Vibrations of Structures
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Lecture – 34
Special Problems in Plate Vibrations - I

We have been discussing about the vibrations of plates. So we have looked at the rectangular and the circular plates in our previous lectures. Today, we are going to discuss some special problems in vibrations of plates. So, before we do that now as, you have seen in the previous lectures that even for some simple problems the analytical solution becomes very complex and sometimes with increasing complexity of the geometry or the boundary conditions etc., the analytical solution becomes intractable.

In such situations, we would like to have approximate methods for discretizing the dynamics of plates and performing modal analysis which we obviously, expect them to be approximate. But then, we can; there is always a scope of improving the accuracy of these methods and we have seen such approximate methods in this course. Today, we are going to look at two examples based on which we are going to solve using the approximate method.

Before we start discussing the examples, let us look at the variational formulation of plate dynamics. Till now, we have not discussed this variational formulation. We have discussed only the Newtonian formulation of plate dynamics. Now, this variation formulation is little cumbersome for the plates. So let us see at least how it is formulated? But then if you go through the variation formulation of plate dynamics, then you will also see that the boundary conditions are obtained naturally in the process.

So, you can cross check, that the boundary conditions that we have discussed based on the Newtonian formulation and based on Born–von Karman's boundary condition for free edge; you can obtain them directly from the variation formulation. So let us a start with a little bit of discussion on the variation formulation for dynamics of plates.

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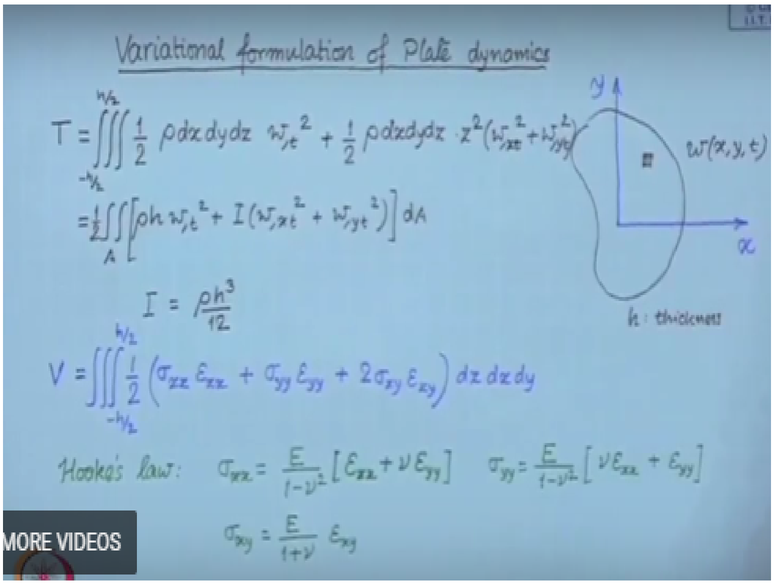

So, let us consider a plate. Now, here when we do the variational formulation, we first write down the kinetic energy of the plate. So, the kinetic energy is for this little element is given by one half, if rho is the density then dx dy dz, where z is the coordinate perpendicular to the plane of this figure that is plane of the paper; so, this is the mass of this little element times its transverse velocity's square.

Now, in addition, if you also want to consider the rotary inertia then you have in addition to this half rho; now this is the mass times z square; so mass times the distance from the neutral plane square; so that will give the moment of inertia of this little element at height z from the neutral axis times the angular velocity square; so now this angular velocity can be written as, in the y, it is written as del w del x del del t of del w del x whole square.

So this is the contribution from the rotary inertia and if you now integrate this over the complete volume of the plate, then we can obtain the total kinetic energy of the plate. Now, since this field variable, this field variable tracks the transverse displacement of the neutral plane of the plate, so this is independent of z; and if density is uniform etc., so with all those nice conditions, there is nothing in the integrant that is function of z.

So, z can be integrated out and written - So, this z goes from minus h by two to plus h by 2. So, this z coordinate goes from minus h by two to plus h by two, where h is the thickness of the plate. So, then if I perform that integral, I can simplify this expression; so is the integral over area now of the plate; that is the kinetic energy. Now, this I, which is the moment of inertia of -per unit area, so that is the moment of inertia per unit area of the plate.

Next, we will write down the potential energy expression for this little element. Now, we have considered that this little element is subjected to in-plane stresses. So if you have in-plane stresses then the strain energy stored because of deformation is given by half times the stress times the strain. Now, we have the normal stresses times the strain in the x plus normal stress in y times the strain in the y and plus two times, the in-plane shear stress and the corresponding shear strain of the element; now this is per unit volume.

Now, I have to again integrate. The thickness goes from minus h by two to h by two; and x, y go over the domain of the plate. Now, here I have to finally obtain these expressions in terms of the displacement field variable which is w; so to do that we first write down the constitutive relation which is Hooke's law. From Hooke's law, we can write; you have seen this before; so, the stress, normal stress in terms of the normal strains in x, similarly in y, and the shear stress in terms of the shear strain.

So, if you use these expressions in the potential energy x and of course, we also have to write the strain displacement relation which we have also written out before.

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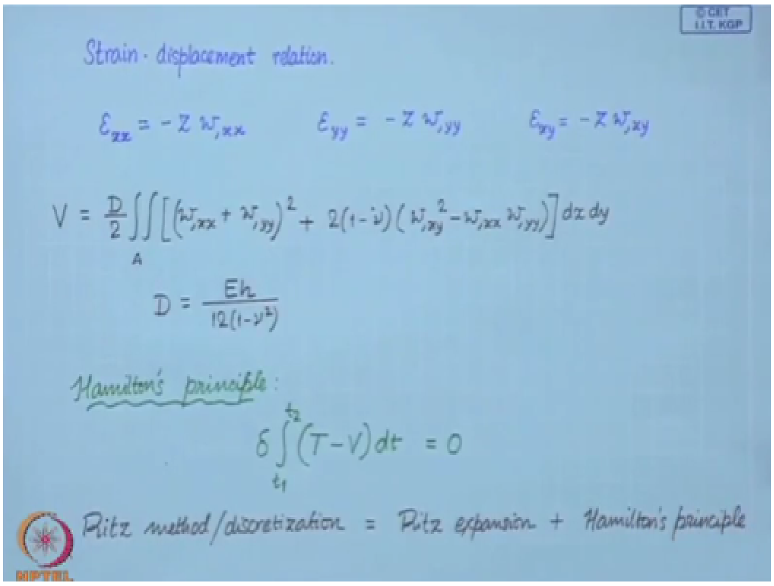

So, from the displacement kinematics, so we have seen these expressions before. Now, we will substitute these strains in terms of the displacement field variable in here and finally, the stress terms will be put here. So if you do that and make the final simplification, then upon integrating over the thickness direction, this is the expression of the potential energy, the strain potential energy, where this D is - here of course E is the Young's modulus and nu is the Poisson's ratio.

Now, with the obtained expressions of kinetic and potential energy, we can now write the - use the Hamilton's principle to derive the equation of motion. So Hamilton's principle says that, this variation must vanish. So we have these expressions of the kinetic and potential energy and if you want to derive the equation of motion, then you must follow the procedure that we have discussed in case of other structure elements like a strings, membranes etc.

This will closely follow that of the membrane; only the terms are more complicated or complex here and because of that the procedure is straight forward, but cumbersome. So, one can derive the equation of motion and the boundary conditions are also obtained along with this. So, that is the advantage of using the variational formulation. So, now, using this then we can also use the discretization using Ritz.

So, based on, based on this Hamilton's principle and Ritz expansion; so Ritz method requires, the Ritz expansion plus the Hamilton's principle; so these two lead us to the discretization of the dynamics.