

Fluid Machines.
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Lecture-1.

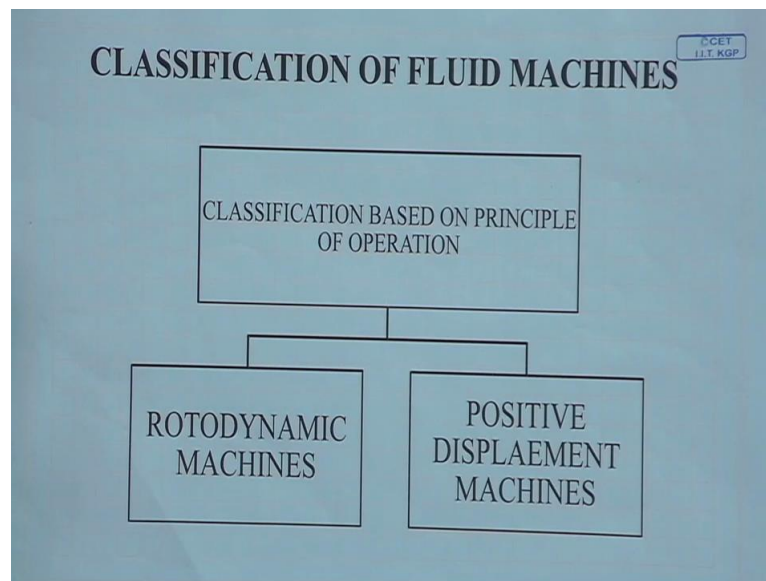
Definition Of Fluid Machines and Energy Transfer in Fluid Machines Part I.

Good morning and welcome you all to this first session of the course on fluid machines. Now as I have already mentioned in the brief introductory session of this course that fluid machine is a device that converts the stored energy of fluid into mechanical energy and vice versa. The stored energy of fluid usually appears in the form of pressure, velocity or temperature or inter molecular thermal energy by virtue of its temperature, while the mechanical energy is obtained by a rotating shaft.

Now the use of fluid machines, already I mentioned is very wide in the industrial applications. The major applications pertain to electric power generations, aircraft and rocket propulsions and in varieties of medium and small scale industries. In electric power generations as you know turbines are used which are the main important components or power producing component of the unit. For aircraft propulsion, compressors are used, centrifugal compressors and axial flow compressors.

Again for industries where high pressure air is required and this is used in almost all process industries, the centrifugal compressors are used. Similarly for transmission and distribution of water or liquid, circulation of liquid, pumps are incorporated in industrial applications and almost all industrial applications are involved in such applications or such operations of transmission and distribution and circulation of liquid. So therefore we have already appreciated starting from a very routine industrial application to a very high-tech industrial application, the fluid machines, turbines, pumps and compressors, they are used.

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Now, at the outset of this course, with this short introductory idea as the preface, I will start with the classification of fluid machines. Let us start with the classification of fluid machines. Now fluid machines are classified in various ways depending upon the different aspects. The 1st classification is based on the principle of operations and based on these principle of operation, we classify the machines into 2 groups, one is Rotodynamic machines and other is positive displacement machines.

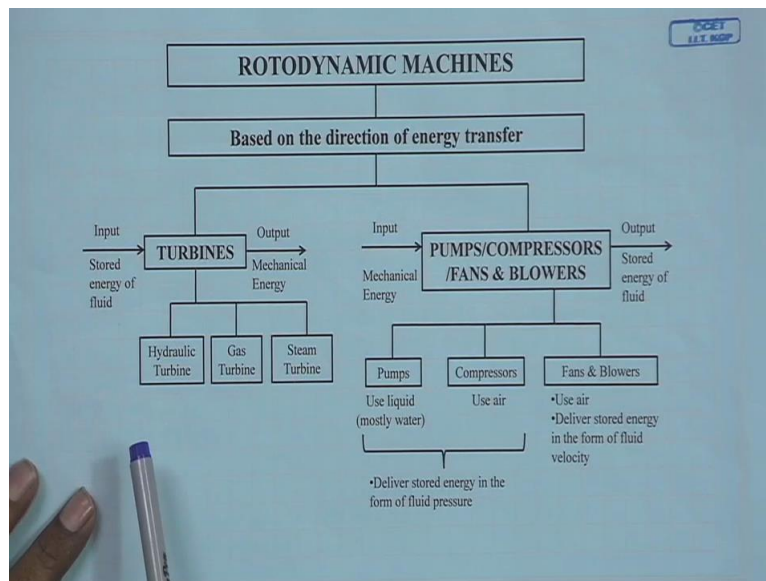
Now Rotodynamic machines are those machines where the energy conversion between the or energy transfer between the fluid and the machine takes place due to a rate of change of angular momentum of the fluid in course of its continuous flow through a number of blade passages comprising one or more moving rows of blades. That means in this category Rotodynamic machines, there should be or there has to be a continuous flow of fluid through blade passages and there has to be a relative motion between the fluid and the blade.

And this way we can tell that the principle of Rotodynamic is based on the principle of fluid dynamics. Whereas in a positive displacement machine, what is done is that some mass of fluid is taken as a closed system or and entrapped under some some locations of the machine and then by the physical displacement of one of the boundaries, its volume is changed and accordingly the pressure change is manifested. So, or the vice versa, pressure, high pressure is being converted to mechanical energy.

So in this case this energy interaction takes place by changing the volume of a given amount of or given mass of fluid as a closed system by a physical movement of the boundary, that is

why the name positive displacement machine has come or is given. This type of machine based on this type of principles are seen in the form of a reciprocating piston cylinder. That means this principle is utilised by executive reciprocating motion of the piston in a cylinder while entrapping certain amount of air or certain amount of water within it. And those machines are known as reciprocating engines or reciprocating pumps or compressors depending upon the direction of the energy transfer, that I will tell afterwards.

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So based on these 2 principles of operations, 2 different principle of operations we define one as Rotodynamic machines and another class as positive displacement machine. Now the Rotodynamic machines, we will discuss mostly the Rotodynamic machines in this course which is based on the fluid dynamical principle of rate of change of angular momentum of a continuous flow of fluid through rotating blade passages can be classified like this based on the direction of energy transfer.

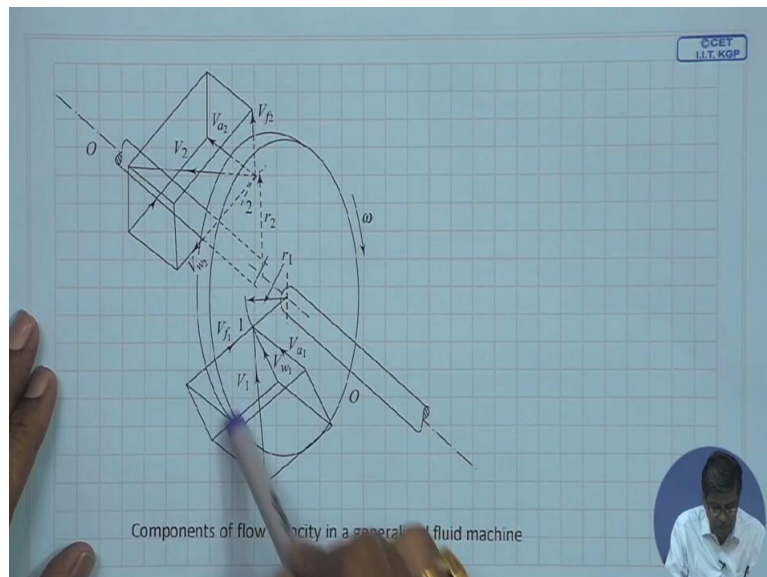
If you see the classification is like this, where the input energy is stored energy of the fluid and the output energy is the mechanical energy, those machines are known as turbines. And depending upon the types of fluids used, they are classified as hydraulic turbine which is water, gas turbines which is gas, which are generally the mixture of air and the products of combustion by burning the fossil fuel for this purpose and the steam turbines which use steam, that is used in thermal power stations.

So these are the turbines, hydraulic turbines, gas turbines and steam turbines operate mostly on the same principle but use different types of fluids. Now the fluid machines where the

input energy is mechanical energy, that means the machines which absorb mechanical energy or take mechanical energy as input and deliver stored energy of fluid as output are known as pumps, compressors, fans and blowers. Actually there is no generic name like turbines, so they are termed by different names and different names have meaning otherwise depending upon the different fluids.

For example pumps which use liquid, mostly water. Compressors that use air and in both the machines deliver stored energy in the form of fluid pressure. So high-pressure liquid and high-pressure air we get at the output from pumps and compressors, where fans and blowers, they use air but there the stored energy of fluid is in the form of velocity, so they deliver high velocity air, that means high flow of air, which comes from fans and blowers. So this is the preliminary classifications of fluid machines.

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Now we will develop or deduce a very simple expression of the energy transfer between the fluid rotor, and machine rotor, fluid machine's rotor and the fluid. So this thing we will do now, let us see this picture, now where we have shown the components of flow velocity in a generalised fluid machine. Now this is a generalised rotor or a rotor of a generalised fluid machines representative of a typical rotor of a fluid machine which is shown as a circular disk rotating with a constant angular velocity ω and this is mounted on this shaft.

Now here we consider a radial location $R1$ where the fluid enters to the machine, that is the inlet to the fluid. Now fluid enters at all points on the radial location $R1$, so $R1$ has the radial locations where the fluid takes, comes into the machine, that is the inlet of the fluid. Now

there are certain assumptions in this deduction. The first assumption is that we will consider the fluid which is entering into this machine at different radial locations, that means over the azimuthal locations at the radial location R_1 is uniform, that is uniform over the azimuthal location, over the periphery, that means the fluid inlet is uniform with azimuthal location.

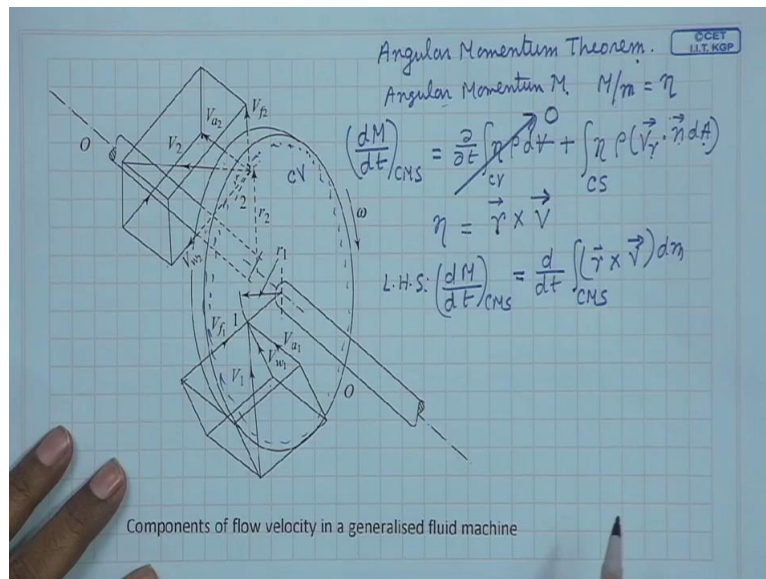
That means that any point here 1, the fluid velocity V_1 is the representative fluid velocity of inlet. Similar is the outlet where we define a outlet at a radial location 2 where the fluid velocity V_2 represents a, representative velocity of the outflow velocities since the outflow velocity along this periphery or along the, above the azimuthal location is uniform throughout. So this uniform flow velocity is one assumption. Next assumption is that flow is steady.

That means the mass flow rate across any cross-section at any radial location is same, there is no mass accumulation, no mass depletion within the rotor, the flow is in the at the steady state, no mass, no energy accumulation or depletion within the rotor. And another very important thing you have to know that here we will deduce the expression for the energy transfer under these assumptions with another basic consideration is that we are not going to solve any fluid mechanics or the fluid dynamics or the flow field for the fluid flow here.

That means given a prescribed inlet flow field, we are not finding out the outlet flow field. Here we will assume both the inlet and outlet, our flow field is given. So the solution of the flow of the fluid within the rotor is not of our concerns, that is very complex and that requires a CFD tool, so therefore that is beyond the scope of this course, this is usually done in an advanced course on fluid machines or Turbo machines.

So therefore we will accept the solution, the flow velocity at the outlet at a radial location of the rotor and this flow is uniform over the entire azimuthal direction, similarly at another radial location where a fluid enters into the machine is uniform over the entire azimuthal location. And at any point at the radial location, the inlet velocity is the representative of the velocity, all the velocities at all the points are representative of the mass flow at the inlet, similarly that at the outlet for outlet velocity, and we consider the flow to be steady.

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So with these assumptions now we will derive the equations for energy transfer. And that is based on the principles of rate of change of angular momentum, that is angular momentum theorem, that is known as angular momentum theorem. Angular momentum theorem. Now look what is the conservation of angular momentum? If you consider the conservation of angular momentum, therefore we can see that if we consider a control mass system, we can tell that the rate of change of angular momentum of the control mass system if the angular momentum is referenced with respect to an inertial frame of reference equals to the moment or torque exerted on that controlled mass system.

Now if we apply this same conservation principle to a control volume, because here in fluid machines we will apply this principle for a control volume considering rotor itself as a control volume. So now we know that when these conservation principles for any extensive property of a control mass system is applied to a control volume, there is a theorem, we invoke a theorem known as Reynolds transport theorem.

Which makes the link between the statement of conservation of such an extensive property for this change, for a control mass system to that it change for a control volume system, this property may be mass, maybe linear momentum, maybe angular momentum. Let us consider, at present we are dealing with the angular momentum.

For an angular of momentum, if we invoke the Reynolds transport theorem, it states like that if you recall that the Reynolds, according to this Reynolds transport theorem, the rate of change of angular momentum of a control mass system equals to the rate of increase of

angular momentum within a control volume + net rate of a flux of angular momentum from the control volume, that is control at its surface, that this control surface.

So this is the Reynolds transport theorem and this is valid for linear momentum, this is valid for fast also. So therefore we can write this, therefore this theorem like this and before that if we write the angular momentum as M , angular momentum we write angular momentum, angular momentum nomenclature as M and M per unit mass as η .

Then we can write the theorem, the Reynolds, according to Reynolds transport theorem that the rate of change of angular momentum for a control mass system equals to the rate of change of angular momentum, if this is η , angular momentum per unit mass $\eta \rho DV$ for a control volume + this is the rate of increase, time rate of change of angular momentum within the control volume DV is the elemental volume of the control volume, + the rate of change of angular, rate of, net rate of flux of angular momentum across the control surface.

What is this, η is the angular momentum per unit mass and this quantity represents the mass flux over a elemental area DA wear \mathbf{V}_R is the velocity vector relative to the control volume. Here definitely I have forgotten to tell you, we define the control volume, the fluid inscribing the rotor, within the rotor we take as control volume, okay. And \mathbf{V}_R is the velocity of the fluid is relative to the control volume, \mathbf{n} is the vector, unit vector directed radial, directed outward of the area A .

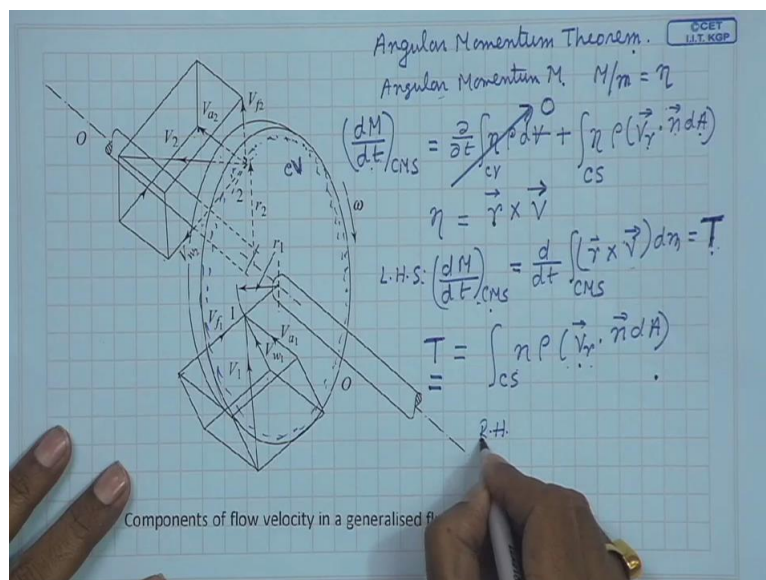
So this is used as the, the area is used as a vector by using this unit vector whose direction is ready, is outward, outward of the area DA , radially outward of the area DA and this is the velocity relative to the control volume. Okay. Now here η , this angular momentum per unit mass can be written as $\mathbf{R} \times \mathbf{V}^2$ where \mathbf{R} is the radial location or the positional vector of the point where we are defining the angular momentum per unit mass and \mathbf{V} is the velocity vector.

If this is referenced with respect to an inertial frame a static frame of reference, then we describe the angular momentum M with respect to a Cartesian, with respect to an inertial frame of reference. Now here you see \mathbf{V}_1 is the velocity at the inlet which has got 3 components, one is the axial direction V_{1a} , another is the radial direction V_{1r} , another is the tangential direction $V_{1\theta}$. Similarly this outlet velocity \mathbf{V}_2 can be resolved into 3 components, one is axial direction V_{2a} , another race radial direction V_{2r} , another is the tangential direction $V_{2\theta}$.

These are the typical nomenclature for the components at inlet and outlet, both the velocities at inlet and outlet can be resolved into 3 respective components axial directions, radial directions and the tangential direction. The radial direction of component is sometimes known as flow velocity. Now with this we see now next, if the flow is steady, this part is 0, that means rate of change of angular momentum within the control volume, the time rate of change, $\frac{dM}{dt}$ is 0 because there is no change within the control volume, that is the definition of a steady flow.

Under steady state, this becomes 0. Now we write the left-hand side, that is the $\frac{dM}{dt}$, that is the left-hand side $\frac{dM}{dt}$ control mass system with this can be written as, this is the $\frac{dM}{dt}$ control mass system can be written as $\frac{d}{dt} \int_{CV} \mathbf{r} \times \mathbf{V} dm$, that is the per-unit mass $\frac{dM}{dt}$ over the control mass system. This can be written in this way over the control mass system.

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And this can be written equal to the torque or moment exerted by the control mass system on the control mass system, exerted on the control mass system provided this is defined or this is referenced with respect to an inertial frame of reference. Okay so if we define the velocity with respect to an inertial frame or static frame of reference and accordingly the angular momentum, we can tell the rate of change of angular momentum for a control mass system. This can be expressed in this fashion, is equal to the torque or moment applied to the control mass system.

And the Reynolds transport theorem, therefore we can write this torque or moment applied, exerted on the control mass system which coincides the control volume at that instant. That

means this is the torque or moment exerted on the control volume itself, so then it becomes the control surface, that means the this quantity $\mathbf{V} \cdot \mathbf{N} \, dA$. That means this becomes the torque or moment exerted on the control volume, control volume is the fluid containing in the rotor, this is the control volume, this is the CV, control volume.

Now we can write the torque or moment exerted on this CV is equal to the net rate of angular momentum, a flux from this CD, this is 0 under steady state, net rate of angular momentum, a flux from the control volume. Now if we designate outflow and inflow area like R1 radial location inflow R2 radial location outflow, if we can, then the RHS can be written, sorry I will take another ...

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$$T = \int_{CS} \eta \rho (\vec{V}_r \cdot \vec{n} \, dA)$$

$$= \int_{A_{outflow}} \eta \rho (\vec{V}_r \cdot \vec{n} \, dA) + \int_{A_{inflow}} \eta \rho (\vec{V}_r \cdot \vec{n} \, dA)$$

$$\eta_{outflow} = r_2 V_{w2} \quad \eta_{inflow} = r_1 V_{w1}$$

$$T = r_2 V_{w2} \dot{m}_{outflow} - r_1 V_{w1} \dot{m}_{inflow}$$

$$\dot{m}_{outflow} = \dot{m}_{inflow} = \dot{m}$$

$$T = \dot{m} (r_2 V_{w2} - r_1 V_{w1})$$

Now you can see here that, can you see, yes. That RHS can be written, let me write this thing again. Now T is equal to, T is equal to $\eta \rho \mathbf{V} \cdot \mathbf{R} \cdot \mathbf{N} \, dA$. Now if we can identify the outflow and inflow area is separately, then we can write his as $\eta \rho \mathbf{V} \cdot \mathbf{R} \cdot \mathbf{N} \, dA$, $A_{outflow} + \eta \rho \mathbf{V} \cdot \mathbf{R} \cdot \mathbf{N} \, dA$, all nomenclature are known to you as I have told A_{inflow} , A_{inflow} . Now here you see since the outflow and inflow, both, let us consider outflow, the radial location is fixed at R2, at all outflow points and we consider that the tangential component of velocity which is perpendicular to the radial location, that is also constant at all point.

This is a very simple assumption based on which we are deducing this expression. So if that is the case, the 1st term becomes equal to, in that case before telling that we write η for example at outflow can be written as $r_2 V_{w2}$, similarly η at inflow can be written as $r_1 V_{w1}$. What is η , η is $\mathbf{R} \times \mathbf{V}$, that is the angular momentum per unit mass,

under this simple condition, simple case, this can be... So without going for any complex vector calculations, we can simply and this is the mass flow rate, this is the mass flow rate at outflow and the mass flow rate at inflow.

If η is constant, comes out of these, so integration of this will be the mass flow rate, so therefore torque can be written, the 1st one can be written as $R_2 V_{W2}$ into \dot{M} outflow because this quantity comes out, so this is the mass flow rate, similarly this is \dot{M} outflow and this is \dot{M} inflow, this quantity, only this quantity. So therefore this will be with a - sign, why - sign because the outflow and inflow finally comes with a - sign, why, because this is not product.

If you consider a control volume, let us have a area DA and let us has a velocity here V_2 , the outlet velocity, this is the normal direction, they are in the same direction. But whereas this DA , where the normal outward is like that, whether the velocity is in this direction. So therefore if you make this dot product, we will see automatically this term will come with a negative sign, so therefore this is the angular momentum outflow, rate of angular momentum outflow, this is rate of angular momentum inflow and with a - sign which will automatically come by this Convention if you make the vector operation in case of a very generalised three-dimensional analysis...

But here it is so simple that η outflow is $R_2 V_{W2}$ and η inflow is $R_1 V_{W1}$, we take it out and simply we just write his, this is \dot{M} outflow and \dot{M} inflow with a - sign. And since it is that steady-state, \dot{M} outflow equals to \dot{M} dot, \dot{M} dot outflow is equal to \dot{M} dot inflow is equal to \dot{M} dot. So therefore we can write T is equal to \dot{M} dot into $R_2 V_{W2} - R_1 V_{W1}$. So therefore we get that the torque, that is or moment applied on this control volume is equal to $\dot{M} R_2 V_{W2} - R_1$.

The nomenclature is that V_{W2} and V_{W1} are the tangential velocity component of the fluid, tangential velocity or the tangential component of the fluid velocity at the outlet, tangential component of the fluid velocity at the inlet, R_2 and R_1 are the radial locations at the outlet and inlet.

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Rate of Energy given to the fluid $\dot{E} = T\omega$

$$\dot{E} = m(r_2 v_{w2} - r_1 v_{w1}) \omega$$

$$\dot{E} = m(v_{w2} u_2 - v_{w1} u_1)$$

$$u_2 = r_2 \omega$$

$$u_1 = r_1 \omega$$

$$H = \frac{\dot{E}}{m g} = \frac{(v_{w2} u_2 - v_{w1} u_1)}{g}$$

$$H = \frac{(v_{w1} u_1 - v_{w2} u_2)}{g}$$

Turbines: $v_{w1} u_1 > v_{w2} u_2$
 Pump/compressors: $v_{w1} u_1 < v_{w2} u_2$

Euler's Equation of Fluid Machines

Now we know that the energy, rate of energy, rate of energy, rate of energy, okay, rate of energy exerted, rate of energy giving rather, giving to the fluid, to the fluid \dot{E} if I write is nothing but the torque or moment exerted on the fluid into the angular velocity. So rate of energy giving to the fluid will be equal to $M\omega$ into $R_2 V_{w2} - R_1 V_{w1}$ into the angular constant angular velocity of the rotor. This can be written as $M \dot{V}_{w2} U_2 - V_{w1} U_1$.

Where U_2 is $R_2 \omega$ and U_1 is $R_1 \omega$ and they represent the rotor velocity, the linear velocity of the rotor which is the radius time the angular speed, this is the at inlet, this is the velocity of the rotor, linear velocity of the rotor, at the outlet this is the linear velocity of the rotor. So therefore, they represent the linear velocities of the rotor. So $V_{w2} U_2 - V_{w1} U_1$. So the rate of energy giving to the fluid is can be written like this. Now usually we express this in terms of the energy per unit weight.

Now if we write the energy per unit weight giving to the fluid E , energy per unit weight given to the fluid E is equal to \dot{E} by $M \dot{G}$, then we can write it as simply an algebraic manipulation, energy per unit weight. Now in fluid mechanics, all these people use these energy per unit weight E which is given as head H , energy per unit weight is known as head. The energy per unit weight has a beauty that its dimension is linear dimension, that means in meter in SI units.

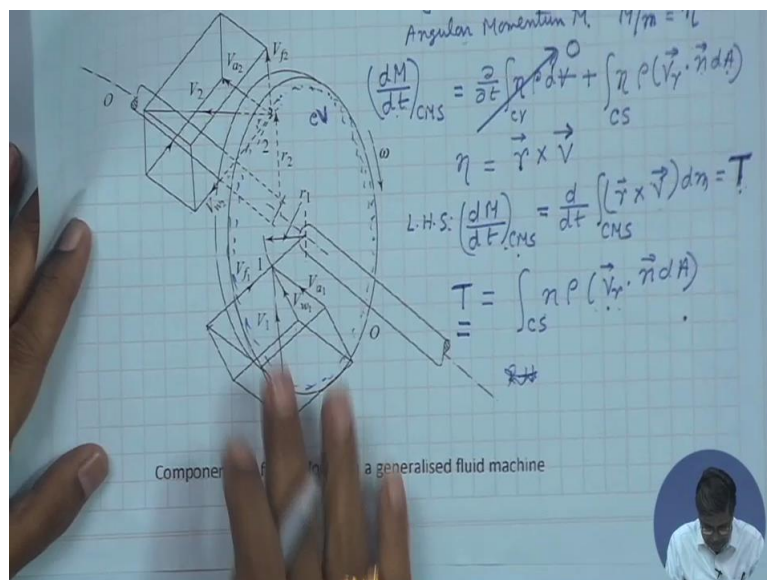
So therefore in hydraulics and in fluid mechanics we use the energy per unit weight as head, usually it is the terminology used in hydraulics, so here I will use in terms of that in fluid machine that had, that is energy per unit weight giving to the fluid is expressed as this. Now

there is a convention, similar to that of thermodynamics that when the energy, mechanical energy or work being delivered by a system or a control volume, it is taken as positive whether it is added to the system is taken as negative.

So here also in this similar convention that when the head, that is energy per unit weight is delivered by the fluid of the machine, that means delivered by the fluid to the machine, then it is considered as positive, while it is giving to the fluid by the machines is considered to be negative. With that consideration H is written as $VW_1 U_1 - VW_2 U_2$ by its sign convention, not from this formula, otherwise this will be $-H$, H is this but by sign convention we write H , that means the positive value of this means the head, that is energy per unit weight has been delivered by the fluid.

That means in case of turbines which deliver mechanical energy, which delivers work, mechanical work VW_2 that means for turbines I write, for turbines therefore $VW_1 U_1$ is greater than $VW_2 U_2$, okay. And for pumps or compressors, for pumps or compressors, just the reverse, $VW_1 U_1$ is less than $VW_2 U_2$, there are negative. So this is the usual sign convention we write that H , that is the head, energy per unit weight delivered by the fluid to the machine is given by this. And this equation is known as Euler's equation, Euler's LERS, Euler's equation of fluid machines.

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Rate of Energy given to the fluid $\dot{E} = T \omega$

$$\dot{E} = m (\tau_2 v_{w2} - \tau_1 v_{w1}) \omega$$

$$\dot{E} = m (v_{w2} u_2 - v_{w1} u_1)$$

$$u_2 = r_2 \omega$$

$$u_1 = r_1 \omega$$

$$H = \frac{\dot{E}}{m g} = \frac{(v_{w2} u_2 - v_{w1} u_1)}{g}$$

$$H = \frac{(v_{w1} u_1 - v_{w2} u_2)}{g}$$

Euler's Equation of Fluid Machines

Turbines: $v_{w1} u_1 > v_{w2} u_2$
 Pump/compressors: $v_{w1} u_1 < v_{w2} u_2$

Euler's, I think you have understood this thing, so with a very simple, in a very simple case with all these assumptions we have derived. So this is a very simple case, actual cases are different but this gives a guideline. But again I tell that the assumptions are like that the fluid the inlet is uniform over the entire azimuthal direction, similarly at the outlet flow is steady and we have the solutions of the flow field at the outlet for a given flow field at the inlet.

But this is at the same time is very generalised expression in a sense that it does not take care of the path taken by the fluid in the rotor and the way the density is varying, so this does not take care of that. So whether density varies or not, whether the fluid path is different or not, whatever path may be taken, this is the expression, under these assumptions as just now I have told, this is the for the head, that is energy per unit weight delivered by the fluid to the machine. $v_{w1} u_1 - v_{w2} u_2$ by G .

The nomenclatures are v_{w1} , v_{w2} at the tangential component of fluid velocity that inlet and outlet, u_1 and u_2 are respectively the rotor velocities at inlet and outlet and this equation is known as Euler's equation of fluid machines and this is the basic equation of energy transfer between the fluid and the rotor and we will reduce many other equations from these equations. These equations will be expressed in terms of different components of the fluid velocities which I will discuss in the next class, thank you.