Vibrations of Structures
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Lecture - 25
Vibrations of Rings and Arches – I

Today, we are going to discuss the vibrations of rings and arches. In the previous lecture we initiated some discussions on the dynamics of curve beams and we had discussed about beams with constant curvature which are in plane. So, before we look into the vibrations of rings and arches let us recapitulate briefly what we discussed in the previous lecture.

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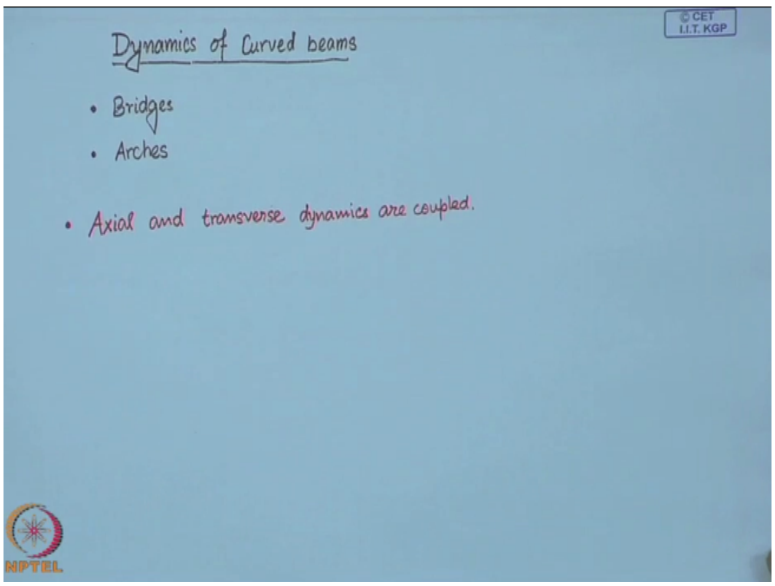

So we considered, so curve beams are found in various civil structures like bridges. They are used in arches and various other places. Now, we looked at the dynamics in the previous lecture on the dynamics of curve beams and we observed that the important aspect of the dynamics is that the axial and the transverse dynamics are coupled. So, they are coupled because of this curvature of the structure of the beams. So, we have made some simplifying assumptions when we formulated the dynamics we assume that the beam is still plane though it is curve in a plane.

We assumed that the deflection is much smaller than the thickness of the beam and the thickness in turn is much smaller than the curvature which was assumed to be a constant. And we also assumed that the other Bernoulli hypothesis holds which means that a cross section of the beam which was initially perpendicular to the neutral fiber remains perpendicular to the neutral fiber, remains flat and perpendicular to the neutral fiber even after deflection.

So, we neglected sheer which means that we considered that the beam is infinitely stiff in sheer. So, with such considerations in the previous lecture we have derived the equation of motion using the variational formulation.

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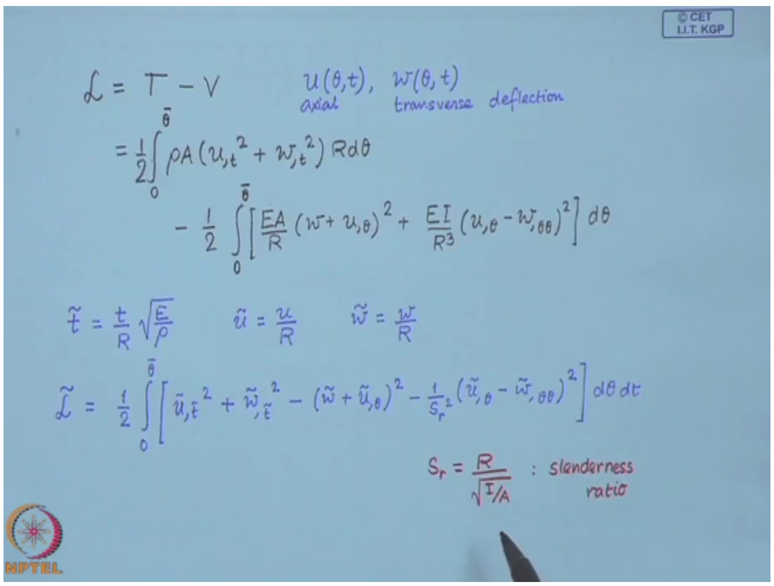

So, we considered the Lagrangian which we wrote as the kinetic energy minus the potential energy and the kinetic energy was one half of rho A. Here these are the field variables so u –so this is the field variable for the axial or circumferential motion deflection and this is the transverse deflection, W is for the transverse deflection. So, this and minus the potential energy we calculated as –so here the angle varies from say zero to whatever angle you have so the angular extent of the beam.

Now, in the previous lecture we also made a simplification based on certain redefinition. So, let us consider some non-dimensionalization. So, the time is none non-dimensionalization so t tilde is the non-dimensional time u is non-dimensionalization using the radius of curvature of the beam. Similarly, w was non-dimensionalize. Now using this non-dimensionalization we can rewrite this Lagrangian. So, this is our Lagrangian here, S r we had defined as the slenderness ratio.

So, this is slenderness ratio which tells us how slenderness the beam is. So, higher the value the more slenderness it is. So, it is the radius of curvature divided by the radius of gyration of cross section about the neutral axis. So, with this Lagrangian we derive the equation of motion.

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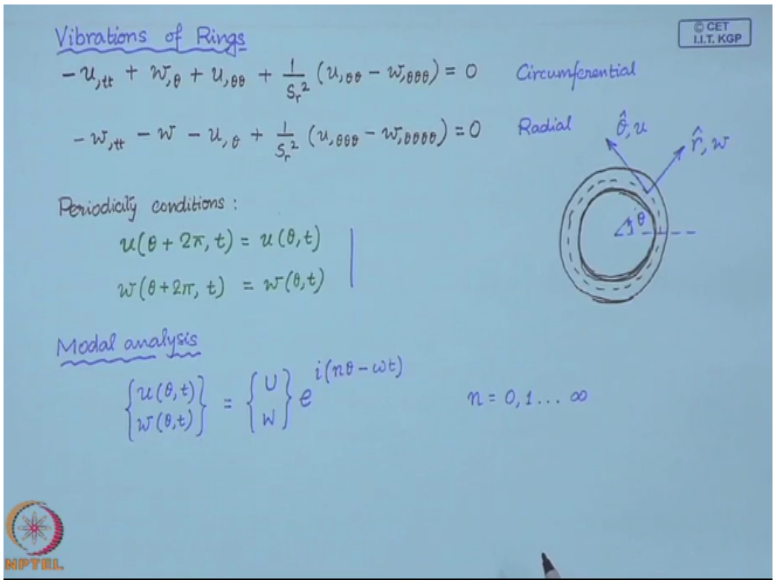

So, this was the equation corresponding to u the circumferential motion and corresponding to the transfer motion. We have these two equations. So, today we are going to first discuss the vibrations of rings. So, the equation of motion so this is the circumferential motion and this is the radial or the transfer motion. So, we imagine that we have a uniform ring. So this is the radial direction and this is the circumferential, tangential direction.

This is the angle theta now the boundary conditions we discussed the boundary condition for the complete ring like this. So, it turns out to be periodicity condition on the field variables. Now, we are going to perform the modal analysis of this system so we search for solutions with the structure. So, we are interested in solutions with this separable structure. Now you see that this is a function of theta and t now it must be periodic in theta.

So, we must have solution of the form like this. Where n can take value zero, one, two etcetera. So, this is to inforce the periodicity conditions. So, satisfy the periodicity conditions that we have written here. So, we search for solutions of this form now here if n is zero then as you can see it becomes independent of theta which means then we are talking about axisymmetric modes. So, modes which are independent of theta. For non-zero value of n we have non-axisymmetric modes.

So, let us see what happens when we consider a solution like this so we substitute the solution in the equations of motion.

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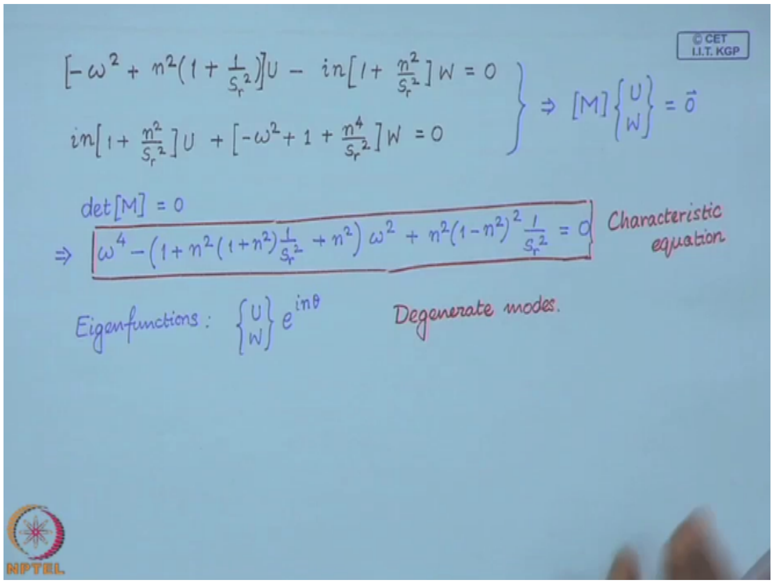

So, if you do that then it can check that upon simplification of these equations. So, this is the first equation. The second equation reads. So, these are the two equations that we obtained by substituting the modal solution found in the equation of motion. Now, for non-trivial solution of u and W this capital U and capital W we must have the determinant of this matrix. So, we can write this as matrix. So, determinant of M must vanish.

So, for non-trivial solutions of U and W. And that leads us to the characteristic equation which can be obtained easily. So, this is our characteristic equation. Now, we have to solve for omega from this equation substitute in these two equations and then solve for these eigen vectors U and W and then we will obtain the eigen functions. So, you see the eigen functions will be complex like this. So U and W themselves can be complex because you have this i here in these equations.

So, U and W, are themselves complex and hence these eigen functions are all complex. Now, we have discussed already that when you have complex eigen functions both the real and the imaginary parts of this can be the eigen functions. So what we can conclude is that for a give eigen frequency we can have more than one eigen function. This is called degeneracy. So, we have multiple eigen functions for a given eigen frequency.

Now, let us see, solve this characteristic equation and try to find out the eigen frequencies and the eigen functions which will characterize the mode of vibration. So, we start with the value n equal to zero. Let us consider

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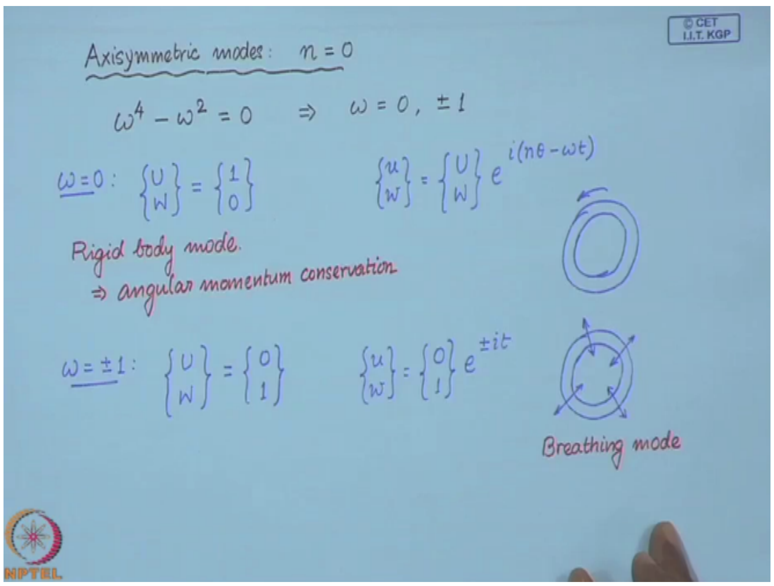

Axisymmetric so n is equal to zero. So if n is equal to zero then you can see straight from here. So, n being zero. So, this is the characteristic equation for axisymmetric modes. So that would imply so you have omega equal to zero and omega equal to plus or minus one. So, let us first look at omega equal to zero. So the eigen vector responding to omega equal to zero turns out to be if you solve the matrix equation then U and W turns out to be one and zero.

Now, this means that see the solution was –so n is zero, omega is also zero. So this term is absent and so the motion is –this is one and this is zero. So, which means it is a motion along the circumferential direction of the ring with zero frequency. So, this is nothing but a rigid body mode. Which implies that angular momentum is conserved. So, this is not a vibratory mode. So, next let us look at the other solution which is omega equal to plus or minus one.

So, in this case if you solve the eigen vector that turns out to be zero and one. So, now there is no motion in the circumferential direction. So here n equal to zero but omega is plus or minus one which means it is in oscillatory mode. So, it will be an oscillatory mode which is only in the radial direction. So, you have something known as a breathing mode. This is sometime known as breathing mode. So, the ring expands and contracts axisymmetrically. So this is a breathing mode.