Vibrations of Structures
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Lecture – 16
Damping in Structures – II

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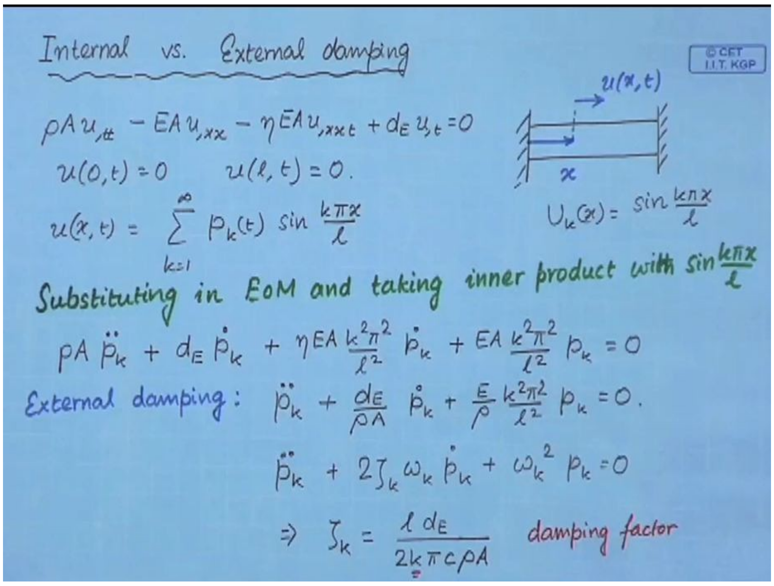

Let us delineate the roles of this internal and external damping term, so let us see how they are actually a little different. So for this, let us consider so we are going to look at the difference in roles of this internal and external damping. So to understand this let us consider this simple example of fixed-fixed bar in actual vibration. So the equation of motion is this. The boundary conditions are given by this.

So therefore I can expand as you know that these are Eigen functions of this fixed-fixed bar. So when you substitute this expansion and take the inner product so we substitute this expansion in the solution and take inner product with sin k pi x over l and that filters out the k-th quotient in the expansion. So what we are going to get, this term is for the external damping. This term is for the internal.

So this is the external and that is the internal damping and this is what we are going to get. Now let us look at this equation one by one, so suppose that there is no internal damping. In that case so when we have only external damping I can write this as which can be written as and if you compare then zeta k can be written as this. So you see that this k appears in the denominator of this damping factor.

So, which means that for high values of k this damping factor is actually very low which tells us that external damping is inefficient in damping out higher modes? So, higher modes are damped less compared to the lower modes.

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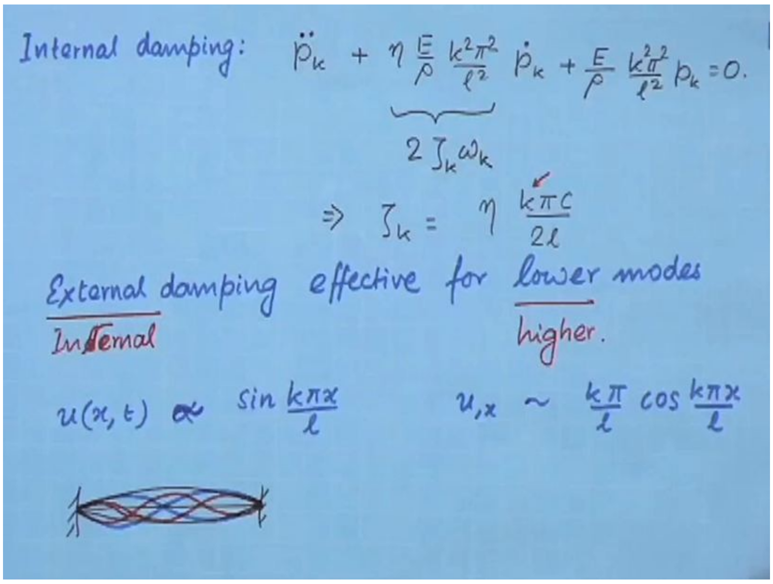

We will look at the reason for this now let us consider the internal damping case, in that case I can write and once again if you represent this term as two zeta k omega k then now you see that k appears in the numerator. Therefore, for higher modes, higher values of k, zeta k is high goes higher so which means higher modes have better damping because of internal dissipation. So internal damping is more effective for higher modes while external damping is more effective for lower modes.

So, external damping is effective for lower modes while internal damping is effective for higher modes.

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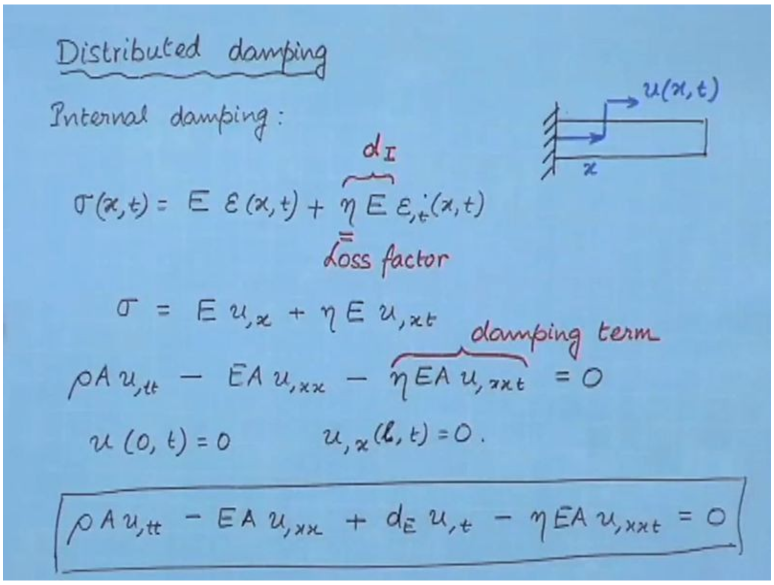

Now to understand this, you have to look at the expression of, say for example the internal damping. So we have seen already that this internal damping, the way we have introduced so it is strain rate dependent on strain rate. Now as you go to higher modes, and if you consider the solution, then del u del x, so rate of change of del u del x with time.

So if you see this solution is say for example for this bar you have terms for expansion of u like this. So del u del x so higher the value of k higher is the contribution from this strain rate term. So for this reason higher the mode, higher will be the effectiveness of the internal damping. On the other hand, if you consider this external damping, if you look at say for example, the first mode. So the first mode of the bar or the string looks like this, the second mode looks like this, the third mode looks like this.

So higher the mode more will be the nodes and therefore the effective motion of the system in the ambient fluid will be lesser and lesser. So, the effective damping because of external damping reduces as you go to higher modes.

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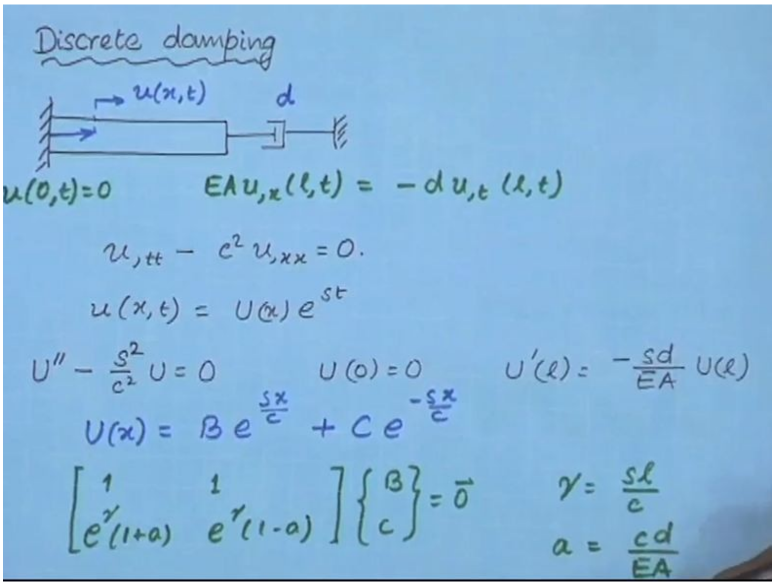

Now, let us look at discrete damping. As I have already mentioned this discrete damping occurs when we attach for example, an external dashpot to a continuous system at a particular point, say for example, Stockbridge damper is typically used in high tension cables, so that is attached to a particular point on the cable. And that damps the vibrational energy of the high tension cables so this is discrete damping or lumped damping.

Other than that you can have lumped damping when you have connections or joints, say for example when you have riveted connections between two structural elements, in that case damping is localized at the connection or at the joint. So for such cases what you have is the discrete damping. So let us look at an example, this is a bar in actual motion, and here we have a dashpot attached to this end of the bar. So I can write the boundary conditions of this problem.

So this is a fixed end and at this end I have the force because of the damper. The equation of motion is like this. Let us search a solution of the form like this so we have discrete damper at this end and we have introduced this damping not in the equation but in the boundary condition which we can always do. Now once you substitute this solution form in the equation of motion, we obtain an Eigen value problem.

The solution of this Eigen value problem can be written in this form. When we use the boundary conditions, so these two boundary conditions can be written in this form where gamma is s times l over c and this a. Here the small c is of course the speed of actual waves in the bar. So we have to solve this system the solution for nontrivial solution of b and c.

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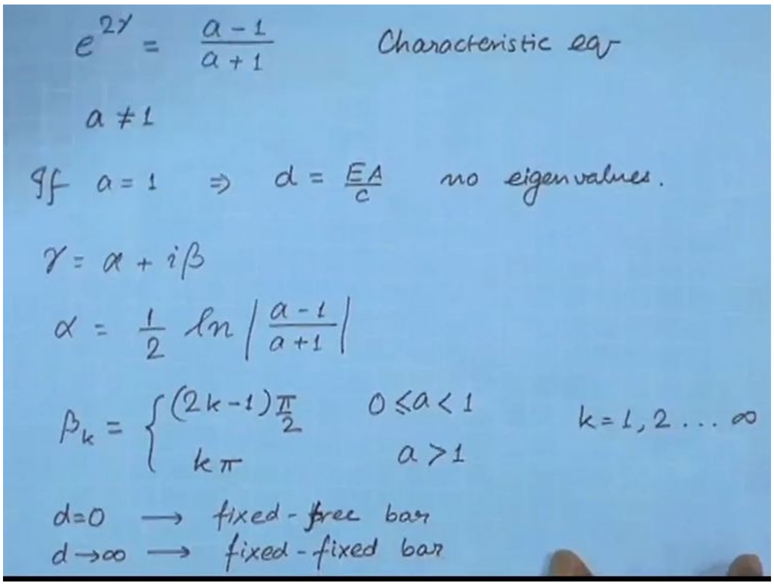

So for nontrivial solutions of b and c, the determinant of this matrix must be zero which leads us to as you can very easily find out that this is the equation which is the characteristic equation. And which can be solved for gamma when a is not equal to 1. When a is equals to one, so this can be solved when a is not equal to one. If a is equals to one which implies that the damping so the adjustable parameter is the damping.

There are no Eigen values for this problem when the damping is tuned to this value. No Eigen values would imply that there are no solutions, but then we have considered solutions of this very special form. So there are no solutions of this very special form, but then there are solutions which we have to understand in terms of wave propagation. Now that we will keep for a later discussions right now let us solve when a is not equal to 1.

And for solving that we can consider gamma to be a complex number of this form. And then alpha plus i beta so alpha can easily be solved, as log natural of and beta is index now that is so there are infinitely many values of beta which are now indexed with k and for various ranges of values of a we have these solutions and you can check, if you put d as zero then you can get the Eigen values of fixed free bar.

If you put d as infinity, then it is fixed-fixed bar that can be checked from the solutions of gamma that we have obtained.

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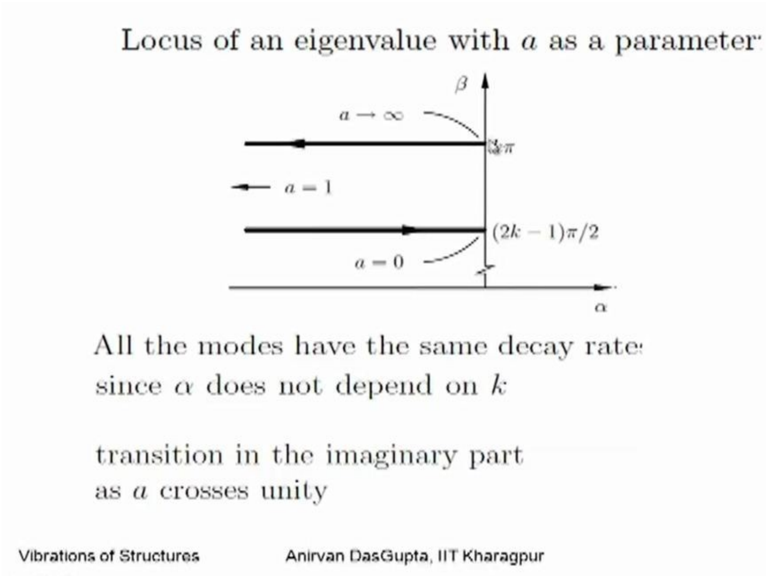

Now, here in this slide, I have plotted the variations of alpha and beta, so the solutions of gamma, the real part and the imaginary part. As a varies so when a is zero, we have a solution here. So a will be zero when the damping is zero which means it is a fixed free bar. While a is infinity when d is infinity so this is a fixed-fixed bar. And then a goes to one this goes to infinity, as we have already seen.

So this is the locus of the solution as a varies. So as you can see that as all modes have the same decay rate as alpha does not change or does not depend on k so all modes will have the same decay rate. There is a jump from this value to this value as a crosses one. So we have considered one case of bar with boundary damping.

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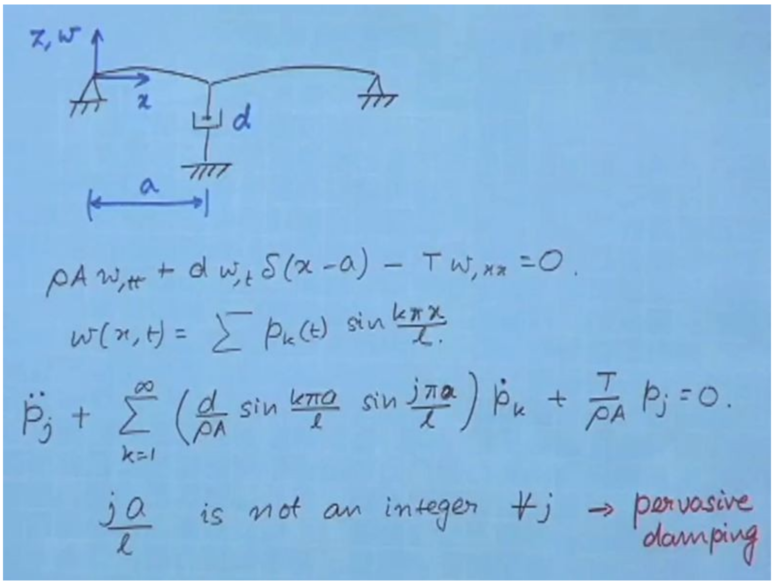

Let us look at another example of a string with concentrated damper or a lumped damper at x equal to a. So in this case you can write down the equation in this form. If you use the modal expansion substitute in the equation and take inner product with j-th Eigen function. We obtain this. Now if you consider that this j times a over l is not an integer. So if you can adjust this a, such that this is not an integer for all j then you will find that all modes are damped whereas if that this does not happen then some of the modes will not be damped.

So for this situation, when this is not an integer, what we have is known as pervasive damping. So when the damping is pervasive, all modes will be damped whereas if you put this damper at the middle of the string you will find that the second mode is never damped. So the second mode, the fourth mode, these modes will not be damped. So the damping is done non-pervasive.

So, in today's lecture what we have studied is this we have introduced the damping and we have seen two forms of damping, internal and external. We have studied distributed damping and lumped or concentrated damping. We have delineated the effects of internal and external damping. So, with that we conclude this lecture.