Vibrations of Structures
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Lecture - 26
Vibrations of Rings and Arches – II

Next, let us consider the case n equal to one so now we are talking about non-axisymmetric modes of course because there is theta dependence as soon end is non-zero we have theta dependence.

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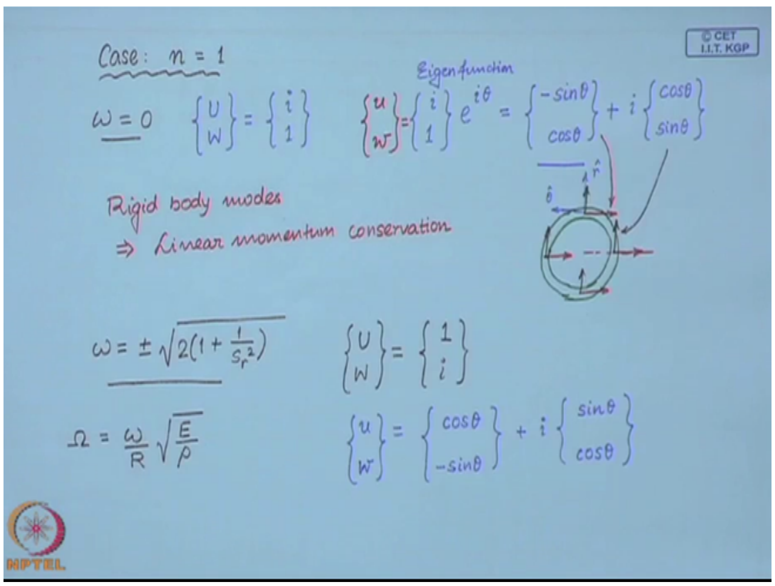

So in that case if you substitute n equal to one in the characteristic equation once again you will find that the first solution is omega equal to zero. So this is one solution for n equal to one, omega equal to zero is again a solution. So, that you can see directly from here once again. So if n is equal to one so this term drops out in the characteristic equation so once again omega equal to zero is a solution and then there is another solution which can be determined from here.

So, the first solution if omega equal to zero again we suspect that this is a rigid body mode which it is. So if you calculate once again the eigen vectors they turn out to be i and 1. So, let us see what this means. So this get multiplied the eigen functions are nothing but –so these are the eigen functions. And that can be written as –so if you consider this solution then let us see how is he information. So, this is a ring.

So this is the deflection of the center, the neutral fiber now sin theta with a negative so this implies that so this is the theta equal to zero (()) (3:44) So, this is u motion and this is the w motion. So u is zero here where as w is plus one and then you will find that at this point theta equal to pi by two. So this is minus one now minus one would mean because the axis here is like this.

So minus one would mean this and similarly you can find out that this represents the motion like this. Similarly, this eigen function vector will represent a motion like this. So these are nothing but the rigid body modes in the x direction and in the y direction. So, these are again rigid body modes which implies linear momentum conservation. Now let us consider the other solution for n equal to one we show that there are two solutions one was zero the other one turns out to be this.

So for this the eigen vectors happen to be given by this complex notation so this turns out to be following this notation so if you multiply this with exponential i theta so here we have so these the eigen function vectors. So, these are actually the non-dimensional frequency if you want to find out the dimensional frequencies they are given by –so these are the dimensional frequencies. So these are the eigen functions that we have for this mode. Now let us have a look at this mode.

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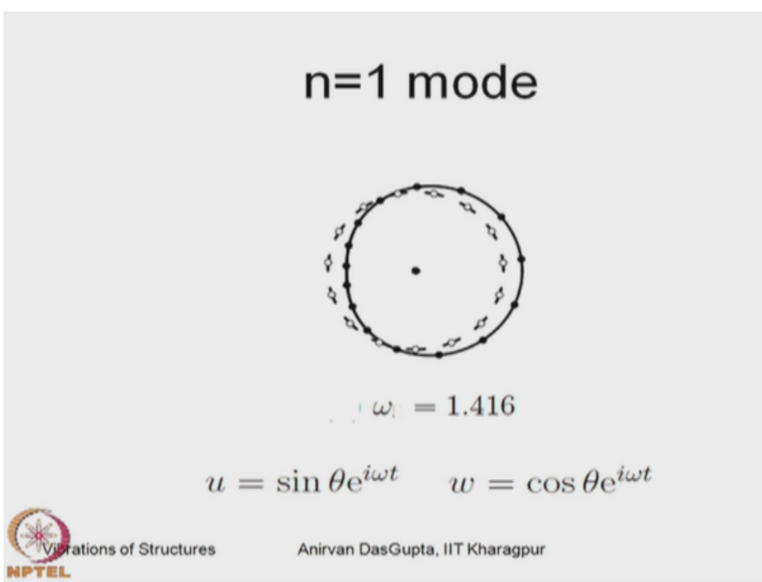

So, this shows the mode because of some expect ration problem it might seem a little like an eclipse. But actually this dash one actually a circle which is the undeform position of the ring. And this black one shows the mode corresponding to the imaginary part as you can see here. So, let us understand this motion before we look at this picture again.

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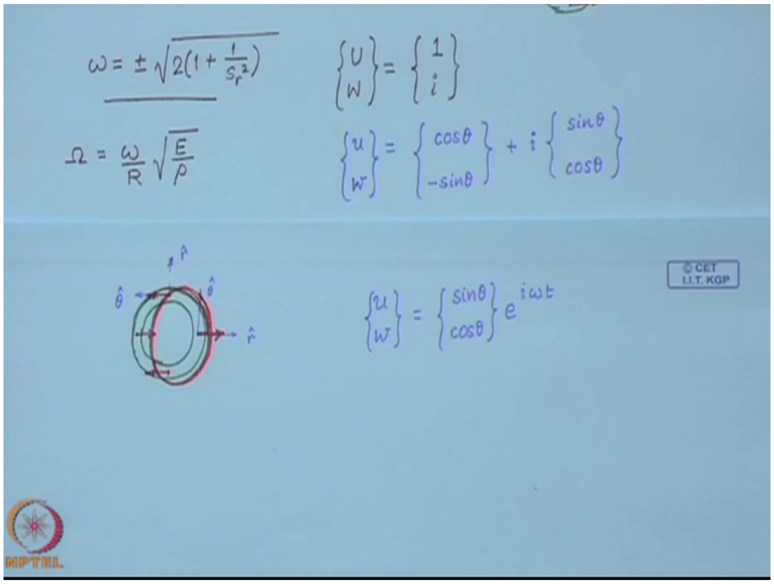

So, let us consider this imaginary part. So once again let me draw the ring. So if you consider this imaginary part of the solution so this as the eigen function so the motion can be written as – where omega is of course given by this eigen frequency circular eigen frequency. So in the circumferential direction you have this as sign theta. So which means here there is no circumferential motion but there is this radial motion.

Here you have circumferential motion as you increase so you have circumferential motion. Here the circumferential motion is the maximum it is positive plus one so in this direction when you come here this motion is purely radial. There is no circumferential motion at this point similarly you have circumferential motion in the negative direction here at this point and no radial motion here. So which means that you expect the ring to –so the center line of the ring will look like this.

It seems that the center at least the geometric center of the figure has shifted. I mean this is exaggerated figure of course. It seems that the geometric center has shifted but actually the center of gravity of the ring is not shifting because of combination of radial and circumferential motion. So, the particles are actually moving in this direction circumferential while radial motion is like this. So, we will look at this figure once again now.

So, here you can see this is the undeformed ring and I have drawn these empty circles to indicate the particles before deformation and these filled circles are these particles after or these material points after deformation. So you can see that there is a clustering of these material points here. So you have compression here and rarefaction here. So, that finally its momentum conserved mode.

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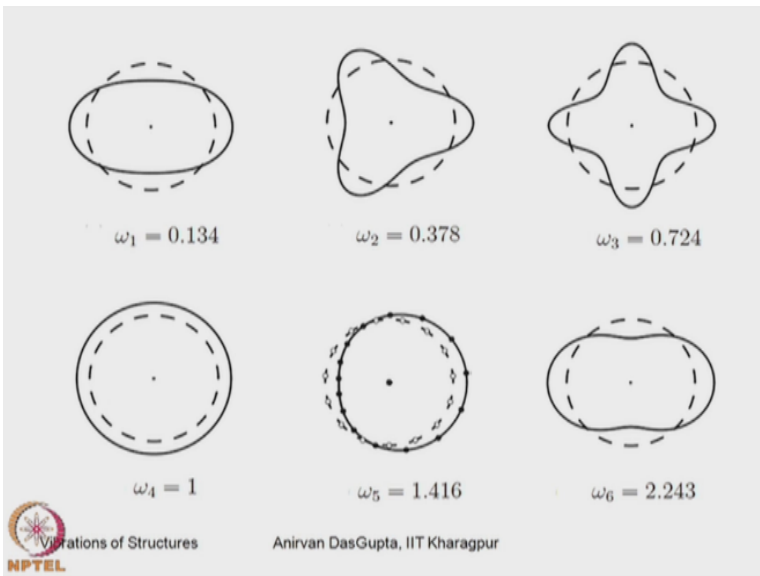

So, continuing this way you can then solve for higher modes but then let us see something interesting. Now, we have discussed as yet till now this mode this mode which is the breathing mode and this mode. But they are not the lowest modes. The lowest mode appears to be like this once again there is some aspect ratio problem. So this dashed one is the undeform ring and this is the deformed configuration.

So you see in this mode it is moving out here and moving in here and vice versa. So, this is going to oscillate like this. So there is a phase difference of between this and this which is pi. Here, you have the higher modes so you can see the circular eigen frequency non dimensional. This is the next higher and this is the fourth mode and this is the fifth mode and so on. So, you can calculate all the eigen frequencies of various modes and also plot the eigen functions.

Now, looking at these figures it might seem that these are nodal points but this has to be checked properly because now we have not only radial motion but also circumferential motion. So, if a node is considered to be a point at which there is no motion then this might not be nodes. For example, here this is definitely not a node. So, occurrence of actual nodes has to be checked by looking at motion of material points on the ring.

So, till now we have been discussing about vibrations of rings. Next, we will discuss vibrations of arches. So, circular arches.

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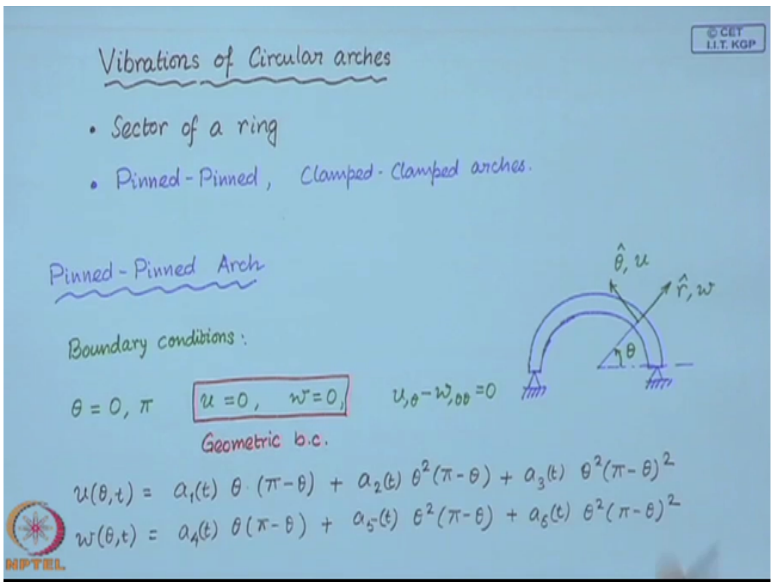

So, a circular arch is nothing but a sector of a ring. We will consider two examples of arches. One is the Pinned arch and clamped arches. Now, as you will realize that the equation of motion for arches, for the curve beams they are coupled and complicated. So what we are going to do for the case of arches is that we are going to solve this approximately using the Ritz procedures. So let us see what are the boundary conditions?

Because we need to choose admission functions for application of the Ritz method. So, let us look at first the Pinned-Pinned arch. So this is a schematic representation of a Pinned-Pinned semicircular arch. Now the boundary conditions, if you look back on the discussion we had on the boundary conditions of the curve beams then as theta equal to zero and theta equal to pi we must have u equal to zero which is the circumferential motion and w equal to zero.

And we also must have –so we have six boundary conditions as we had discussed. So, here we have zero displacement at these two points and this is the zero moment condition at these two ends. So, we have these as the geometric boundary conditions. So to choose now we have to choose admissible functions so we can admissible functions so we expand our field variables so the way I have chosen for this problem.

So, the first admissible function so this is zero at theta equal to zero and also zero at theta equal to pi. So, I can now construct –so I have taken a three term expansion for u and similarly three terms expansion for w using the same admissible functions. Now these expansions we substitute in the I the Lagrangian.

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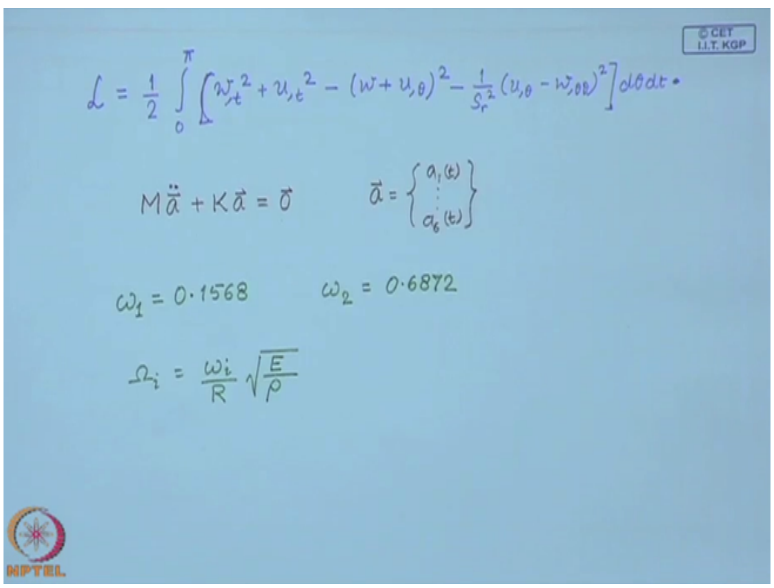

So, we have a semicircular arch so zero pi. So this is our Lagrangian so we substitute this expansion here and do the integration over theta and finally we obtain the discretized equations of motion in terms of these coordinates A. So, we will obtain these vector a is a 1 to a 6 and we can perform the model analysis and obtain the circular eigen frequencies. So for this arch Pinned-Pinned arch.

The none dimensional circular, the first two none dimensional circular eigen frequencies are obtained as. Now, from here you can calculate the dimensional circular eigen frequency.

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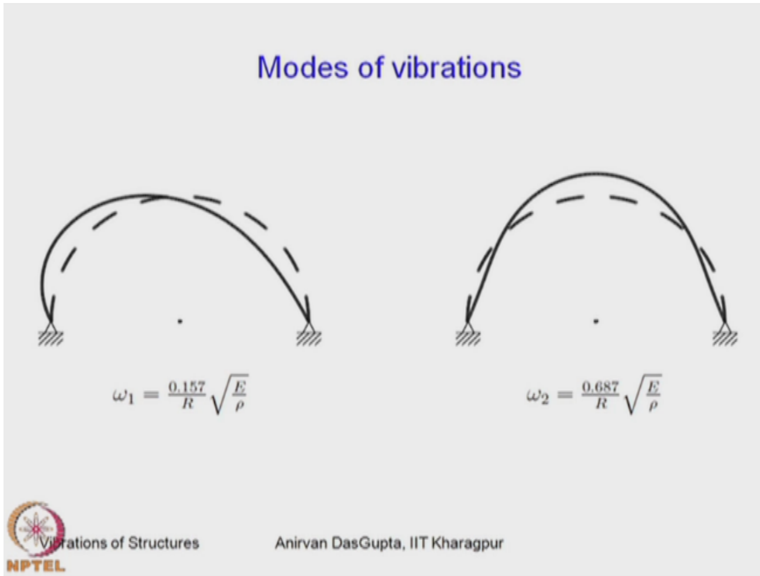

Now this figure shows the first two modes of vibration of the Pinned-Pinned arch. So you can see that so this is the asymmetric mode and this is the symmetric vibration mode which is having a higher frequency.

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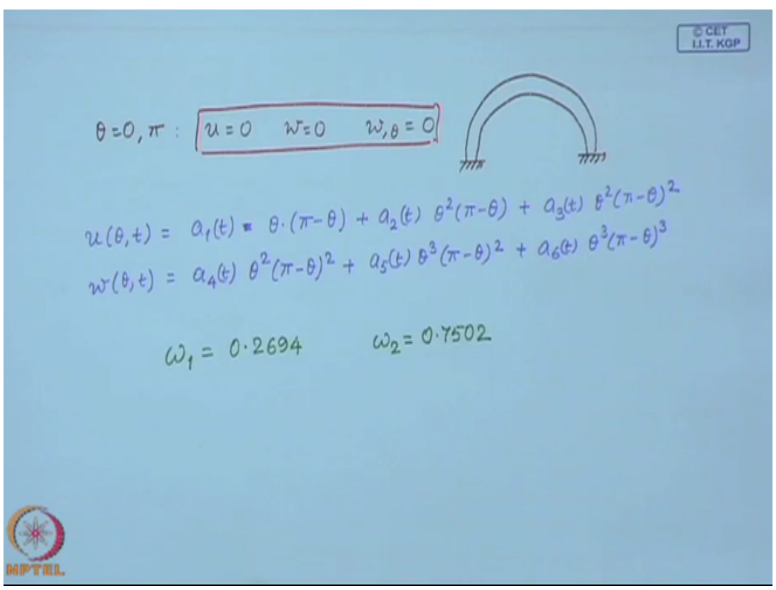

Now, in a similar manner you can perform for Clamped-Clamped arch here the boundary conditions at zero and pi happen to be like this. Now, here all the boundary conditions are geometric boundary conditions. So, in view of this we have this expansion. Now here because you have this slope condition as well so delta w, delta theta condition so the admissible function for w must be taken like this.

So, theta square pi minus theta whole square. So, if you once again substitute this in the expansion and calculate the discretized equations of motion and further calculate the eigen frequencies they turn out to be –so the none dimensional circular eigen frequencies appear as –so these are higher than the Pinned-Pinned case as we expect. So here this figure shows the modes of vibration again this first mode is the unsystematic mode.

And this is the symmetric mode with higher frequency. So, let us recapitulate what we have discussed today. We discussed the vibrations of rings and arches. So, we have looked at some interesting results in the vibrations of rings and we have considered two kinds of semicircular arches and using Ritz method we have determined the Eigen frequencies and modes of vibrations so with that I conclude this lecture.