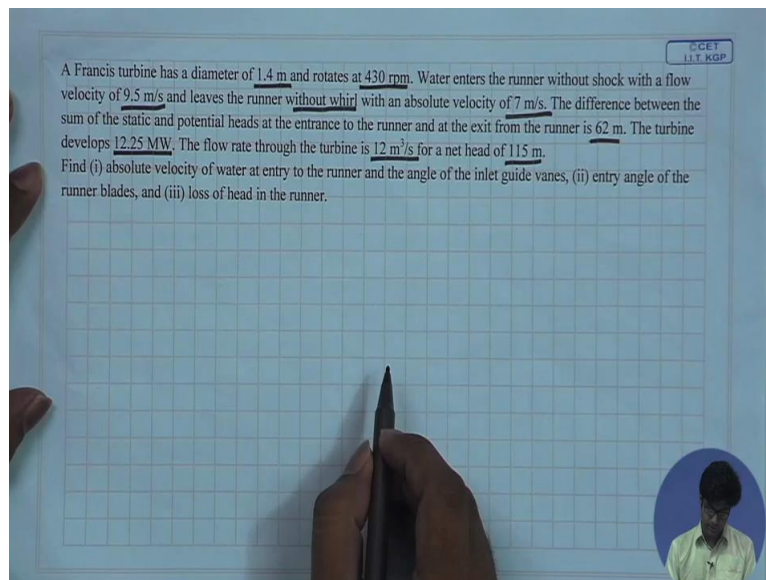


**Fluid Machines.**  
**Mr. Sayan Das.**  
**Teaching Assistant.**  
**Department Of Mechanical Engineering.**  
**Indian Institute Of Technology Kharagpur.**  
**Tutorial-3.**

Today I will be doing 2 problems on Francis turbine. The 1<sup>st</sup> will be a basic problem on the concept of Francis turbine and 2<sup>nd</sup> will be based on the draft tube. So let us start with the 1<sup>st</sup> problem. Let me read out the problem. A Francis turbine has a diameter, or the runner diameter of the Francis turbine is given as 1.4 metre and it rotates at 430 rpm. Water enters the runner, so the rotational speed of the runner is 430 rpm and the diameter of the runner is 1.4 meter.

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Water enters the runner without shock with a flow velocity of 9.5 metre per second and leaves the runner without Whirl and absolute velocity of 7 metre per second. The difference between the sum of the static and potential heads at the entrance to the runner and at the exit from the runner is 62 metres. The turbine develops a power of 12.25 megawatts, the flow rate through the turbine is 12 metre cube per second with a net head of 115 metres. So we have to find out the absolute velocity of water at the entry to the runner, the angle and the angle of the inlet guide vanes.

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A Francis turbine runner has a diameter of 1.4 m and rotates at 430 rpm. Water enters the runner without shock with a flow velocity of 9.5 m/s and leaves the runner without whirl with an absolute velocity of 7 m/s. The difference between the sum of the static and potential heads at the entrance to the runner and at the exit from the runner is 62 m. The turbine develops 12.25 MW. The flow rate through the turbine is 12 m<sup>3</sup>/s for a net head of 115 m. Find (i) absolute velocity of water at entry to the runner and the angle of the inlet guide vanes, (ii) entry angle of the runner blades, and (iii) loss of head in the runner.

1)  $D = 1.4 \text{ m}$   
 2)  $N = 430 \text{ rpm}$   
 3)  $V_{f1} = 9.5 \text{ m/s}$   
 4)  $V_2 = 7 \text{ m/s}$   
 5)  $\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = 62 \text{ m}$   
 6)  $P = 12.25 \text{ MW}$   
 7)  $Q = 12 \text{ m}^3/\text{s}$     8)  $H = 115 \text{ m}$

$V_1 = \sqrt{V_{w1}^2 + \dots}$

Also we have to find out the entry angle of the runner blades and the loss of head in the runner. So 1<sup>st</sup> let us see the velocity diagram for the Francis turbine at the outlet and the inlet. So this is the velocity diagram at the inlet. This is the tangential velocity at the inlet or the whirl velocity at the inlet, this region represents the peripheral velocity of the runner blade at the inlet. This angle represents the inlet guide vanes angle and this angle represents the runner blade angle at the entry or the inlet.

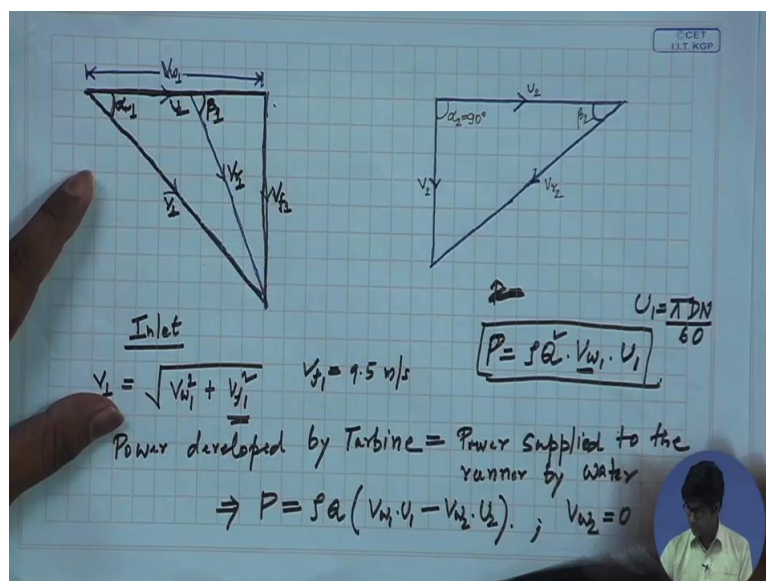
This is the relative velocity or VR1 at the inlet, this is the flow velocity at the inlet or VF1 and this is the absolute velocity at the inlet of the runner which is V1. So this is the inlet velocity diagram, so basically in the 1<sup>st</sup> part we are required to find out the absolute velocity

at the inlet. The absolute velocity of water at entry to the runner, that is  $V_1$ . So 1<sup>st</sup> let us see what are the parameters that are given.

So 1<sup>st</sup> the diameter of the runner is given as 1.4 metres, 2<sup>nd</sup> the rotational speed of the runner is given as 430 rpm, next it is given that the water enters the runner without shock with a flow velocity that is  $V_{f1}$ , the flow velocity at the inlet is given as 9.5 metres per second. The absolute velocity is given at the outlet at 7 metre per second. Next it is said that the difference between the static and the potential heads at the entrance to the runner and at the exit is 62 metres, that is the sum of the static and the potential head, this is the potential head or the static, sorry, static head at the inlet to the runner is given by  $P_1$  by  $\rho G$ .

And the static ad, sorry the potential head at the entry is given as  $Z_1$ . That is the elevation at the inlet to the runner - the static head at the outlet to the runner is  $P_2$  by  $\rho G$  + the potential head at the exit of the runner is given by  $Z_2$ . So this whole difference is given as 62 metres. Next the power developed from the turbine is given as 12.25 megawatts. The flow rate is given as 12 metre cube per second and the net head available is given as 115 metres.

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So from this velocity diagram we see that the absolute velocity at the inlet is square root over the tangential velocity at the inlet whole square + the square of the flow velocity at the inlet. That is we are applying Pythagoras theorem to this triangle. So the flow velocity at the inlet is given as 9.5 metre per second. So we are required to find out the tangential velocity at the inlet. In order to find out we use the basic relationship that is the power developed, power developed by turbine is equal to the power supplied to the runner by water.

So the power is equal to  $P$  is equal to  $\rho Q$  into  $V_{w1}$  into  $U_1$ ,  $U_1$  is the peripheral velocity of the runner blade at the inlet -  $V_{w2}$  into  $U_2$ . Now in the problem it was said that the flow at the outlet of the runner blade exits radially, that is the tangential component is 0. So, we are left with, now in this relationship, the only unknown right now is the peripheral velocity of the runner blade at inlet, that is  $U_1$ .

The other quantities that is  $Q$  is given in the problem, we know the density of water is 1000 KG per metre cube and we are also required to find out the tangential velocity of the, at the inlet. The power  $P$  developed is given, so  $U_1$  is given as  $\pi DN$  by 60.

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$$U_1 = \frac{\pi DN}{60} = 31.5 \text{ m/s.}$$

$$V_{w1} = \frac{P}{\rho Q U_1} = 32.4 \text{ m/s}$$

$$V_1 = \sqrt{V_{w1}^2 + V_{f1}^2} = 33.76$$

$$\alpha_1 = \tan^{-1} \left( \frac{V_{f1}}{V_{w1}} \right) = 16.34^\circ$$

$$\beta_1 = \tan^{-1} \left( \frac{V_{f1}}{V_{w1} - U_1} \right)$$

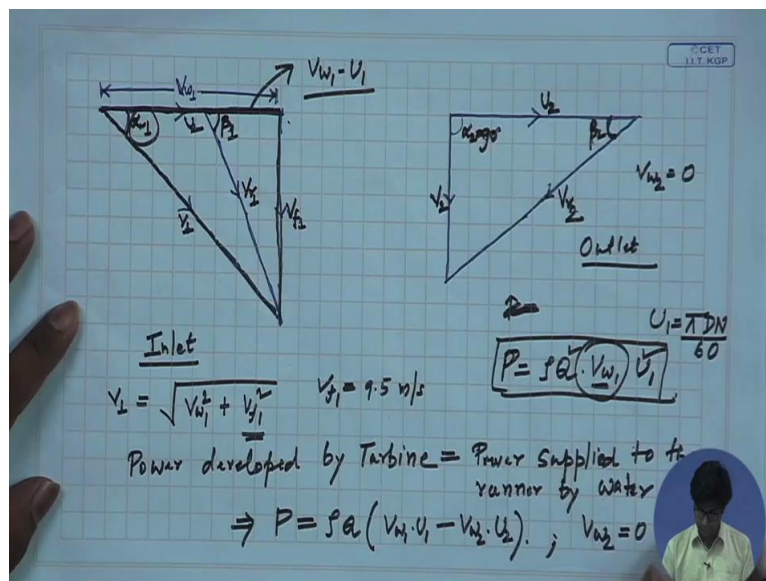
$$= 84.6^\circ$$

That is  $U_1$  is equal to  $\pi DN$  by 60 where  $D$  is the diameter of the runner blade which is given,  $N$  is the rotational speed of the runner which is also given, the diameter of the runner is given as 1.4 metres and the rotational speed of the runner is given as 430 rpm. Substituting the values we get the value of peripheral velocity of the runner blade as 31.5 metre per second. So from the relation, from this relationship right now we know the value of  $U_1$ .

So we find out the value of tangential velocity at inlet as  $P$  by  $\rho Q$  into  $U_1$ . So substituting the value we ultimately find the tangential velocity at inlet as 32.4 metre per second. So finally returning to the basic formula for the absolute velocity at the inlet,  $V_{w1}^2 + V_{f1}^2$ , so substituting the values here, we ultimately get the fluid velocity at inlet as 33.76. Also we are required to find out the value of  $\alpha_1$  which is the inlet guide vanes angle and clearly from this velocity diagram we can see that  $\alpha_1$  is equal to the tan inverse of  $V_{f1}$ , that is  $\alpha_1$  is equal to tan inverse of  $V_{f1}$  by  $V_{w1}$ .

Substituting the values we ultimately get the value of the inlet guide vane angle as 16.34 degree. Next we are required to find out the runner blade angle at the inlet to the, at the entry. That is beta 1. So beta 1 is given as tan inverse, inverse of, tan inverse of  $V_{f1}$  divided by this part, this line which is actually  $V_{w1} - U_1$ . So beta 1 is given as tan inverse of, sorry  $V_{f1}$  by  $V_{w1} - U_1$ . We know the flow velocity at the inlet, we also know the tangential velocity as well as the peripheral velocity of the runner at the inlet. So substituting the values we ultimately get the value of beta as 84.6 degree.

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So right now let me return to the outlet velocity diagram, outlet velocity diagram. Since we are told that the velocity at the outlet or the water exits the runner radially, so guide vanes angle at the outlet is 90 degree and  $V_{w2}$  is equal to 0. This is the runner blade angle at the exit, this is the peripheral velocity of the runner at the exit, this is the relative velocity at the exit and this is the absolute velocity of water at the exit. So finally what we are required to find out is the loss of head in the runner.

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Head Available at the inlet of the runner ( $H_1$ )  
= Head Available at exit of the runner ( $H_2$ )  
+ Head lost in the runner ( $h_f$ )  
+ Head for work done by turbine. ( $H_w$ )

~~Suffix~~ Subscript '1' → Inlet to Runner  
Subscript '2' → Outlet

$$\underline{H_w} = \frac{V_{w1} \cdot U_1 - V_{w2} \cdot U_2}{g} = \frac{V_{w1} \cdot U_1}{g}$$
$$h_f = H_1 - H_2 - H_w$$

For that we write the basic relationship of the head available, that is head available at the inlet of the runner is equal to head available at exit of the runner + the head lost in the runner + the head that is used up for the work done by the turbine. So if we represent the inlet to the runner by suffix 1 or subscript 1, that is subscript 1 represents the inlet to the runner and subscript 2 represents the outlet to the runner.

So the head available at the inlet, we can represent it by  $H_1$ , the head level, available at the exit of the runner we are representing by  $H_2$ , the head lost in the runner, that we are going to find out is  $H_f$  and let this be represented by  $H_w$ . So the head that is used up for the work done by the runner is actually the work done per unit weight of the fluid, which is actually given by  $V_{w1} U_1 - V_{w2} U_2$  by  $G$ . Again this is 0 as  $V_{w2}$  is 0, so we are left with.

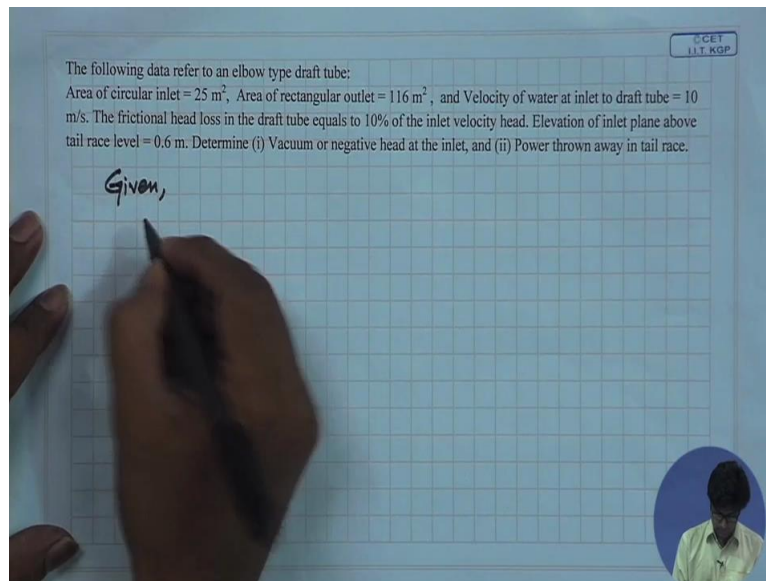
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$$H_1 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$
$$H_2 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$
$$H_1 - H_2 = \left[ \left( \frac{p_1}{\rho g} + z_1 \right) - \left( \frac{p_2}{\rho g} + z_2 \right) \right] + \left( \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right)$$
$$= 62 + \frac{1}{2g} (33.76^2 - 7^2)$$
$$h_{fj} = H_1 - H_2 - H_w = 13.45 \text{ m}$$

We know the values of  $V$ ,  $W_1$ ,  $U_1$  and  $G$ , so we can find out the value of  $HW$ . So we can write  $HF$  as  $H_1 - H_2 - HW$ . So  $H_1$  now can be written as  $P_1$  by  $\rho G + V_1$  square by  $2G + Z_1$ ,  $H_2$  is  $P_2$  by  $\rho G +$  that is the pressure head, + the velocity head at the exit + the potential head. So  $H_1 - H_2$  is you can write it as  $P_1$  by  $\rho G + Z_1 - P_2$  by  $\rho G + Z_2 + V_1$ , that is the difference in the velocity head. Now this part, it is given as 62 metres in the problem.

So and we are, and the velocity, absolute velocity at the outlet is also given in the problem, it is given as 7 metres. And  $V_1$  I think we have found out, yes,  $V_1$  was found out as 33.76, so 33.76 square -  $V_2$  value is 7, so 7 square. So we get the value of  $H_1 - H_2$ . So ultimately we get the value of  $HF$  as  $H_1 - H_2 -$  of  $HW$ . All the values we have already found out, so substituting all the values, we ultimately get this lost in head as 13.45 metres.

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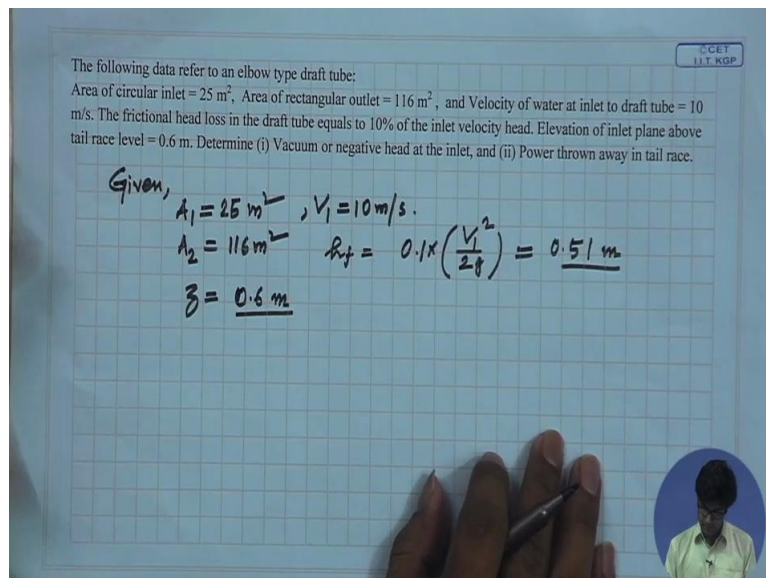
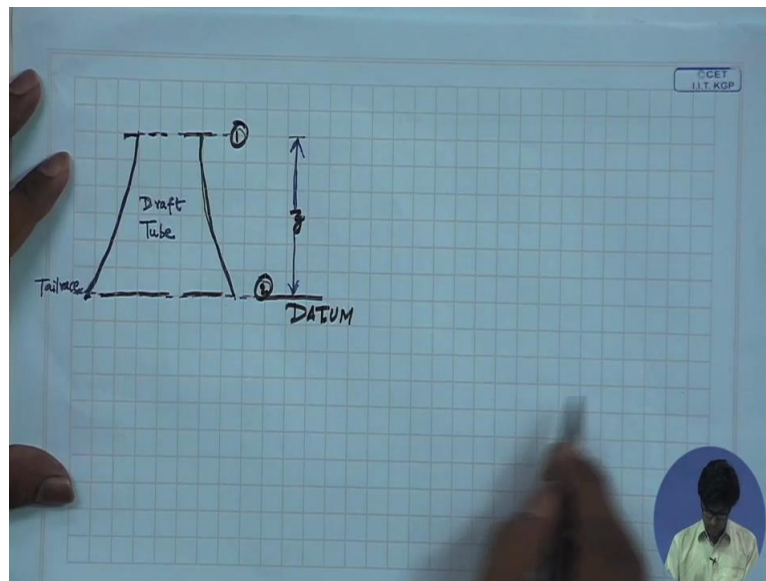


So we have found out all the quantities in problem 1, now let us move onto the next problem which is based on draft tube. So let me read out the problem. The following data refers to an elbow type draft tube, the area of the circular inlet is 25 metres square, the area of the rectangle outlet is 116 metre square and the velocity of water at inlet to the draft tube is 10 metre per second. The frictional head loss in the draft tube equals 10 percent of the inlet velocity head, elevation of the inlet plane above the tale rest level is 0.6 metres.

So we have to determine the vacuum or the negative head at the inlet and the power thrown away in the tail rest. So let us write down the quantities that are given. Before that let me show you a schematic of the draft tube.



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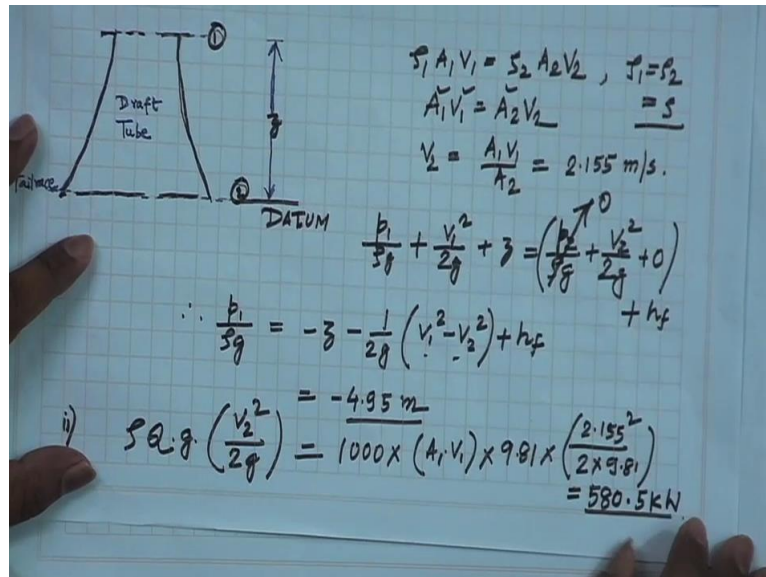


So this is the draft tube, this is the tail rest and we will be considering this as the datum. This is section 1 or the inlet to the draft tube and we are considering the tail rest as the section 2, this is the elevation of the inlet of the draft tube above the tail rest level which is  $Z$ . So since we are denoting the inlet section of the draft tube by subscript 1, we write the area at the inlet  $A_1$  as 25 metre square and area out the outlet is given as 116 metre square. The velocity of water at the inlet to the draft tube is given as  $V_1$  is equal to 10 metre per second.

And the frictional head loss in the draft tube equals to the 10 percent of the inlet velocity head. That is we know the inlet velocity head, that is  $H_f$ , we know the inlet velocity head is  $V_1$  square by  $2g$  and 10 percent of this, it is  $0.1$  into  $V_1$  square by  $2g$ . So this is the head lost

in the draft tube, sectional head loss. And if we find out the value, it comes out as 0.51, we know the value of  $V_1$ , 0.51 metres. Next we are given that the elevation of Z, of the inlet of the draft tube above the tail rest is 0.6 metres.

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So 1<sup>st</sup> we have to find out the negative head at the inlet. So let us apply Bernoulli's equation between these 2 points. Before that let us apply the law of conservation of mass between these 2 points. So we can write at the inlet we can equate that, we can equate the mass flow rate at the inlet and the outlet and we can write  $\rho_1 A_1 V_1$  is equal to  $\rho_2 A_2 V_2$ . Since  $\rho_1$ , since the density is constant, we are considering constant, so we can finally write  $A_1 V_1$  is equal to  $A_2 V_2$ .

We know the inlet area and the outlet area, we know the velocity at the outlet, sorry we know the velocity at the inlet, so we can find out  $V_2$  as  $A_1 V_1$  by  $A_2$  which comes out to be 2.155 metre per second. So now let us apply the Bernoulli's equation between section 1 and section 2. So we can write  $P_1$  by  $\rho g$  or the pressure head + the velocity head at the inlet + the potential head is equal to  $P_2$  by  $\rho g$  +  $V_2$  square by  $2g$  +, since we are considering the datum at the section 2 itself, so the elevation is 0 + the head lost which is  $H_f$ .

Now since all the pressure, we are considering all the pressure in this, pressure head in this as the, all the pressures that we are considering in this equation are the gauge pressures, that is the pressure above the atmospheric level. So since the at the section 2, this section is exposed to the atmosphere, so this is nothing but the atmospheric pressure. So you can take this as 0.

So ultimately the pressure head at the inlet can be written as  $-Z - \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + H_F$ .

So we know the value of  $Z$ , it is given as 0.6 metres, we know the value of both  $V_1$  and  $V_2$  and we have to find out the value of  $H_F$  as 0.51 metres. So substituting all the values we ultimately get the value of the negative pressure head as - 4.95 metres. So there is a suction pressure at the inlet and suction pressure at the inlet of the draft tube is 4.95 metres. Next we are required to find out the power that is thrown away in the tail rest.

So as you can see that the come at the exit of the draft tube, the velocity is not 0. So whatever energy at the exit is lost, is lost due to the kinetic energy at the outlet. So the power that is lost, or the power lost due to the kinetic energy can be written as, this is the 2<sup>nd</sup> part,  $\rho Q$  into  $G$  into  $V_2$  square by 2 into  $G$ . So substituting all the values here, we know the value of  $Q$ , it is 1000 into  $Q$ , that is, what is the value of  $Q$ , okay, the value of  $Q$  is nothing but the product of  $A_1$  into  $V_1$  into 9.81,  $V_2$  we have found out as 2.155 square by 2 into 9.81.

So this comes out as 580.5 kilowatts. So this power is lost due to the velocity head present in at the exit of the draft tube. So with this, this ends the 2<sup>nd</sup> problem.