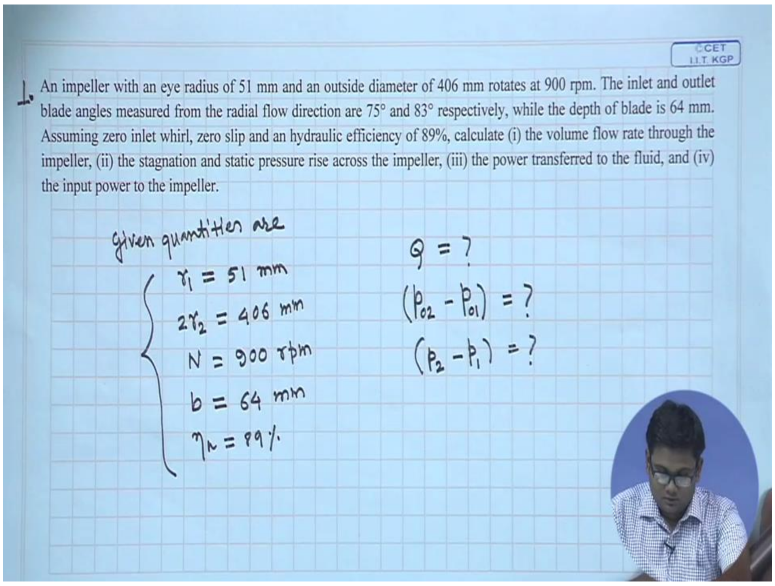
Fluid Machines.
Mr. Subhadeep Mandal.
Teaching Assistant.
Department Of Mechanical Engineering.
Indian Institute Of Technology Kharagpur.
Tutorial-5.

(Refer Slide Time: 0:44)

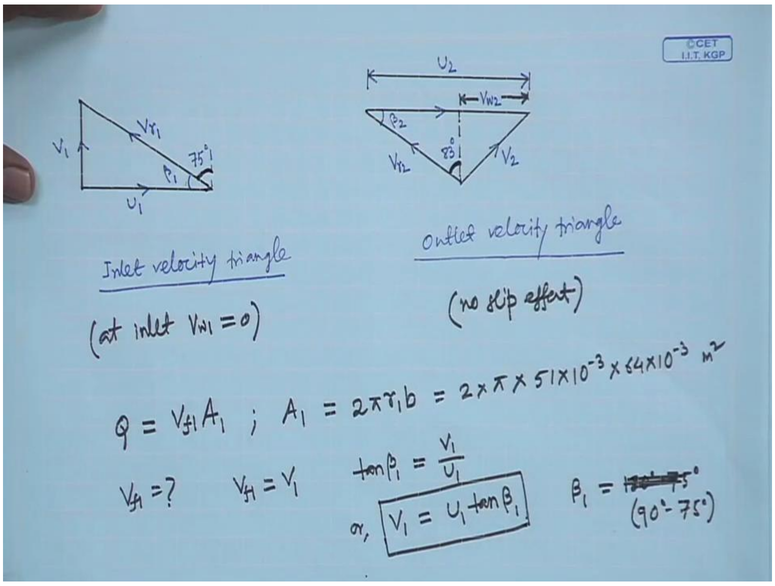


So welcome to this session of the course fluid machines. In today's tutorial class we are going to solve problems related to centrifugal pumps. So let us start with problem number 1. So the problem statement is given here. So the statement is as follows. And impeller with an eye radius of 51 millimetre and an outside diameter of 406 millimetre rotates at 900 rpm. The inlet and outlet blade angles measured from the radial flow direction are 75 degree and 83 degree respectively, while the depth of the blade is 64 millimetres.

Assuming 0 inlet whirl or swirl velocity, 0 slip and hydraulic efficiency of 89 percent calculate the volume flow rate through the impeller, the stagnation and static pressure rise across the impeller, the power transferred to the fluid and the input power to the impeller. So let us 1st note down there important given quantities in the problem. So given quantities are the eye radius with this 51 millimetre, so the eye radius is the radius of the pump at the inlet. Out outside diameter, so R2, 2 R2 is 406 millimetre where R2 is the radius at the outlet of the pump, pump impeller.

Rotational speed N is 900 rpm, width of the blade is 64 millimetre and hydraulic efficiency is given as 89 percent. So these are the given quantities. Now we have to determine the volume flow rate through the pump, stagnation and static pressure rise. So let us represent stagnation pressure at the outlet as P02 and stagnation pressure at the lead as P0 1. Then the rise across, rise of stagnation pressure above the impeller is P0 2- P01. So we have to determine this, similarly we have to determine the static pressure rise which is P2 - P1, power transferred to the fluid and also the input power to the impeller.

(Refer Slide Time: 3:32)

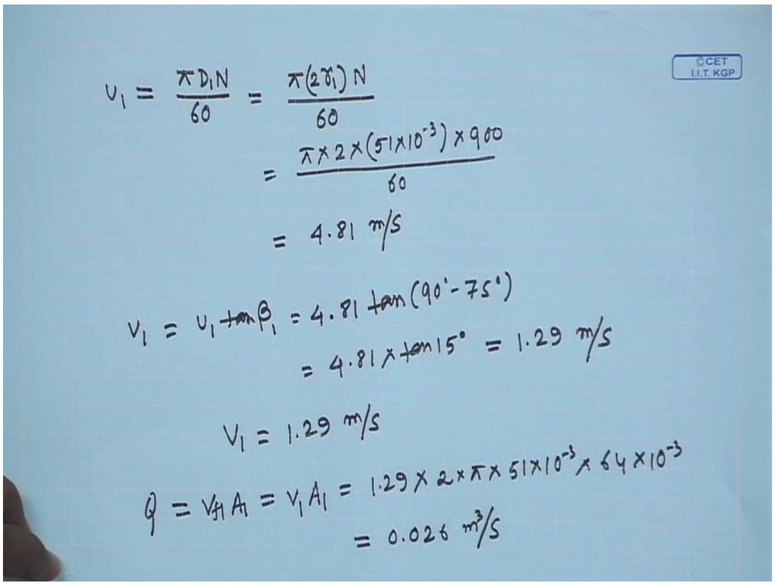


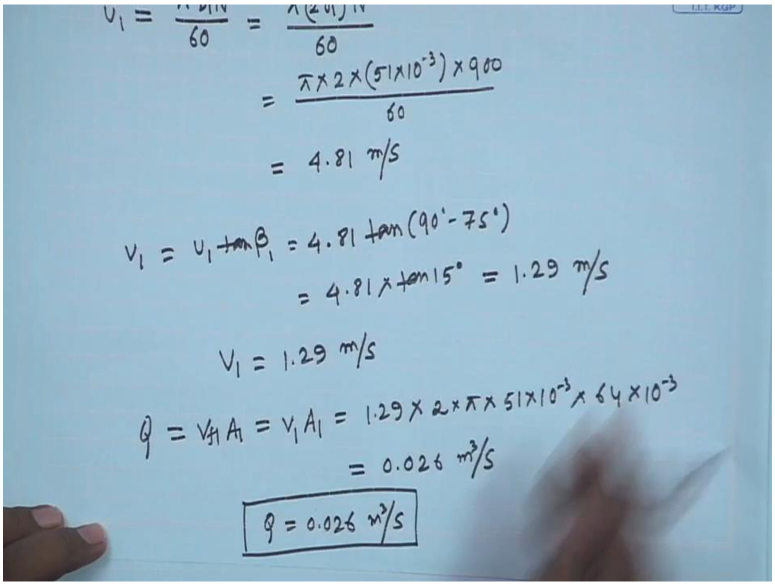
So let us 1st look into the velocity triangles. So this is the inlet velocity triangle and the blade angles are represented here, so the inlet and outlet blade angles are measured from the direction, radial direction as 75 degree and 83 degree respectively. So radial velocity makes an angle with the flow direction as 75 degree. So this is the blade angle here at the inlet. Similarly radial velocity at outlet makes an angle 83 degree, 83 degree with respect to the flow direction. So this is here 83 degree.

Now it is the inlet velocity triangle, we have used the fact that there is 0 inlet whirl or 0 inlet swirl velocity. So at inlet, V W1 is 0, that we have used to draw this diagram. Similarly at the outlet we have used the fact that 0 slip, so the radial velocity leaves at an angle with, which is specified by the blade angle. The presence of slip, this VR2 maybe deviated from this angle, so in the absence of slip we have drawn this diagram, so no slip effect. So with this consideration now we determine the flow rate through the pump.

So flow rate Q will be the flow velocity, say at the inlet times the cross-sectional area at the inlet. So here we are using subscript 1 to represent inlet quantities and subscript 2 to represent outlet quantity. Now A1 is the cross-sectional area at the inlet is to pie R1 times B, B is the width of the blade. So this is 2 times pie times R1 is given as 51 millimetre. So 51 times 10 to the power -3 and B is given as 64 millimetre. So this is the cross-sectional area in metre square.

(Refer Slide Time: 7:05)





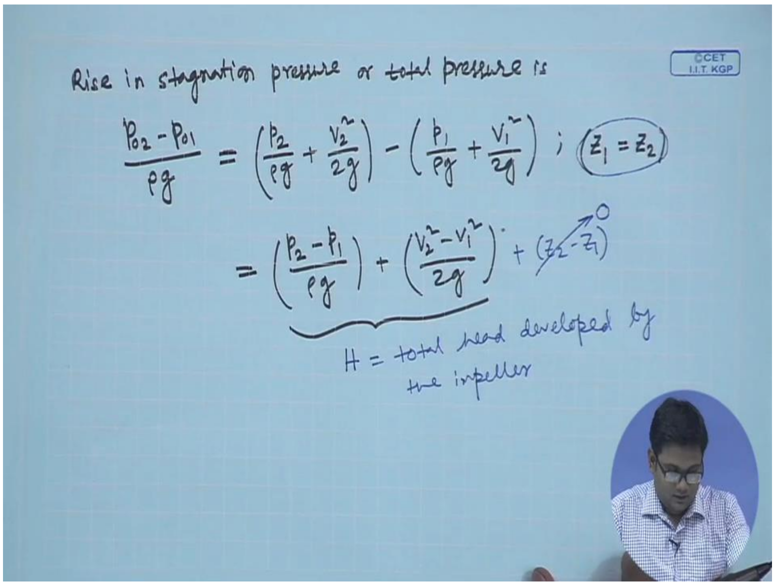
Now what is VF1? Now here as there is no swirl component of velocity, VF1 is same as V1. Now from the velocity triangle we can relate V1 to U1 in terms of beta 1. So tan final beta 1 will be V1 by U1. Or V1 is U1 times of beta, beta 1. Now we have to determine U1 and beta-1. Beta-1 is nothing but 180 degree - 75 degree or 90 degree - 75 degree. So this total angle is 90 degree, this much is 75, so beta is 90 - 75 degree.

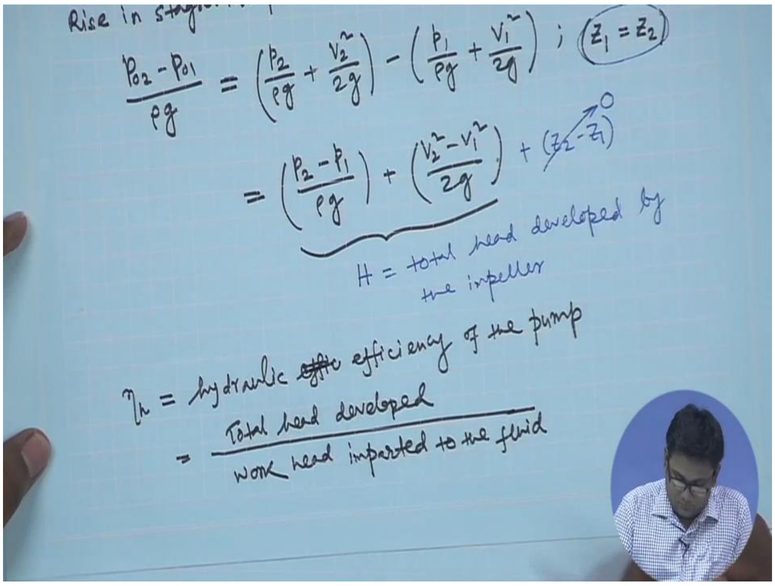
Now to determine U1, so U1 is peripheral velocity, so U1 is peripheral velocity of the impeller at inlet, that can be determined by this rotational speed of the pump. So pie, D 1 is the inlet diameter which is 2 R1, N is the rotational speed in rpm. So pie times 2 times R1 is given as 51 millimetre, so 51 times 10 to the power -3. N is given as 900 rpm divided by 60. So U1 is 4.81 metre per second. Now I will substitute U1 tan beta 1 in this relation to obtain V1.

So V1 is U1 tan beta-1 is equal to A 4.81 times tan of 90 degree -75 degree, so which is 4.81 times tan 15 degrees equal to 1.29 metre per second. So inlet flow velocity is 1.29 metre per second. So the flow rate, expression of which we have determined earlier as Q equals VF1 A1. Where VF1 is A1 because there is no swirl component of the flow. So Q which is VF1 A1 is same as V1 A1. So 1.29 times A1 is 2 times pie times 51 into 10 to the power -3 times 64 into 10 to the power -3.

So this you can obtain as 0.026 metre cube per second. So the 1^{st} result that we are interested in is the flow rate, now we have obtained it. The 2^{nd} quantity of interest in this problem is the rise in stagnation pressure across impeller, so let us find that.

(Refer Slide Time: 9:56)



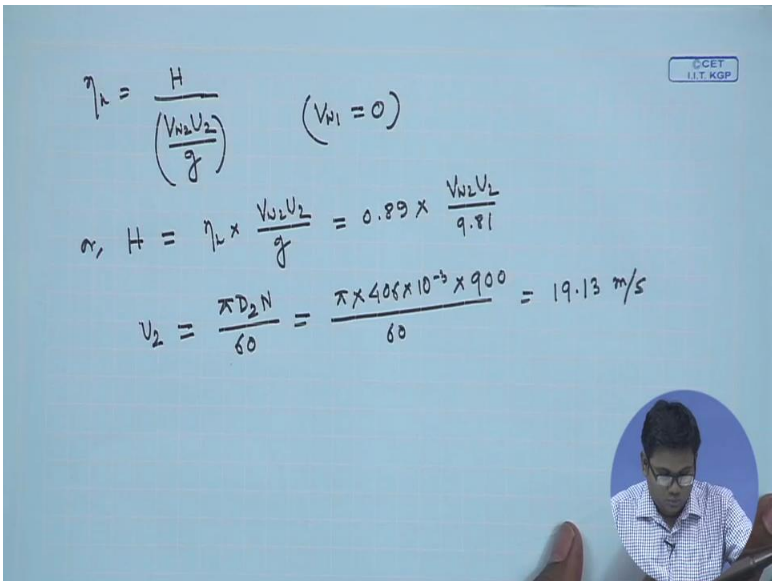


So rise in stagnation pressure or which is also called total pressure is P0 2 - P0 1, let us represent it in terms of head. So P0 2 - P0 1 over rho G is P2 by rho G + V2 square by 2G, this is the stagnation pressure head at the outlet. And P1 by rho G + V1 square by 2G, this is the stagnation pressure head at the inlet of the impeller. So here we are assuming one thing that the elevation at the inlet is same as the elevation at the outlet. So this is one consideration.

Now we can rearrange this expression in this way. P 2 - P1 by rho G + V2 square - V1 square by 2G. This is nothing but the H which is total head developed by the impeller, impeller. In a more general case, this will contain Z2 - Z1 but as we are assuming Z1 is equal to Z2, so this will be 0. So only pressure and velocity heads will be there in the total head expression. Now we have to find this total head. Regarding this we can use the hydraulic efficiency which is 89 percent.

So let us write the definition for hydraulic efficiency. Hydraulic efficiency of, efficiency of the pump. Hydraulic efficiency of the pump is given in this way, it is the ratio of total head developed by the pump over the work head imparted by the rotor to the fluid. So total head developed, developed by the work head imparted to the fluid.

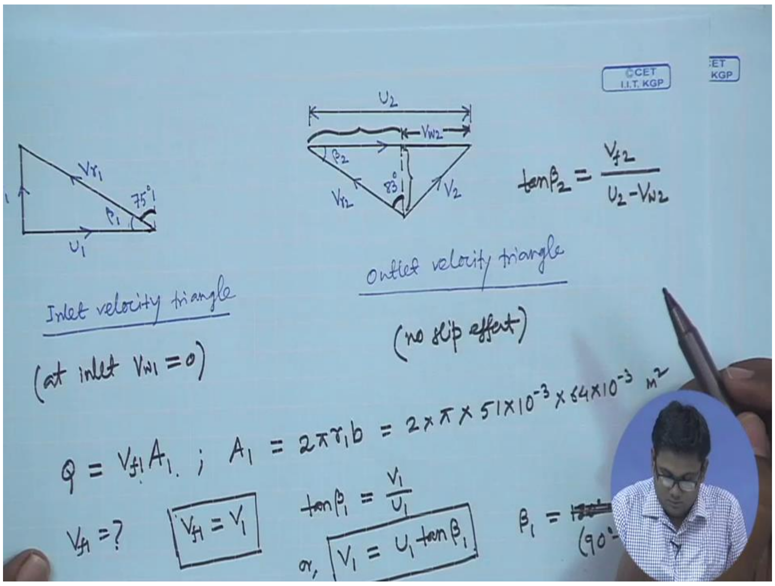
(Refer Slide Time: 13:03)

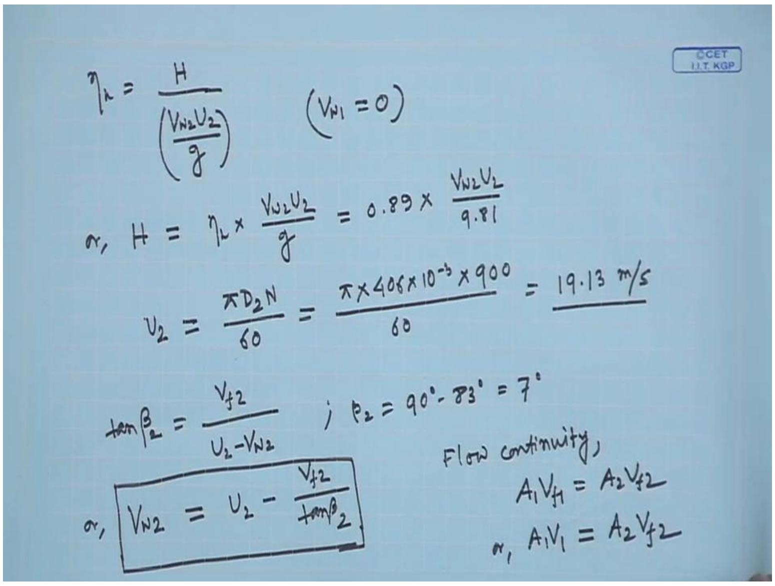


So here the total head, total head developed is H and the work head imparted to the fluid will be VW2 U2 by G. Here also we are using the fact that the swirl component at the inlet is 0. So from here we can obtain H as Eta hydraulic times VW2 U2 by G. So Eta hydraulic is given by 0.89 but VW2 U2 are not known to you now. So let us 1st find VW2 and U2. So determination of U2 is very straightforward. Pie D2 N by 60 where D2 is the diameter, outlet diameter of the impeller.

So outlet diameter of the impeller is given as 406 millimetre. So 406 times 10 to the power -3, N is 900 rpm over 60. This you can obtain as 19.13 metre per second. Now to determine VW2, let us look into the velocity triangle.

(Refer Slide Time: 14:25)

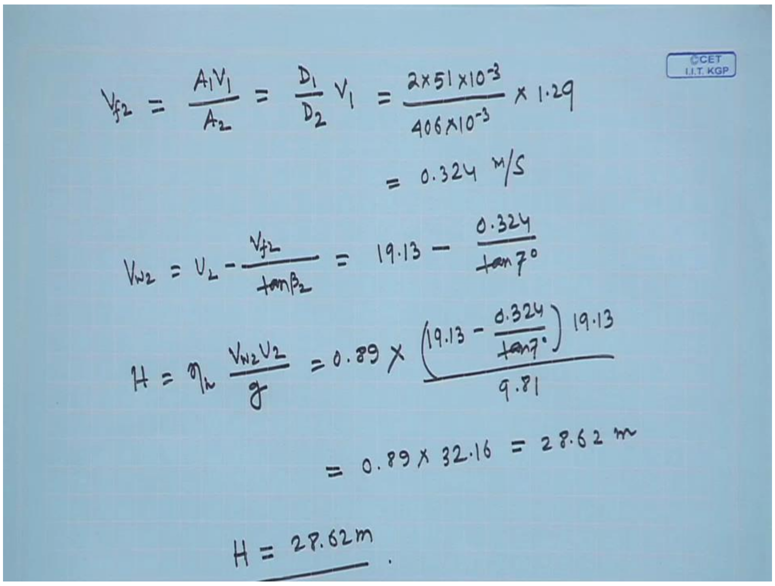




So VW2 can be obtained from this triangle, if you look into the tan of beta-2. So tan of beta-2 will be this height which is VF 2 or flow velocity at outlet over U2 - VW 2. So U2 is this much and VW2 is this, so this distance will be U2 - VW 2. So tan beta 2 is equal to VF 2 by U2 - VW 2. So tan beta 2 is equal to VF 2 by U2 - VW 2. Beta-2 is nothing but 90 degree - this angle, so beta-2 is 90 degree - this angle, so beta-2 is 90 degree - 83 degree equal to 7 degree.

Now and from this relation we can say VW2 equals to U2 - VF 2 by tan beta-2. So now we have obtained U2 as 19.13 and expressed VW2 in terms of U2 VF 2 and beta-2. Beta-2 is known, now how to relate VF 2? Now from the continuity of flow, so flow continuity, flow continuity gives that inlet flow will be same as the outlet flow at steady state. VF1 is nothing but V1 because there is no swirl component, so this is equal to A2 VF 2.

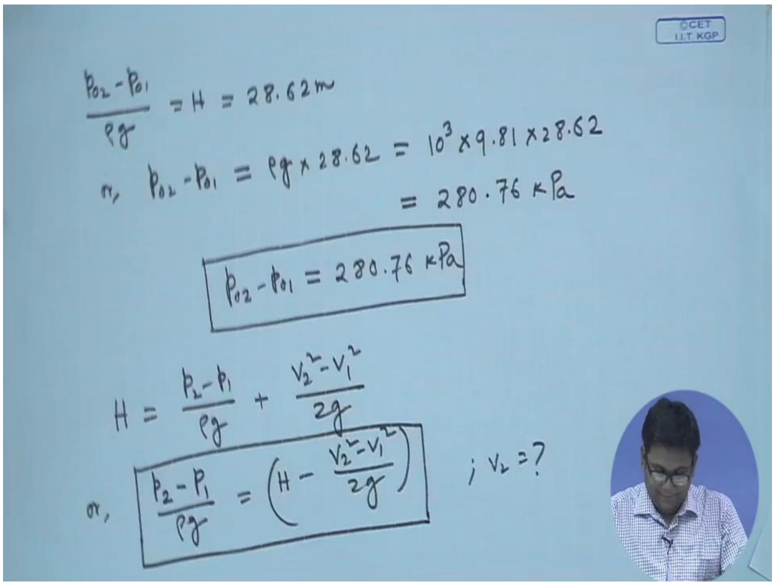
(Refer Slide Time: 16:29)



So using this relation we can write VF 2 equals to A1 V1 by A2. Now V1 we have previously obtained as 1.29 metre per second and ratio of area, area ratio will be nothing but ratio of respective radius or diameter. So diameter at inlet, diameter at outlet and V1. Diameter at inlet will be twice of radius, so 2 times 51 into 10 to the power -3 and diameter at outlet is 406 into 10 to the power -3 and V1 is 1.29. This gives V F2 as 0.324 metre per second. Now we substitute V F2 and U2 and beta-2 in this relation to obtain VW2.

So VW2 is U2 - VF 2 by tan beta-2. U2 we have obtained as 19.13, V F2 is 0.324 and tan of 7 degree. So this is VW2. Now head is hydraulic efficiency times VW2 U2 by G. So let us substitute all the quantities. VW2 is 19.13 - 0.324 over tan 7 degree times U2, U2 we have obtained as 19.13 over G, 9.81. So this will be 0.89, this quantity, head imparted by the impeller comes as 32.16. So this will be 28.62 metres. So head developed is 28.62 metres.

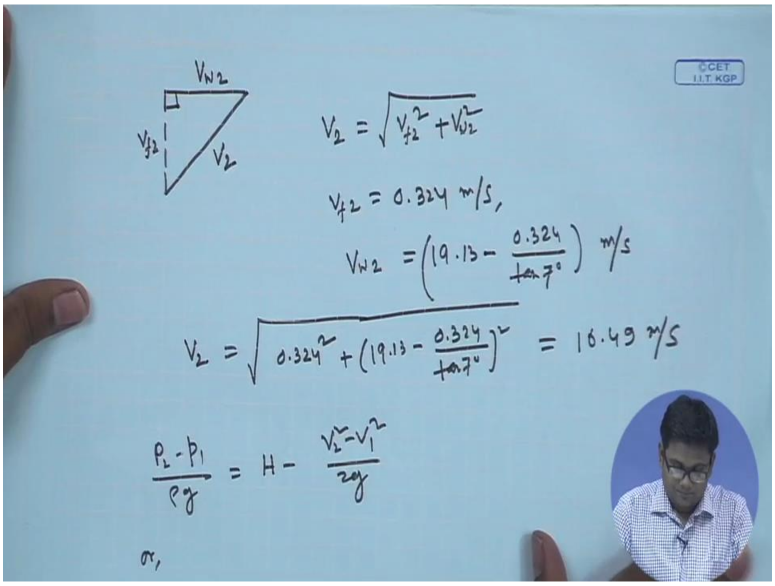
(Refer Slide Time: 19.09)



Now previously we have related this head developed with the rise in stagnation pressure. So rise in stagnation pressure head is same as the total head developed by the impeller. So P0 2 - P0 1 over rho G is H which now we know 28.62 metres or P0 2 - P0 1, which is the desired quantity of interests is rho G times 28.62. So rho for water 1000, G is 9.81, 28.62. So this you can obtain as 280.76 kilopascals. So rise in stagnation pressure across the impeller is 280.76 kilopascals.

Another desired quantity of interest is the rise in static pressure. So rise in static pressure can be obtained here. So H is P2 - P1 by rho G + V2 square - V1 square by 2G. So this relation keep the 2 - P1 by rho G equals to H - V2 square - V1 square by 2G. So this relation can be used to obtain the rise in static pressure. So before that we know H, we know V1, so we have to determine beta-2. To determine we do, let us 1st going to the velocity triangle.

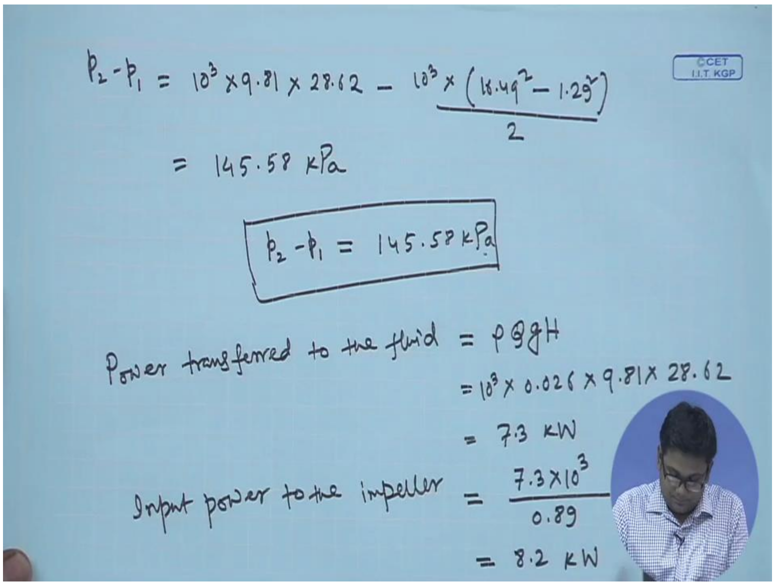
(Refer Slide Time: 21:31)



So V2 can be obtained from this triangle as square root of this which is VF 2 or let me draw the triangle separately. This is V2, this is VF 2, this is VW2, this angle is 90 degrees. So V2 will be square root of V F2 square + VW 2 square. We have previously determined VF 2 using the flow continuity as 0.324, so let us write this term here metre per second. And VW2 we have obtained in terms of U2 and VF 2 as this. So let us substitute this, 19.13 - 0.324 by tan 7 degree, so this will be also in metre per second.

So V2 can be obtained by substituting this 0.324 square +19.13 - 0.24 over tan 7 degree square. So this you can obtain as 16.49 metre per second. So now we can substitute V1, V2 in the expression of static pressure rise. So this was the expression for static pressure rise in terms, in head, so static pressure rise P2 - P1 rho G will be H - V2 square - V1 square by 2G. Or in terms of pressure, this is rho G H - rho V2 square - V1 square by 2.

(Refer Slide Time: 23:44)

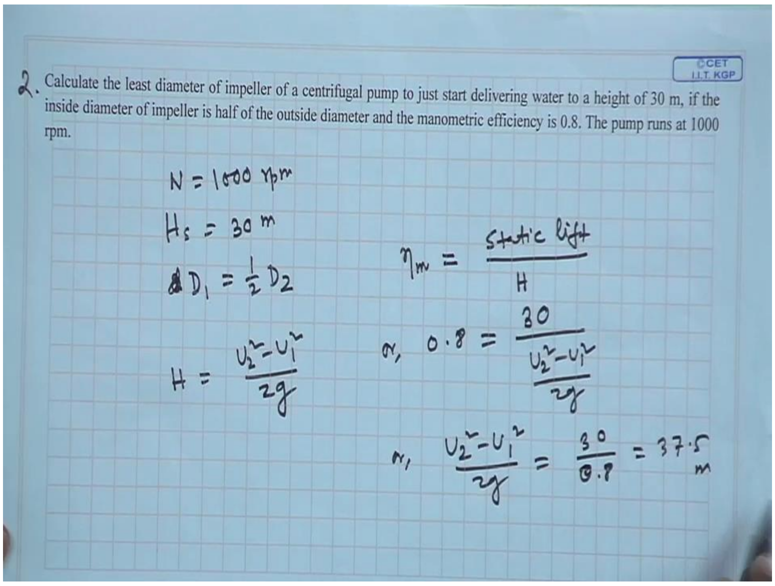


So let us substitute all the quantities. So P2 - P1 will be rho is 1000, G 9.81 and H we have obtained previously, so H, value of H is 28.62 - rho 1000, V2 is 16.49 square - V1. So V1, value of V1 we have obtained previously. So V1 is obtained as 1.29 metre square. V1 is 1.29. So substituting all these quantities we finally obtain P2 - P1 as 145.58 kilopascals. So P2 - P 1, which is the static rise of pressure across the impeller is 145.58 kilopascals.

Now let us move towards determining the other quantities. So next we have to determine the power transferred to the fluid. Now power transferred to the fluid is given by, power transferred to the fluid is rho Q GH. Where rho is the density of water, Q we have obtained as 0.026, G 9.81 and H is 28.6 U2. So this will be 7.3 kilowatts. Another quantity of interest is the input power to the impeller. So input power to the impeller will be power given to the fluid over the hydraulic efficiency.

So power given to the fluid is 10 to the power, 7.3 times 10 to the power 3 and hydraulic efficiency is 0.89. So this will be 8.2 kilowatts. So this is the input power to the impeller which is the power transferred to the fluid over the hydraulic efficiency.

(Refer Slide Time: 26:37)

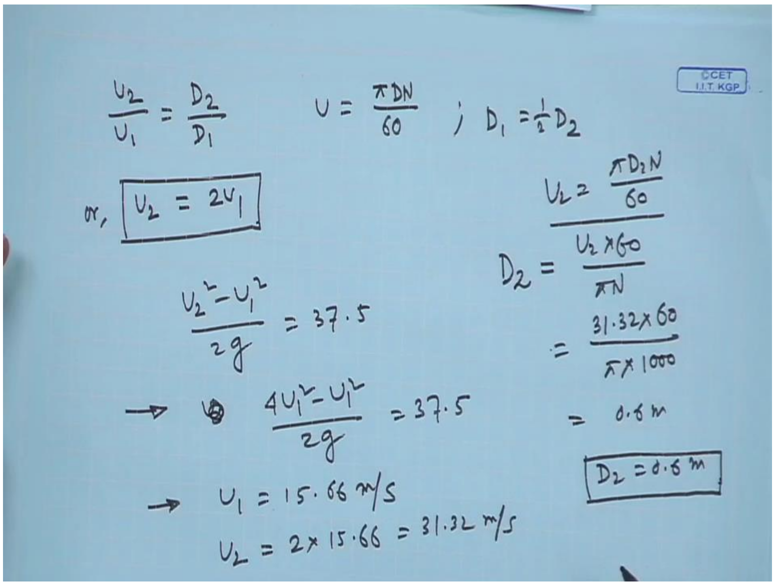


So let us move solve our next problem. So it is a simple problem related to centrifugal pump. So this is problem number 2. Calculate the least diameter of impeller of a centrifugal pump to just start delivering water to a height of 30 meter. If inside diameter of impeller is half of the outside diameter and manometric efficiency is 0.8 and pump runs at the speed 1000 rpm. So pump runs at a speed of 1000 rpm, static lift is 30 metres and the inside diameter D1 is half of the outside diameter, these are the given quantities.

Now we have to find the least impeller diameter of the centrifugal pump to just start the delivery. Now in this case one thing to note here is that the pump is fully primed, there is no air inside the system and the, when the flow starts, at this situation the absolute velocity at the inlet and outlet and also the relative velocities are negligible or equal to 0. So only the impeller speed is present. So the head developed by the impeller will be the centrifugal head.

So when the pump is just starting, at this point the head developed by the pump is centrifugal head. So let us just find the centrifugal head H which will be U2 square - U1 square by 2G. Now the manometric efficiency is given by the static lift, static lift over the head, centrifugal head, that is U2 square - U1 square by 2G. So from here we can, so manometric efficiency is given as 0.8, static lift is 30 meter and let us substitute expression for H.

(Refer Slide Time: 29:01)



So we can obtain U2 square - U1 square by 2G is equal to 30 by 0.8 which is 37.5 metres. Now U2 and U1, this can be related to the diameters because in this way because U, speed at any radial location will be pie DN by 60 where is the diameter. So U2 by U1 is D 2 by D1. And in the problem it is mentioned that D1 is half of D 2. So here we can obtain U2 is twice U1. Let us substitute this in the previous expression, so we have previously obtained U2 square - U1 square by 2G is 37.5.

So this gives U2, this now we are substituting U2 in terms of U1. So 4 U1 square - U1 square by 2G is 37.5, so this will give you one as 15.66 metre per second. So U2 will be 2 times 15.66 will be 31.32 metre per second. Now the quantity of interest here is the least impeller diameter, least diameter of the impeller which is the, let us find the outer diameter because the inner diameter will be half of that.

So outer diameter can be written as U, so we know U2 is pie D2 N by 60. So D2 will be U2 times 60 by pie N. So let us substitute U2 pie and N is given as 1000, so D2 is 0.6 metres. So this is the least diameter, outer diameter of the impeller required to make a flow through the pump having a static lift of 30 metres. So with this I am ending today’s lecture, thank you.