given by, that is we have to find out this work, okay, so the expression for work done, work input per KG of water per second is actually given by this expression.  $V_{w2} U_2 - U_1 V_{w1}$  by G.

We know that  $V_{w1}$  is 0, so what we are left is  $V_{w2} U_2$  by G. Now  $U_2$  is obtained, which is available velocity of the impeller blade at the outlet from this expression. We know both the values of N and D, so separating them we get the value of  $U_2$  and 31.4 metre per second.  $V_{w2}$  on the other hand can be obtained from the velocity diagram.

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The impeller of a centrifugal pump is 0.5 m in diameter and rotates at 1200 rpm. The outlet is at an angle of  $30^\circ$  to the tangent at outlet tip. If the measured velocity of flow at the outlet is 5 m/s, find the work input per kg of water per second. Find the theoretical maximum height to which the water can be raised if the pump is provided with whirlpool chamber which reduces the velocity of water by 50%.

Given  $D = 0.5 \text{ m}$ ,  $N = 1200 \text{ rpm}$ ,  $\beta_2 = 30^\circ$   
 $V_{f2} = 5 \text{ m/s}$ ,  $W = ?$

$$W = \frac{V_{w2} U_2 - U_1 V_{w1}}{g}; \quad V_{w1} = 0$$

$$= \frac{V_{w2} U_2}{g}; \quad U_2 = \frac{\pi D N}{60} = 31.4 \text{ m/s}$$

$$= 72.78 \text{ m}$$

ii)  ~~$V_{f2}$~~  Loss in absolute velocity at the outlet  
 $= 0.5 \times V_{f2}$

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$$V_{w2} = U_2 - V_{f2} \cot \beta_2$$

$$= 22.74 \text{ m/s}$$

$$V_{w1} = 0 \quad V_2^2 = V_{w2}^2 + V_{f2}^2$$

$$V_2 = 23.38 \text{ m/s}$$

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Loss in velocity head at outlet =  $\frac{(0.5 \times V_2)^2}{2g}$

Ideally Total Head Developed by pump

= Total work input per unit weight of water

$$H = 72.78 \text{ m}$$

Maximum Height to which the water can be raised =  $H - \frac{(0.5 \times V_2)^2}{2g}$

$$= 65.87 \text{ m}$$

That is this portion as you can see from this diagram is given as  $U_2$  - of this portion which is  $V_{f2} \cot$  of  $\beta_2$ . So substituting the values, we know the value of  $U_2$ ,  $V_{w2}$ ,  $V_{f2}$  to, that is the flow velocity at the outlet is given as 5 metre per second and  $\beta_2$  is 30 degree. So if we substitute the values, we get the value of  $V_{w2}$  as 22.74. So substituting the value of  $V_{w2}$  and  $U_2$  here, we ultimately get the work done per KG of water or weight of water, per unit weight of water as 72.78 metres.

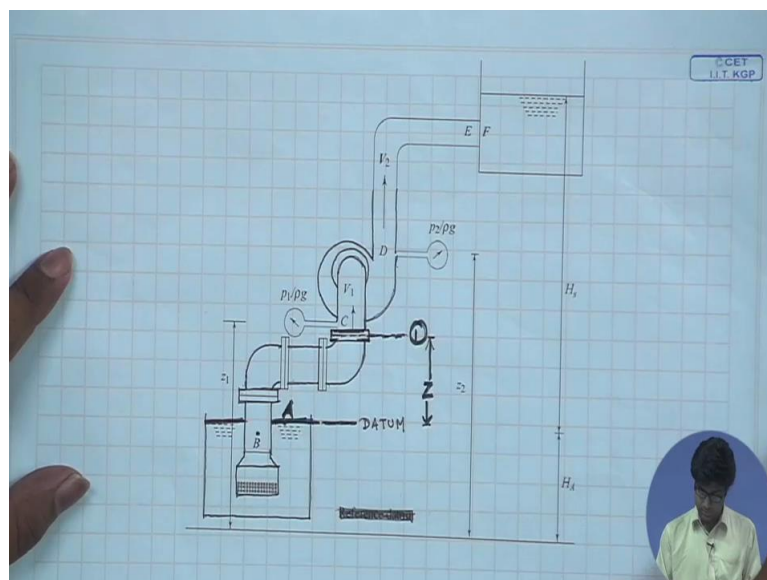
Next we are required to find out the maximum height to which the water can be raised if the pump is provided with a whirlpool chamber which reduces the outlet absolute velocity by 50 percent. That is for a 2<sup>nd</sup> part, we have said that the outlet velocity is reduced by 50 percent, so actually loss in absolute velocity at the outlet is 0.5 into  $V_2$ , so the corresponding loss in velocity head at outlet is 0.5 into  $V_2$  whole square by 2G.

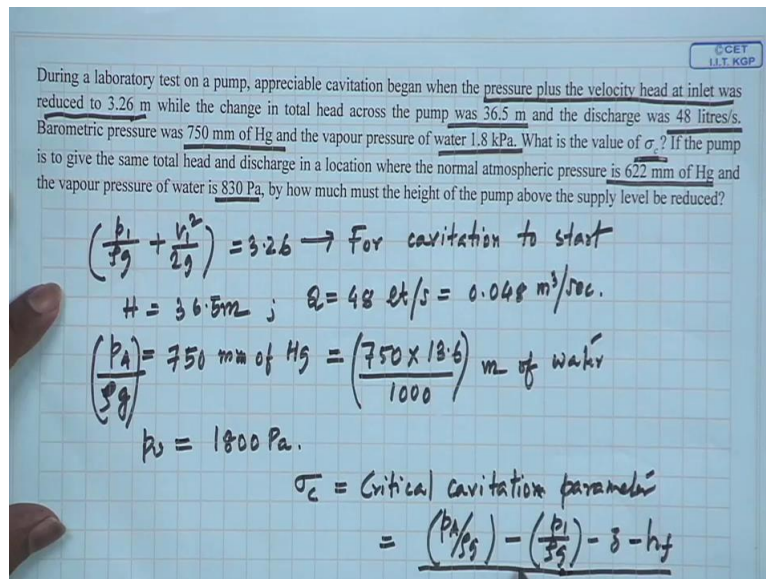
Now we can write that the total head that is developed by the pump is equal to the total work input, that is in the ideal case, in the ideal case we can write that total head developed by the pump is equal to the total work input given or total work input per unit weight of water. This quantity we just found out which was 72.78 metres, so the total head in the ideal case we can say is actually 72.78 metres. But there was lost in velocity head which amounts to  $0.5 V_2^2$  whole square by  $2G$ .

So the net head available or the maximum or the net head which or the height to which the water was, water can be raised is, that is the maximum height to which water can be raised is actually  $H - 2 \text{ into } G$ . So that in the net head developed is actually the maximum height to which the water can be raised. Now the unknown head is  $V_2$ , that is the absolute velocity at the outlet, so we again refer to the velocity diagram at the outlet and we can see from the velocity triangle, from this triangle that  $V_2$  is equal to or rather  $V_2^2$  is equal to  $VW^2 + VF^2$  square.

We know the value of  $VW^2$  and  $VF^2$  is given in the problem as 5 metres, so substituting those values were ultimately get the value of  $V^2$  as 23.38 metre per second. So we substitute the value of  $V_2$  in this expression, we know the value of  $H$ , so the height that we are required to find out comes out as 65.87 metres. So this is the end of problem 1.

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Now let us move onto the next problem regarding cavitation. Let me read out the problem. During a laboratory test on a pump, appreciable cavitation began when the pressure + the velocity head at the inlet was, pressure + the velocity head at the inlet was reduced to 3.26 metres while the total change in head across the pump was 36.5 metres and the discharge was 48 litres per second. The barometric pressure was 750 millimetre of mercury, that is the atmospheric pressure, 750 millimetre of mercury and the vapour pressure of water was a 1.8 kilopascals.

So we have to find out the value of Sigma C which is actually the critical cavitation factor or critical cavitation parameter. Next we have to find, if the pump is to give, is to give the total same and total head and discharge in a location where the normal atmospheric pressure is 622 millimetre of HG and the vapour pressure of water is 830 pascals, by how much must the height of the pump of the supply level be reduced. So before starting the problem let me show you a schematic of the pump.

So as you can see here, this is the water level in the reservoir, we denote it by A, this section. And we will consider this as the datum. This is the runner, sorry, impeller inlet, this section we denote as 1 and this elevation or the height between the impeller inlet and the reservoir water level is denoted by Z. So all the parameters, all the variables which denote that, which is at the inlet of the impeller are denoted by subscript 1 and the parameters of the variables at the reservoir water level are denoted by subscript A.

So let us write down the parameters that are given. First we are given that the pressure +, the pressure head at the impeller inlet + the velocity head at the impeller inlet, the sum of those 2

is equal to 3.26 for cavitation to begin. So for the cavitation to take place, this should be the total head or the sum of the pressure head or the velocity head at the impeller inlet for cavitation to start. Next we are given the net head change across the pump as 36.5 metres, the discharge is given as 48 litres per second which is 0.048 metre cube per second.

The atmospheric pressure is given as and we know that this, since this is exposed to the atmosphere, so the pressure head or the pressure at this section is actually the atmospheric pressure itself. Another point to note is that this, at the reservoir water level, at the reservoir, the water is almost stationary. So the velocity head at this portion is also 0. So the atmospheric pressure is given as 750 millimetre of mercury or actually this is the atmospheric pressure head which can be written as 750 into 13.6 divided by 1000 meter of water.

So this comes out as, okay let us leave it like this only. Next the vapour pressure at this, at this atmospheric, pressure and temperature is given as 1800 pascals. So we are required to find out the value of Sigma C which is the critical cavitation parameter and it is given as the pressure head at the reservoir water, reservoir water surface - it is the atmospheric pressure, - the pressure head at the impeller inlet - the elevation that is the elevation between the impeller inlet and the reservoir - the head loss due to friction in the pipe connecting the reservoir and the impeller inlet.

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$p_1 = p_v = 1800 \text{ Pa.}$   
 $\frac{p_1}{\rho g} = \frac{1800}{9.81 \times 1000} = 0.183 \text{ m}$   
 $\frac{v_1^2}{2g} = 3.26 - 0.183 = 3.08 \text{ m}$   
 Bernoulli's equation:  
 $\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + 0 = \left(\frac{p_1}{\rho g}\right) + \frac{v_1^2}{2g} + z + h_f$   
 $\Rightarrow \frac{p_A}{\rho g} - \frac{p_1}{\rho g} - z - h_f = \frac{v_1^2}{2g} \rightarrow \sigma_c = \frac{v_1^2}{2g} / H = \frac{3.08}{36.5}$   
 $z + h_f = \left(\frac{p_A}{\rho g}\right) - \left(\frac{p_1}{\rho g} + \frac{v_1^2}{2g}\right) \rightarrow 3.26 = 6.93 \text{ m}$

That is HF is the frictional head loss and divided by total head. So we are required to find out this quantity. In the 2<sup>nd</sup> part, we will be discussing that later. So 1<sup>st</sup> let us deal with the 1<sup>st</sup> part. So for cavitation to take place, the inlet pressure at the impeller is equal to the vapour

pressure, that is 1800 pascals. So the inlet pressure, so the inlet pressure head in the impeller is, which comes out as 0.183 metres. Next since we are given, the sum of the pressure and the velocity head at the inlet, so from this we can get the velocity head at the inlet by subtracting the pressure head which we just obtained.

So velocity head at the impeller inlet is obtained as - the pressure head at inlet, which is 3.08 metres. Now let us apply the Bernoulli's equation between this section and the impeller inlet. So, so if we apply the Bernoulli's equation, we get, Bernoulli's equation  $P_A$  by  $\rho G$ , that is the pressure head at the reservoir + the velocity head at the reservoir +, since this is the datum, so height elevation is 0, this is also 0 as we already talked about, that is the velocity at the reservoir level is 0.

Next is equal to the pressure head at the reservoir inlet, sorry pressure head at the impeller inlet, the velocity head at the impeller inlet, this will be  $2 \cdot V_1^2$  by  $2G$ ,  $V_1^2$  square by  $2G$  + the elevation, that is  $Z$  + the head loss. So from here we can write  $P_A$  by  $\rho G - P_1$  by  $\rho G - Z - H_f$  is equal to  $V_1^2$  square by  $2G$ . Also from here we can write  $Z + H_f$  is equal to  $P_A$  by  $\rho G - p_1$  by  $\rho G + p_1^2$  square by  $2G$ . So this we know as 3.26 which was given in the problem.

3.26 and this too is given in the problem,  $P_A$  by  $\rho G$ , this was given in the problem. So substituting the values here, we get this value of the potential head + the head loss at the impeller inlet as 6.93 metres. Now if you can recall the expression for the critical cavitation parameter was this. So we just derived that this expression is nothing but the velocity head at the impeller inlet. So from here we can write  $\sigma_c$  is  $V_1^2$  square by  $2G$  whole divided by  $H$ .

So we know the value of  $H$  and  $V_1^2$  square, that is the velocity head at the inlet to the impeller, we have already found out as 3.08 metres. So substituting the values, the head was given as 36.5 metres. So  $\sigma_c$  comes out to be 0.084.



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$p_2 = p_1 = \frac{1000}{9.81} \times 830 \text{ Pa}$  at the impeller inlet remains same  
 Bernoulli's eqn,  $\left(\frac{p_A}{\rho g}\right) = \left(\frac{p_1}{\rho g}\right) + \frac{V_1^2}{2g} + z_1 + h_f$  (Velocity Head =  $\frac{V_1^2}{2g}$  same)  
 $(z_1 + h_f) = \left(\frac{p_A}{\rho g}\right) - \left(\frac{p_1}{\rho g}\right) - \frac{V_1^2}{2g} \rightarrow 3.08$   
 $z_1 + h_f = 5.29 \text{ m}$   
 $\frac{830}{9.81 \times 1000}$   
 Height of the pump should be reduced above the supply level should be reduced by  $\rightarrow (z_1 + h_f) - (z_1 + h_f)$

Now let us move onto the next part. In the next part the atmospheric pressure was given as 622 millimetre of HG, that is atmospheric pressure head is, sorry, 622 into 13.6 divided by 1000 metre of water and the vapour pressure of water in this case was given as, which is also equal to the pressure at the inlet to the impeller, 830 pascals. So we have to find out by how much the height of the pump above the supply level should be reduced. So let us find out the elevation for this case. So we again apply the Bernoulli's equation, just like in the previous case.

So for the reservoir we have only  $P_A$  by  $\rho G$ , that is the pressure head equal to  $P_1$  by  $\rho G + V_1^2$  square by  $2G + Z$ , let us denote this  $Z$  by  $Z_1 + HF$ . Now it is given here that the, the head remain the same and also the discharge remains the same. So since the discharge remains the same as in the previous case, this signifies that the velocity is also the same or in other words the velocity head at the inlet, at the impeller inlet remains the same. That is  $V_1^2$  square by  $2G$ , this is because the discharge remains the same.

So also since the pump is the same as in the previous case, so the head loss due to friction in the pipe connecting the impeller inlet and the reservoir is also the same. So  $HF$  is also the same as in the previous case. So we have the head as well as the discharge and also the velocity head at the inlet of the impeller same. So from here we find out  $Z_1 + HF$  which is  $P_A$  by  $\rho G - P_1$  by  $\rho G +$ , when  $\rho$  is the density of water, sorry - of  $V_1^2$  square by  $2G$ .

This is the same as in the previous case. This is 830 divided by 9.81 into 1000 and this we have already found out which is 622 into 13.6 by 1000. So we substitute,  $V_1^2$  square by  $2G$ , in

the previous case we found out as 3.08 metre. So we substitute these values, that is 3.08 metres. Substituting these values we get, we get the value of  $Z_1 + HF$  as 5.29 metres.

So the net, so the height of the pump should be reduced, that is above the pump, height of the pump above the supply level or above the reservoir water level should be reduced by the height that is  $Z_1$ , sorry  $Z + HF -$  of  $Z_1 + HF$ .

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The image shows a whiteboard with handwritten mathematical work. The equations are as follows:

$$h = (Z - Z_1) = (6.93 - 5.29) \text{ m}$$
$$= \underline{\underline{1.64 \text{ m}}}$$

To the right of the equations, there is a note:  $\rightarrow (Z + h_f) - (Z_1 + h_f)$ . In the top right corner of the whiteboard, there is a small logo that reads "SCET IIT KGP". A person's hand holding a pen is visible at the bottom right of the whiteboard.

Now that is the height that  $H$  is equal to, required height is  $Z - Z_1$  since  $HF$  is the same, this cancels out. Now we are found out  $Z + HF$ , the initial  $Z + HF$  as 6.93, that is 6.93 - of  $Z_1 + HF$  we just found out as 5.29 metres. This is actually the difference of  $Z + HF -$  of  $Z_1 + HF$ . So this comes out as 1.64 metres. So this is the required height. With this I end this 2<sup>nd</sup> problem, so this ends today's lecture, thank you.