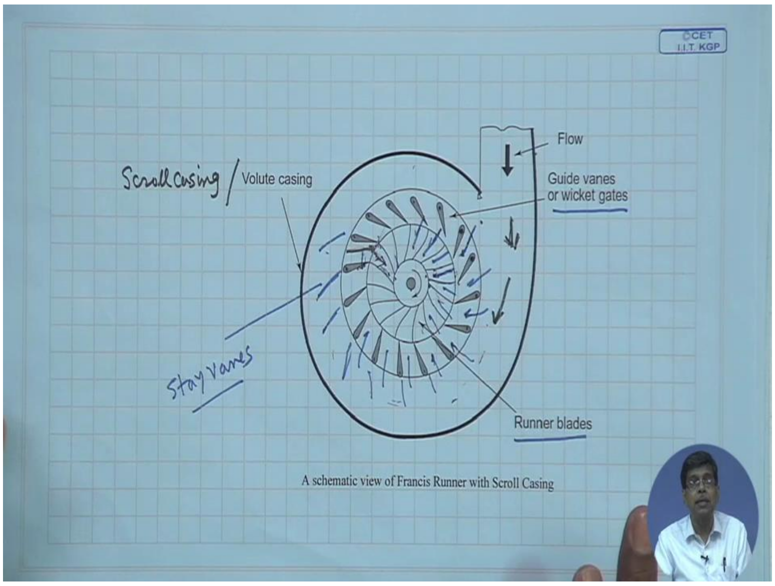
Fluid Machines.
Professor Sankar Kumar Som.
Department Of Mechanical Engineering.
Indian Institute Of Technology Kharagpur.
Lecture-12.
Analysis of Force Part II and Power Generation.

Good morning and welcome you all to this session of the course on fluid machines. Last class we started the discussion on Francis reaction turbine and we discussed the basic components in the Francis reaction turbine which is a typically radial inward flow turbine. We have recognised the guide vanes, stay vanes, the runner blades, the purpose of the draft tube and the different terminologies as far as that energy per unit weight, that is the head of the machine is constant, the head across the turbine, net head produced in the work or the gross head and how a draft tube increases the net head producing the, the turbine by keeping the turbine at a higher altitude from the tail rest level.

(Refer Slide Time: 1:38)



All these things we discussed. Now we will analyse the force on the one blade of a turbine and our expressions for power develop and the blade efficiency, hydraulic efficiency. So to start with I will again refer to this figure to know, though this is a very simple figure in a two-dimensional plane. Let us consider this figure, another turbine, the runner of the Francis turbine, you see these are the runners of the Francis turbine, these are the, the shape is little bit complex as compared to the runner of a Pelton wheel. Now see the fluid flows in this direction.

This is the direction of the fluid flow, the fluid enters like this and then flows within the blade passages. So at inlet, the flow is mainly have components in radial direction and tangential direction. But while it comes out of the runner, the flow becomes purely radial, I will be telling the reason behind it and at the same time the passage is a converging passage. And as the flow takes place through the runner, the tangential and radial flow ultimately is being converted to a radial flow and the flow area, the cross-sectional area of the flow converges.

And this is the height of the runner blade, so flow takes place like this. If we consider this as a horizontal plane, this is the vertical shaft, Francis runner, the flow takes place like this. So at the outlet of the runner, the flow becomes purely radial and then it takes a 90 degree turn and in the vertical plane, that is the parallel to the axis of the shaft, that is the axial direction it is bent, that means this takes a 90 degree bend and then it flow through the draft tube to the tail rest.

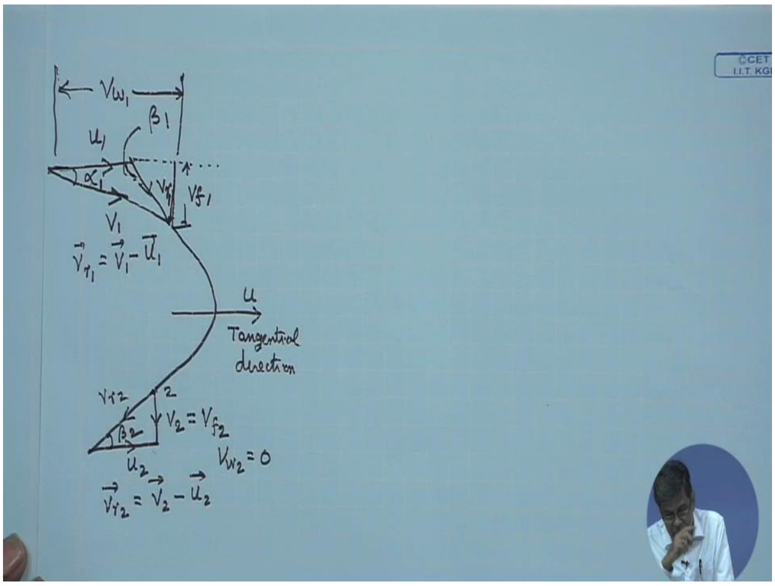
Now at the exit of the runner the flow is to be radial because of the 2 things, one is that the runner, as you know, the discharge from the runner is a waste through the, though we have a draft tube which reduces further the kinetic energy but kinetic energy at the runner also has to be kept minimum. At the same time we have seen that the cavitation not to occur, the velocity at the runner inlet if you see, at the runner outlet or inlet to the draft tube has to be low.

If you see the earlier equations, then you will see there is likelihood of cavitation if the velocity at the inlet of the draft tube or at the runner outlet is high. So therefore to keep the velocity as minimum as possible at the runner outlet, to keep the kinetic energy out at the outlet minimum and also to avoid cavitation we design the blade in such a way that the fluid velocity becomes minimum and that is, that can be achieved provided the velocity becomes purely radial.

That means the absolute velocity is in the direction of the radius becomes radial, there is no tangential component of the absolute velocity of the fluid at the runner outlet. So runner is designed this way. So with this concept in the background, let us draw a Francis runner blade, that means if this blade, this is the blade and this is the height for example, the height of the blade which also varies from inlet to outlet.

I will explain afterwards that this increases from inlet to outlet, the height of the blade, however if this is the height of the blade, that means if we take a cross-sectional view like this from the top if you see the blade by the flow enters at a higher radius, radial location and comes out at a lower radial location and if we take the sectional view from the top, a blade looks like this.

(Refer Slide Time: 5:16)



A blade looks like this, if you see a blade, this is the typical blade shape and this direction is the tangential direction, tangential direction, tangential direction, tangential direction. Okay. That means this is the direction of the rotor speed U. Now let me 1st draw the velocity triangle at the outlet. So the radial velocity will be gliding along this will be having the same angle as that of the blade at the outlet and as I have told, the outlet velocity will be purely radial, there will be no tangential component, this is V2.

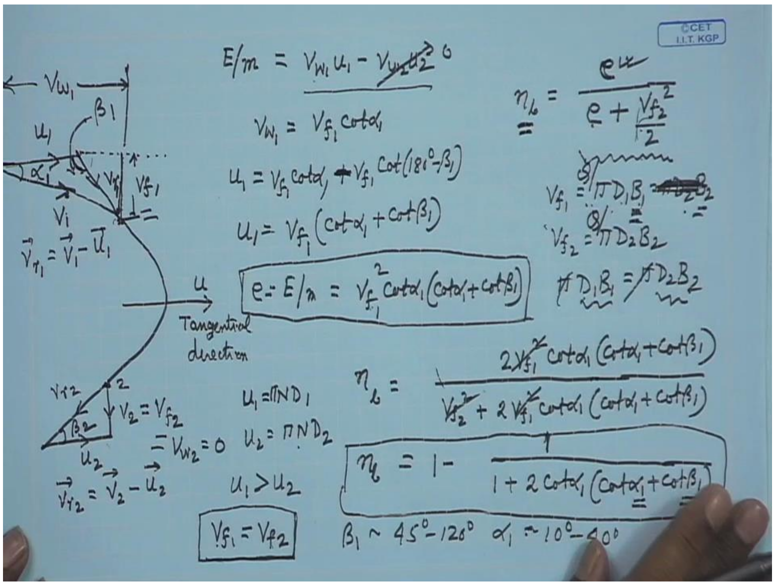
And if this is V2, so this is VR2 and this is U2. 2 is this section at the outlet. And this velocity wrangle I have told you earlier that VR2 is nothing but the difference between V2 and U 2. That means V2 is VR2 + U2. So VR2 class U2 gives the V2. And V2 is purely radial, it does not have any tangential component of velocity and it can be written as equal to V F2, this is the nomenclature we use for the radial component of flow velocity which is known as velocity of flow, V F2.

Now at the inlet, the diagram will be like this. This is the again coinciding here, another thing I will like to tell you that the angle the relative velocity makes with the tangential direction, let it be beta-2 which is the blade angle at the outlet. Similarly one can draw the velocity diagram at the inlet like this, this is U2, this is U1, this is VR1, rather I, okay, VR1 and this is V1. Okay here also if you write, you will see that VR1 is equal to V1 - U1 so that V1 U1 + VR1 is V1. U1 + VR1 is V1.

Now this angle we define or this angle whatever you call, if this angle is defined as the angle beta 1, that is the angle at the inlet of the runner at the inlet. That is the angle of the relative velocity at the inlet. And the angle which the absolute velocity at the inlet means the nomenclature usually give in Alpha, Alpha 1, where Alpha 2 is 0 in this case at the outlet. That is the velocity, absolute velocity makes with the tangential direction, this is the tangential direction is nothing but the angle of the guide blade.

Because the guide blade directs the fluid to the runner. So from the guide blade, the fluid velocity relative to the velocity means that guide blade or guide vanes are Static, so therefore this is that lead velocity, this is the angle of the guide vanes. And here we have a flow component of velocity, so VR1 this velocity is, this is VF 1 and this one is therefore, this one is therefore V W1, that means the tangential component of the inlet velocity V1. The outlet velocity tangential component is 0, here VW2 is 0.

(Refer Slide Time: 9:47)



Okay, this is the nomenclature for the velocity triangles or the velocity at a diagram at the inlet and outlet of a Francis turbine. Now here 2 things are very important, Francis turbine, inlet is having a velocity which has substantial tangential and radial component of velocity. As it flows through the turbine, the flow becomes, flow is mostly in tangential and radial direction but at the outlet the tangential component reduces to almost 0 and the flow comes in a purely radial direction, that is absolute velocity direction is radial, that is perpendicular to the tangential direction.

This has no component in the tangential direction. So this is the diagram. Now let us find out the value of head per, energy per unit mass, not head, head is energy per unit weight, necessarily I am not using this G, so energy per unit mass that is being delivered to this runner by the fluid as it passes through it. So this we know is given by, we use this Euler's turbine equation, VW2 U2.

Since in this case this is 0, this becomes V W1 U1. Here one thing you have to remember that you one is given by, what is U1, U1 is given by N D1 pie N D1, sorry one pie will be there and U2 will be given by pie ND2, okay. Since D1 is more than D2 and N is constant, U1 is always higher U2. So while drawing this vector diagram, one has to take care of that U1 should appear bigger than U2, that is just for a sense. Now therefore VW 1 U1, now VW 1 can be expressed, now it will be trigonometric, the fluid mechanics concept ends here.

Nor little bit trigonometry afterwards, now VW 1 is VF1 cot alpha-1, VF1 cot Alpha 1. And U1 can written as, that is V W1 - this one. VW 1 is VF1 cot alpha-1 that means this one - this one, this is VF1 cot this angle, 180 degree - beta-1. So if we denote this obtuse angle as beta-1, it will be + VFI cot 180 degree - beta-1 which is - - - sorry, which is - - - this will be +. So this will be cot alpha-1 + cot beta-1.

So this is U1, U1 can be expressed from this trigonometry triangle geometry, U1 like that and VW 1 like that. So therefore we can write E by M is equal to VF 1 square cot alpha-1 into cot alpha-1 + cot beta-1. Cot alpha-1 + cot beta-1, that is the energy per unit mass, this we can denote as small U, that is being delivered. That is VF1 square cot alpha-1 into cot alpha-1 + cot beta-1.

This is the head, not head, energy per unit mass, head is energy per unit weight delivered, VF1 square cot alpha-1 cot alpha-1 + cot beta-1. Now here we define a blade efficiency Eta B as this energy delivered. That means this energy delivered to the rotor divided by the input energy. Now here input energy is not the kinetic energy, that is the concept. Earlier in Pelton wheel the input energy to the runner or the rotor is the kinetic energy of the fluid because pressure throughout was the same.

That was the atmospheric pressure but here there is also a pressure energy or the static head which changes during the course of its flow. So therefore we cannot find out straightaway from that concept that V1 square by 2, so we have to know the pressure energy or pressure head on static head at the inlet to this runner. So this is rather expressed in this way, this is the energy per unit mass delivered + the energy which is lost here, that is V2. That means V2 square by 2.

And since V2 is equal to VF 2, I can write VF 2 square by 2. That means this is what, this is energy delivered to them and this is the energy lost or energy given out or discharged from the runner. So this energy is added to get the input energy, okay. So input energy has to be found from this equation. So if you do that, if you write that, and then if you just put these values you get Eta B equals to. Now if I write VF 2 square first + 2 VF1 square cot alpha-1, this one, cot alpha-1 + cot beta-1.

That means 2 E + VF square, numerator will be 2 E, that means 2 VF1 square cot alpha-1 cot alpha-1 + cot beta-1. Well, now another concept here we have to take that in this type of reaction turbine, Francis turbine, VF1, that is the flow velocity at inlet is equal to the VF 2. So the flow velocity at the inlet and outlet is made same. And they are made uniform, the flow velocity.

If you assume this, VF1 and VF 2 so that the change of momentum in the radial direction is 0, so to have this, the flow velocity in the radial direction, that means VF1 and VF 2 are kept same. So if we substitute this VF1 equal to VF 2 then this will be cancelled out and if you make a little rearrangement then we can write Eta B as 1 - 1 by, just a rearrangement, 1+2, simple rearrangement cot alpha-1 cot alpha-1 + cot beta-1. Well so this is the final expression of the blade velocity in terms of the guide when angle and the blade angle at the inlet 1 by, 1 - 1 by this.

So it is not necessary that you have to remember this formula at all no need, but you must know how the things are being derived and the concept that is used for defining this thing. Blade efficiency means the runner efficiency, the wheel efficiency, so here we use the energy delivered to the wheel from the Euler’s turbine equation or the runner divided by the energy input at the runner which is found by Output + the loss, loss in the form of kinetic energy at the outlet.

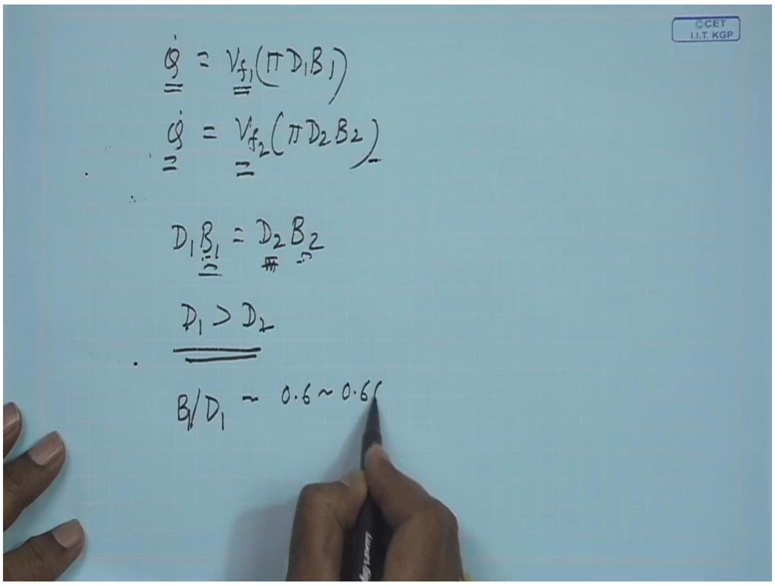
This way we find out this Eta because here we cannot simply use the kinetic energy at the inlet because with respect to the outlet the pressure energy or static head is higher at the inlet. So this change has to be taken into account along with the kinetic energy which is manifested through this. Here is only the concept, afterwards some trigonometric and algebraic manipulation gives you this value. Now usually the value of beta, that means beta-1, that means the angle of the relative velocity at the inlet or the blade angle at the inlet lies between 45 degree to 120 degree, the value of the guide vane is lying between 10 degree to 40 degree.

Now another important concept is that since VF1 is VF 2, now we can write VF1 is pie D1 B1 equals 2 pie D2 B2. Now what is B2? B1 and B2 are the width of the runner or the height of the runner, this is depending upon this configuration. For a vertical shaft, vertical shaft runner, this B is the height of the runner and pie DB represents the flow area. Let us see this way, that here this is a runner, let us consider this is a horizontal plane, this is the runner blade, so this height, this height is B.

So flow velocity, normal area to the flow velocity is pie D times this height. Okay, so D into B therefore in case of a turbine whose shaft is horizontal turbine is in a, turbine is in a vertical plane, this B is the width. So pie DB represents the low area, pie D1 B1 is the flow area, that is the area cross-section area normal to flow velocity at inlet and that at outlet. So VF1 is this, I am sorry, I am sorry and VF 2 is pie D2 be 2, 1st line which should not write this.

Since VF1 is VF 2, so pie D1 B1 is equal to pie D2 B2. So therefore D1 B1 is equal to D2 B2. Now since D1, diameter at the inlet is higher than D2, so B1 is smaller than B2. That means the width or height goes on increasing in the direction of flow, that means from inlet to outlet. That means if you just think the height of the runner, the height of the runner is small here and it goes on increasing at the outlet so that the flow velocity remains same. To make the flow velocity remain same, flow velocity is not this actually, it is Q dot by this.

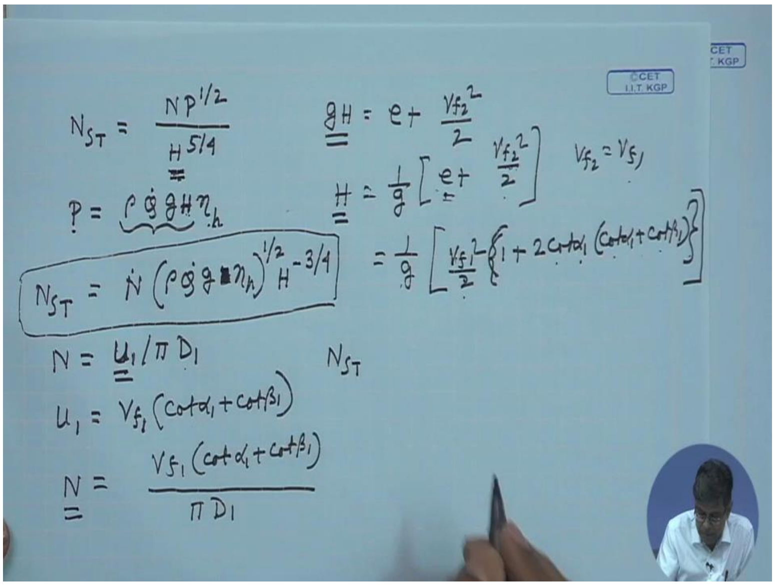
(Refer Slide Time: 21:21)



Q dot by this. I am sorry, Q dot by this pie D1 B1, I can write separately, that is why this problem is there. Q dot can be written as VF1 into pie D1 B1 separately, Q2, Q dot is same written as VF to buy D2 B2. Q dot is same because the flow is steady and if we consider VF1, VF 2 are same, then the conclusion is D1 B1 is D2 B2. That means the width or height, sorry not this one, increases from inlet to outlet because inlet D1 is greater than D2, it is radially inward flow.

That means the inlet diameter is more than the outlet diameter, so therefore inlet width or height is less than that at the outlet. So this is the concept for this geometry. And usually the value of B by D at the inlet, B1 by D1 is kept at between 0.6 to 0.66. Okay.

(Refer Slide Time: 22:28)



Now next we will see the, we will derive the expression of specific speed of such a runner. Similar way as we did for Pelton wheel, specific speed for the turbine, let us write as N P to the power half, H to the power 5 by4. Now P can be written as rho Q dot GH in terms of the hydraulic efficiency. So this is the head available, flow rate rho G and H is the head available, that means this is the total power available times the hydraulic efficiency is the power delivered. Okay.

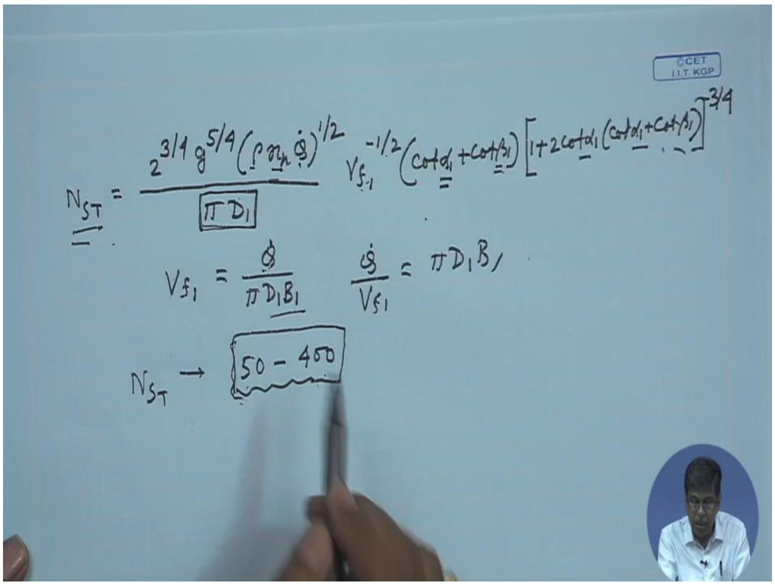
Total head available multiplied with the mass flow rate into hydraulic, the way we did earlier. So if we now put this there, then what we get, we get N P half, we get N rho Q dot GH Eta H half and H if you just cut, Eta H, this H and this H you write H half, H 5 by4, that means H -3 by 4. H to the power half - H to the power 5 by4 I write like this. Now if you write N as U1 by pie D1 where U1 is the blade velocity or runner velocity at inlet and D1 is the diameter at the inlet.

Again we have U1, we found out from the trigonometric relation VF1 cot alpha-1 + cot beta-1. Okay. So therefore we can write N as, just substituting this VF1 cot alpha-1 + cot beta-1 divided by pie D1. So this is the value of N. Now if we have, no before substituting this, we have to find out the value of H. Now we know little GH already E +, how do you know that because earlier we told that the energy per unit mass that is applied to the fluid or available to the turbine or available to the turbine by the fluid by definition this is the H, that is in terms of head.

So GH is E + VF square by 2, then H is equal to 1 by GE + VF 2 square by 2 and this becomes 1 by G, already we know these things, what is this VF1 square by 2, we can take common 1 by G VF1 square by 2 into 1+ 2 cot alpha-1 into cot alpha-1 + cot beta-1. This you can make a 2nd bracket and this is 3rd bracket. This is 1 by G, E E is also VF1 square by 2+, that is already we have taken, we have V F2 square is equal to, here we have used VF 2 is VF1.

So this we have taken common, 1+ E is VF square by 2 into cot alpha-1 into cot alpha-1 + cot beta-1, already we derived earlier. And VF 2 is equal to VF1, so take VF1 square common and then it becomes this. And if you put this value of H and this value of N, finally in this expression of N ST, then you get an expression like this. N ST is equal to, if you put this value, then you get an expression, I write here separately.

(Refer Slide Time: 27:13)



You see it, there is nothing great, if you do, you will get it, it is simple algebra. 2 to the power 3 by 4 , G to the power 5 by4, rho Eta H in terms of Q dot half divided by pie D1, VF1 to the power - half, cot alpha-1 + + cot beta-1 into 1+2 cot alpha-1 cot alpha-1 + cot beta-1 to the power -3 by 4. There is no need of remembering this formula but this is the way this has been derived, one must know. And VF1 is equal to Q by pie D1 B1, one can, this is VF1 Q half -, that means this can be taken together, whereas Q dot by VF1 to the power half.

And Q dot by VF1, Q dot by VF1 can be replaced as pie D1 B1 and another expression will come. So this way one can express this specific speed in terms of guide, inlet guide vane angles and inlet blade angles. Usually these things are known rho, the hydraulic efficiency of the turbine, for a given flow rate, this depends upon this diameter of the rotor and this alpha-1 beta-1. Diameter means diameter at the inlet, diameter of the rotor, this is diameter of the runner blade at the inlet alpha-1, alpha-1 and beta-1.

Now the value of the specific speed usually lies between 50 to 400. Again I tell you the meaning of this is the Francis turbine works most efficiently within this range of specific speed. And the values of alpha-1, beta-1 and the values of B by D at the inlet already has been told earlier. So with these values if you put here, the NST will come as 50 to 400 but if you choose a value of D1. Otherwise we, if we know the specific speed, we can find out the value of D1.

So this is one guiding equation for designing the turbine for a given specific speed. Actually specific speed we have to know for which we are going to design this turbine. Turbine is suitable for this specific speed which is much higher than that of the Pelton wheel. That means this is for a relatively lower head as compared to that of a Pelton wheel. Okay, thank you.