Fluid Machines.
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Tutorial-4.

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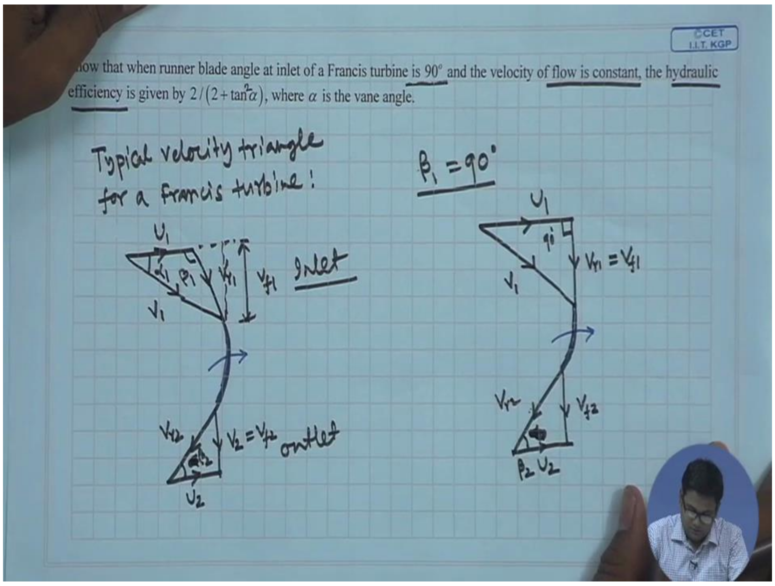

So welcome to this session of the course fluid machines. Today we are going to discuss about few problems related to Francis turbine and Kaplan turbine. So the problem statement for the 1st problem is show that when the runner blade angle at inlet of a Francis turbine is 90 degree and the velocity of flow is constant, the hydraulic efficiency is given by 2 over 2+ tan square Alpha where Alpha is the vane angle. So we need to find out the hydraulic efficiency of the Francis turbine for the particular case in which the inlet blade angle is 90 degree and flow velocity is constant throughout the turbine.

So at 1st let us see a typical velocity triangle for a Francis turbine. So typical velocity triangle for a Francis turbine, so this is the relative velocity at inlet, this U1 is tangential velocity of the runner at the inlet, this is V1, the absolute velocity of the fluid at inlet and this, this part is flow velocity at the inlet. Now the absolute velocity makes an angle alpha-1 with the tangential velocity of the runner which is termed as inlet vane angle. And the relative velocity makes an angle beta one with the tangential velocity of the rotor which is termed as blade angle.

This is the turbine runner which rotates, say in this direction. So this part is inlet and this part is outlet. At the outlet, fluid leaves radially, so V2 is VF 2, this is the tangential velocity of the runner blade at outlet and this is the radial velocity of the fluid at outlet. And this angle can be termed as beta 2 which is the exit blade angle. Now in this particular case for the Francis turbine, the inlet blade angle is 90 degree. So beta 1 is 90 degree. So in this case the velocity triangles will be, so this is the turbine runner.

So for beta 1 equals to 90 degree relative velocity at inlet will be same as the flow velocity at the inlet. This is the absolute velocity at inlet, this is tangential velocity of the runner at inlet. The exit velocity triangle remains same. Now we have to find the hydraulic efficiency of the Francis turbine.

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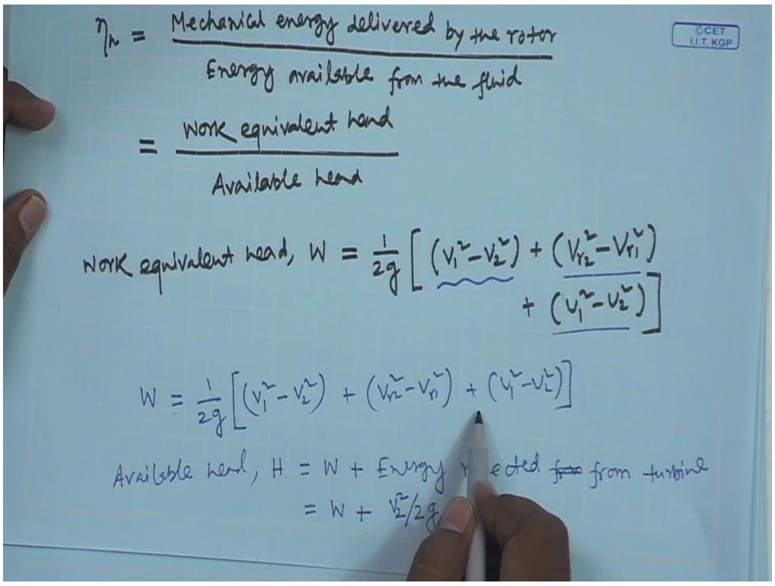

Now let us 1st write the definition of hydraulic efficiency. So hydraulic efficiency can be defined in this way, it is a ratio of mechanical energy delivered by the rotor over the energy available from the fluid. So mechanical energy delivered by the rotor over energy available from the fluid. This definition can also be given in terms of head which is work equivalent head, equivalent head over the available head, available head.

Now the work equivalent head, work equivalent head W is nothing but the energy per unit weight of fluid transferred from fluid to the turbine runner, so which can be expressed in this way, 1 by 2G times V1 square V2 square + VR2 square - V R1 square + U1 square - U 2 square. Where the 1st term, V1 square - V2 square by 2G refers to the change in absolute kinetic energy of the, absolute kinetic energy of the fluid or dynamic head across the turbine.

And these 2, combination of these 2 terms can be explained, this can be written in terms of static heat difference. So W, so this, now the available head, available head which can be termed as H is nothing but W + the energy rejected, energy rejected from turbine at the outlet. So H is W + outlet energy will be V2 square by 2G in terms of head. So let us find these quantities in terms of the inlet vane angle.

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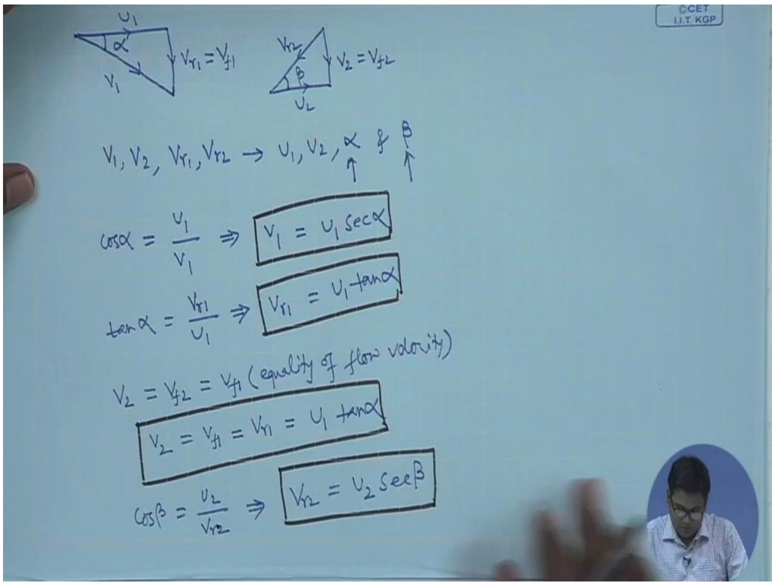

So let us 1st draw another triangle, the inlet velocity triangle. So this is given as alpha which is the inlet vane angle, this is U1, this is VR1, which is also equal to VF1 and this is V1. The outlet velocity triangle is like this, this angle is beta, this is U 2, this is V2 is equal to VF 2 and this is VR2. Now here our main intention is to express V1, V2, VR1, VR2, all these quantities in terms of U1, U2, Alpha and beta. Where Alpha is the inlet vane angle and beta is the blade angle at the exit.

Towards this from this triangle we can write cosine of Alpha will be U1 by V1, so this gives V1 as U1 sec Alpha. And tan Alpha can be written as VR1 by U1, so this gives VR1 equals U1 tan alpha. Now V2 is VF 2 is equal to VF1 as per the definition of the problem. So in problem it is mentioned that the velocity of flow is constant. So flow velocity at inlet will be equal to the flow velocity of outlet. So VF1 equals to VF 2. So using this equality of flow velocity we can write V2 as VF1 which is VR1 equals U1 tan Alpha.

Now to express, so till now we have expressed V1 in terms of U1 and Alpha, VR1 in terms of U1 Alpha and V2. So now we have to express V R2 in terms of U2, U2 and beta. So cosine of beta can be obtained from this velocity triangle as U2 by VR2, this gives V R2 equals U2 sec beta. All the 4 quantities are now obtained. Now let us substitute these terms in the expression of work available head.

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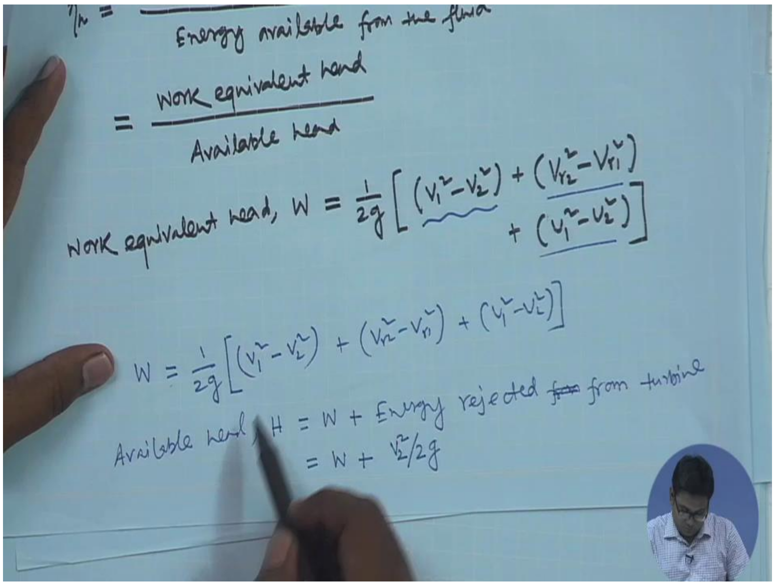

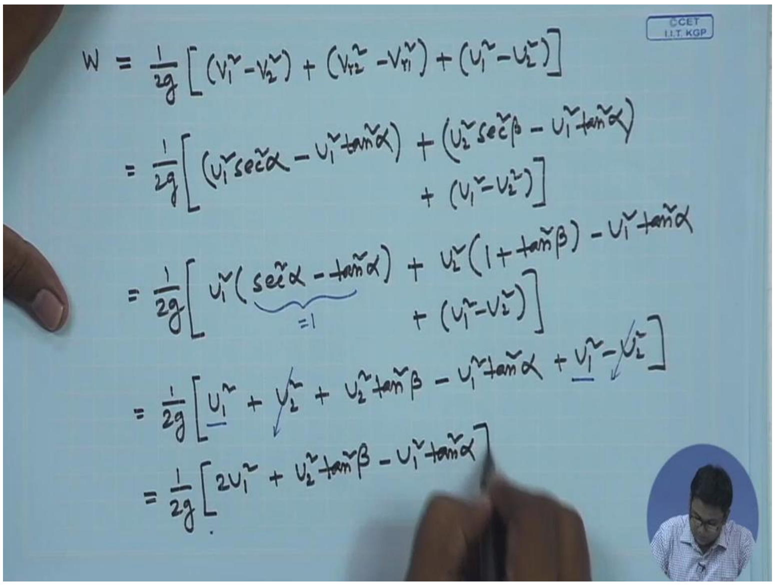

So this is the expression for work available head. Now I am going to substitute V1, V2 and VR2 and VR1. So, let us 1st write again the work available head W is 1 by 2G V1 square - V2 square + VR2 square - V R1 square + U1 square - U2 square. So V1 we have often as U1 sec Alpha, so let us substitute that. So U1 square sec square Alpha. V2 is obtained as U1 tan Alpha, so U1 square tan square Alpha. Now let us substitute VR2 and VR1.

So VR 2 is obtained as U2 sec beta. So U2 square sec square beta, and VR1 is obtained as U1 tan Alpha, so U1 square and tan square Alpha. And we will keep U1 and U2 as it is. So here we can take U1 square common, so this will be sec square Alpha - tan square Alpha. Now we can transform this sec square beta in terms of tan square. So U2 square into 1+ tan square beta - U1 square tan square Alpha + U1 square - U2 square. So this is nothing but 1.

So simplifying this expression we obtain U1 square + U2 square + U2 square tan square beta - U1 square tan square Alpha + U1 square - U2 square. So this U2 square and this U2 square get cancelled and we can say and another thing to note, that there is one U1 square and there is one U1 square. So 1 by 2G 2U1 square + U2 square tan square beta - U1 square tan square Alpha.

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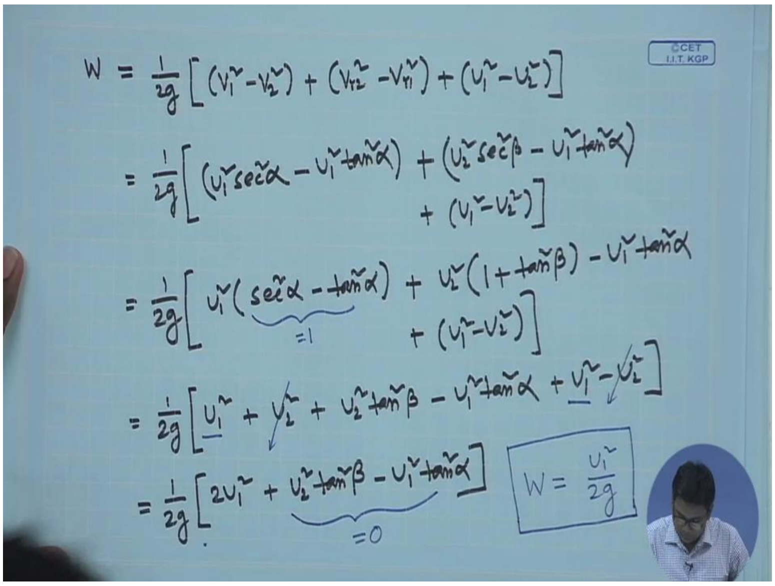

Okay. Now from the velocity triangle let us find out what is U2 tan beta and what is U1 tan Alpha. So U2 tan beta. So from this diagram we can write that U2 tan beta is VF 2 and from this triangle we can write U1 tan Alpha is equal to VF1. But it has been given that VF1 is equal to VF 2, so U1 tan Alpha and U2 tan beta, these 2 are equal. So this term will be 0. This term will be 0. So W is nothing but U1 square by 2G. Now let us substitute this for W in the expression of hydraulic efficiency.

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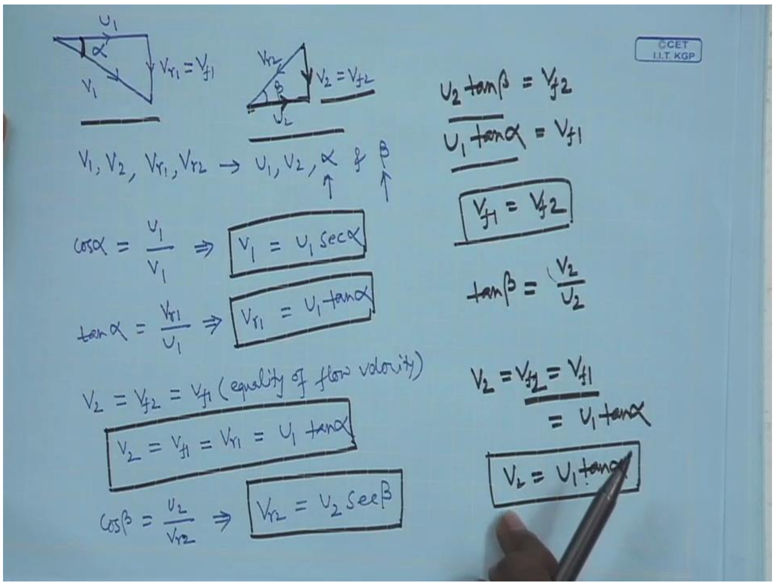

So the hydraulic efficiency is the work available head over the W + H, this is H. So W is obtained as U1 square by 2G and H is W + the head lost. U2 square by 2G. Now there is one small correction here, 2 and 2 will cancel out, so this will be U1 square by G. So here also W will be U1 square by G. And H, the head available is nothing but the work available head + head at the exit of the turbine which is U2 square by 2G. Now from the velocity for angle we can express U2 in terms of, V2 in terms of U2.

So this is U2 and this is V2. So tan beta will be V2 by U2. Or in this case, if we want to express V2 in terms of Alpha, then we can write in this way. So V2 is the flow velocity at the exit VF 2, VF 2 which is equal to the flow velocity at the inlet. And the flow velocity at the inlet can be expressed in terms of the inlet vane angle. So V2 is U1 tan Alpha. So in this way we can express V2 in terms of the inlet vane angle. This definition express, from this triangle we can express V2 in terms of U2 and beta but beta is not given quantity in the problem, so it is useful to use the relation equality of flow velocity and express V2 in terms of U1 and tan Alpha.

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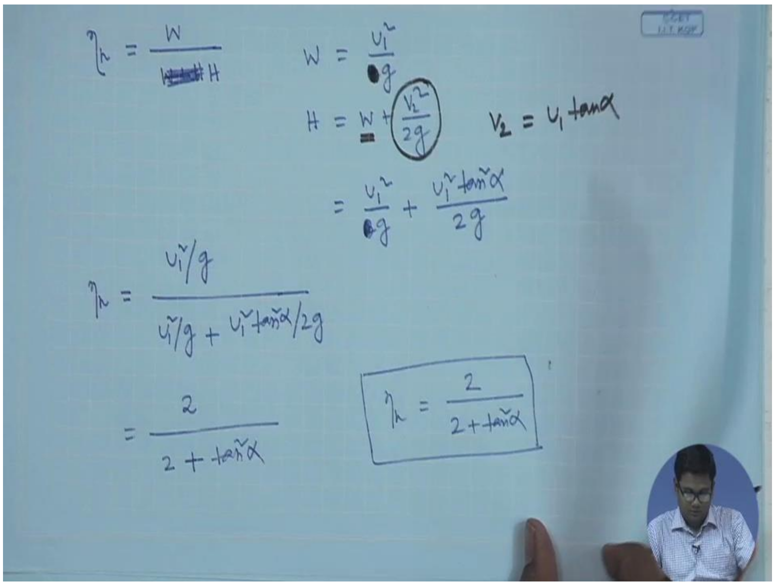

So let us substitute this expression of V2. So V2 is U1 tan Alpha. So H is U1 square by 2G + U1 square tan square Alpha by or U1 square by 2 G, there will not be any 2. So Eta hydraulic is U1 square by G over U1 square by G + U1 square tan square Alpha by 2G. After some simplification we can obtain U by 2 by 2 + tan square Alpha. So Eta hydraulic is 2 over 2 + tan square Alpha. So initially this was the problem, given problem that hydraulic efficiency will be 2 by2 + tan square Alpha.

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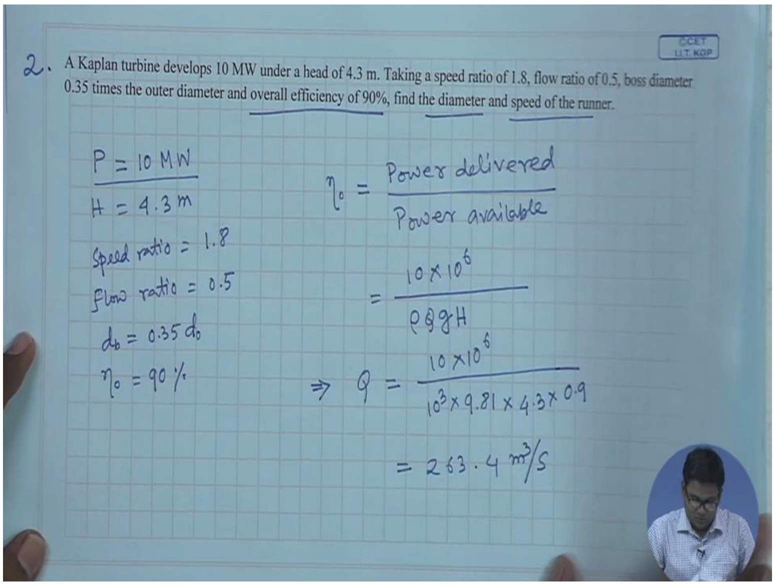

So now we move on to solve the 2nd problem which is related to Kaplan turbine. So this is the problem definition. So a Kaplan turbine develops 100 megawatt of power under the head of

4.3 metre taking a speed ratio of 1.8, flow ratio of 0.5, boss diameter 0.35 times the outer diameter and overall efficiency of 90 percent. Find the diameter and speed of the runner. So we have to find out the diameter and speed of the runner for the given parameters.

So the power output is given as 100 megawatt, head is 4.3 metre, speed ratio, speed ratio is 1.8, flow ratio is 0.5 and boss diameter, let us represent this by DB is 0.35 times the outer diameter, let us represent it by DO. And it is also mentioned that the overall efficiency is 90 percent. To determine the diameter, let us 1st try to obtain the flow rate through the Kaplan turbine. Let us 1st utilised the definition of overall efficiency, so overall efficiency can be written as power delivered over power available.

Power delivered is given as 10 megawatt for this turbine, so this is than into 10 to the power 6 watt. Power available is rho Q G H where rho is the density of the fluid, here we are considering water, Q is the flow rate, G is the acceleration due to gravity and H is the head. Now substituting all the quantities we can determine Q as 10 into 10 to the power 6 is the power density acceleration is 9.81, H is given as 4.3 and overall efficiency is 0.9. So flow rate can be obtained from here as 263.4 metre cube per second.

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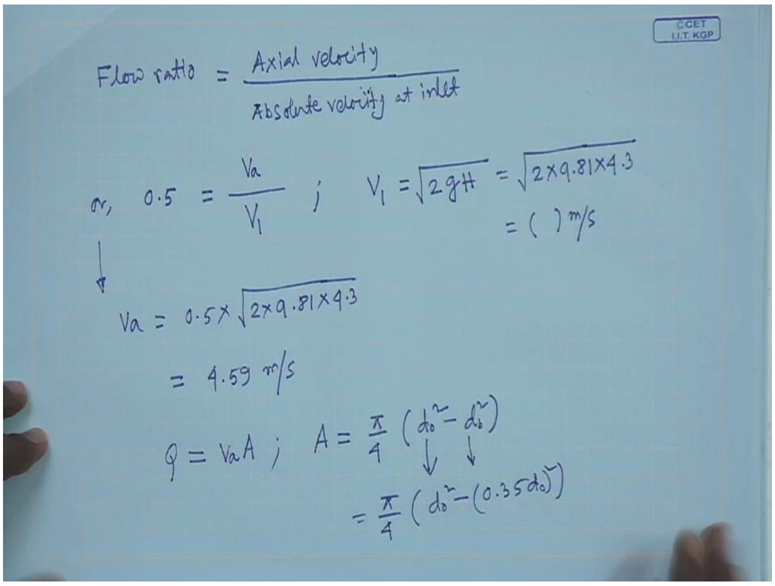

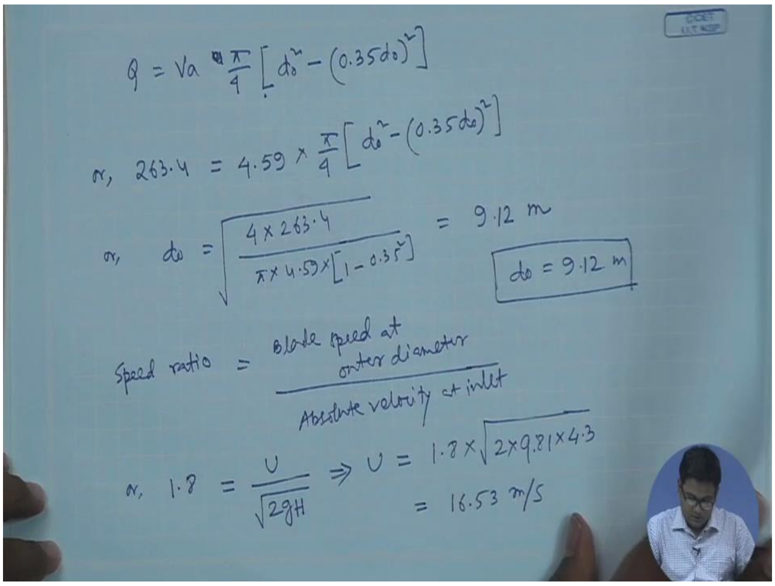

No flow rate is related to the cross-sectional area, so and we can determine diameter from there. So before obtaining diameter, let us define what is flow ratio. So flow ratio, this is just a definition, this represents the ratio of axial velocity and absolute velocity at inlet. So the ratio of axial velocity and absolute velocity at inlet is termed as flow ratio. No flow ratio is given as 0.5, so this is 0.5, axial velocity, let us term this as VA and absolute velocity at inlet as V1.

Now V1 can be obtained from the head given, so H is 4.3 metre. So V1 can be obtained as 2GH equals to 2 into 9.81 into H is 4.3. So V1, you can obtain from this relation in metres per second. Now substituting this V1 we cannot and VA from this relation as 0.5 times 2 into 9.81 into 4.3. This is 4.59 metre per second. So axial velocity is 4.59 metre per second. Now the flow rate through the Kaplan turbine is axial velocity times the cross-sectional area A. Now this cross-sectional area is the annular area over which the flow is taking place.

This can be written as 5 by 4 DO square - DB square. Where DO is the diameter, outer diameter and DB is the boss diameter. Now it has been given that the DB is 0.35 times DO, so let us substitute this. So area is pie by4 times DO - 0.35 times DO square. So flow rate is VA times pie by 4 DO square - 0.35 times DO square. We have obtained the flow rate previously, so flow rate is obtained as 263.4 metre cube per second. So let us just substitute this. So 263.4, axial velocity is also obtained as 4.59 metre per second, 4.59 into pie by4 DO square - 0.35 DO square.

So DO will be, DO can be obtained as 4 times 263.4 by pie into 4.59 into 1 - 0.35 square over root. So this will be 9.12 metres. So one of the results now we have obtained, the diameter of the runner. So this is the diameter, rather outer diameter of the runner. Now we have to find speed of the runner. Now to find this, let us introduce the speed ratio which is the blade speed at outer diameter over the absolute velocity at inlet, this is just a definition.

Now speed ratio is given in the problem as 1.8, blade speed, let us denote this by U and absolute velocity is 2GH. Now substituting these quantities we can obtain U as 1.8 times 2 into 9.81 times H is 4.3. So this you can often as 16.53 metre per second. So this blade speed is related to the rotational speed of the turbine.

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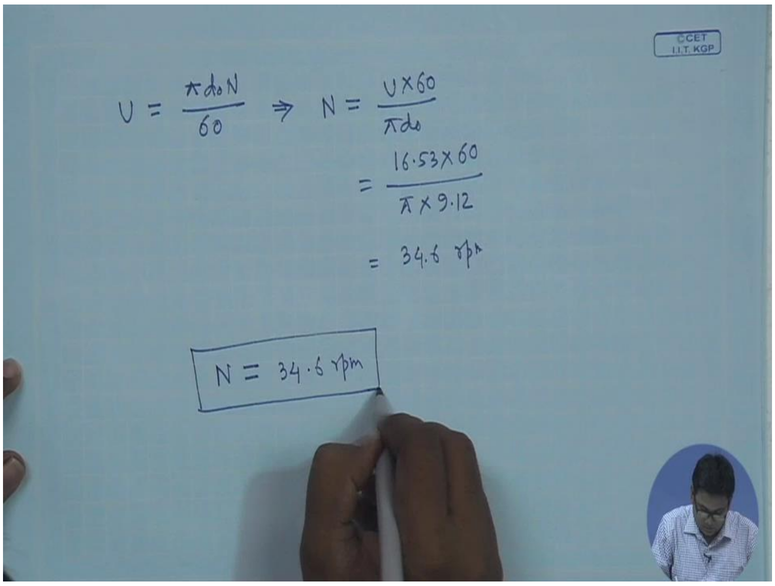

So U is DO N by 60 where N is in rpm. So this relation gives the U into 60 by pie DO. So U we have obtained as 16.53 times 60 over pie times DO is obtained as 9.12, 9.12. This gives 34.6 rpm, so rotational speed of the turbine runner is 34.6 rpm. So this completes our 2nd and the last problem. With this I will end the today's class, thank you.