Vibrations of Structures
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Lecture – 15
Damping in Structures - I

When you consider a structure, a real life structure and if you give it a certain disturbance or some initial conditions so what you observe is the structure response to this disturbance to this initial condition and slowly after sometime the structure stops vibrating. Now in our discussions on initial conditions in this course and in the animations that I have shown once given the disturbance or the initial condition you have seen that the structure is continuously vibrating.

But that is not observed in nature. So we conclude that there is a mechanism inside these structures or because of the interaction of the structure with external world. There is a mechanism which drains out the energy of the structure. So this mechanism of dissipation of mechanical energy is known as damping and in this lecture we are going to look at this mechanism and we are going model put these damping terms in our model so that our model looks more realistic.

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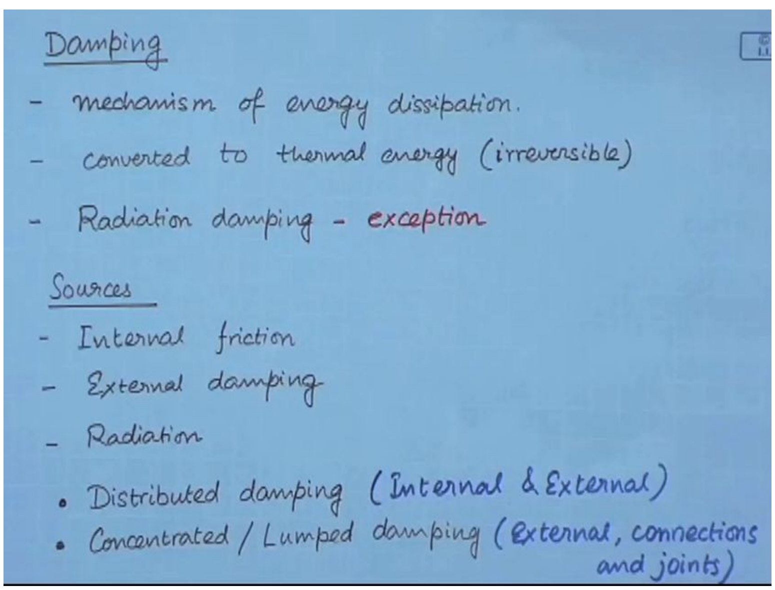

Now there are various kinds of damping. So first what I have said this damping is a mechanism of energy dissipation. Now what's happens to this energy that is dissipated? It gets converted to thermal energy and this is an irreversible process. This conversion of this mechanical energy to thermal energy in these cases is an irreversible process. Now most of the damping mechanism they work on this but there is an exception which is known as radiation damping.

So here this is an exception. This damping mechanism this draining of energy of the structure by radiation damping takes place not by conversion to thermal energy but by radiation of the energy in form of sound in the fluid medium in which the structure is placed. For example, air or in water or in any other liquid or sometimes even from the support points of the structure so wherever the structure is supported from no support is ideal that means no support is rigid.

So there will be some flexibility and continuously energy gets radiated out at the support points. So we have primarily this energy dissipation by conversion to thermal energy but there is another mechanism in which it might also be converted or radiated out of the structure. Now what are the sources of damping? So the first is known as the internal damping or internal friction. So whenever structure is vibrating between the layers of the structure molecular layers.

There is differential because of differential straining this layers they have differential motion that dissipates energy. Now one thing to note is we are considering one dimensional structures now there is no question of any layer in a one dimensional structure but then we are going to have a phenomenological model based on the rates of stretching for example of the structure the rate of strain so strain rate so model internal friction or internal dissipation.

The second source of dissipation is external damping so the structure might be interacting with an external fluid so that provides damping. The third is of course the radiation of energy so this is radiation damping. Now we can have distributed damping or concentrated or lumped damping. Now these internal dissipation and external damping because of fluid they are distributed damping while concentrated or lumped damping occurs in this can occur in external damping.

When we attach a lumped dash part or damper at a particular point on a structure so this is typically occurs when we have that is a Stockbridge damper in high tension cables. So that is the concentrated or lumped damping. This is an external damping and it can happen at connections and joins. So we can have concentrated or lumped damping at connections and joins say for example riveted connections etc.

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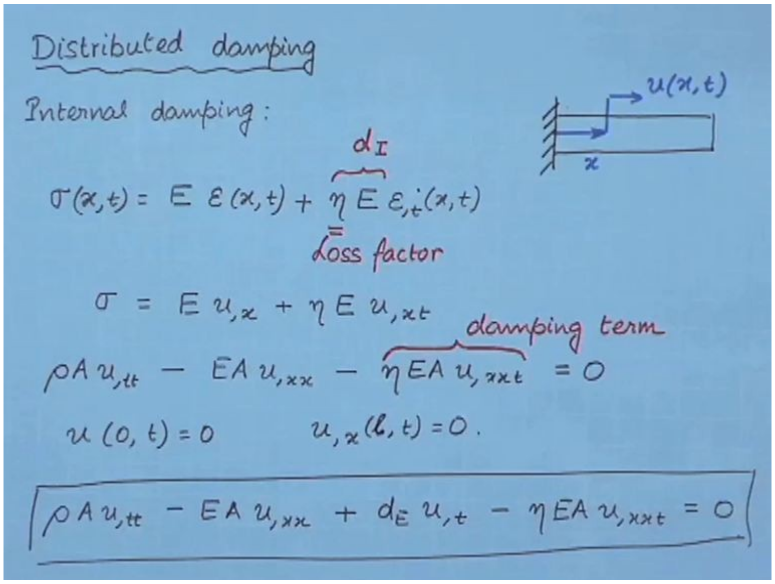

So we have these two kinds of damping. let us being with distributed damping. So let us try to model this internal distributed damping first. Here we are going to discuss the phenomenological model for internal damping so for example in a bar in actual vibration. Now when we derive the equation of motion of such a bar we consider this constitutive relation so like this. Now as I mention that in order to model internal damping for one dimensional system.

We must somehow consider the rate at which this straining is taking place. So we modify our constitutive relation to include the strain rate. Now this term this factor is known as the loss factor. And the product of these two terms may be represented as dI the coefficient of internal damping. So what we have done is we have introduced in our constitutive relation this additional term which we will see how this works as internal damping.

So therefore our stress is given by this. Now when we use this to derive the equation of motion as we have done before we consider uniform cross sectional area so this term you see is E times del u del x this term therefore this term will give us and along with the boundary conditions so this is the damping term. So we have introduced we have obtained this term by introducing this additional strain rate in our constitutive relation.

Similarly, with external distributed damping we have the term like this and together with internal and external damping we have this equation of damp vibration of a uniform bar.

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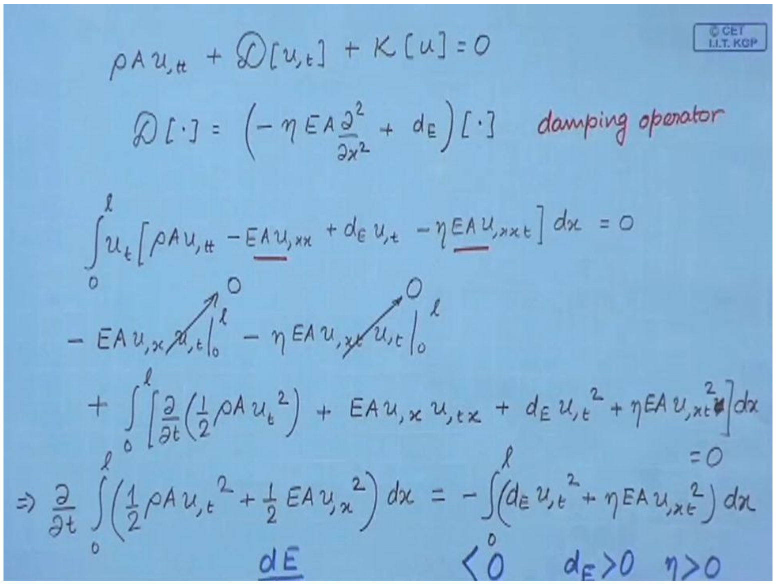

Now this can be this equation may be written in the compact form like this where this damping operator this is known as the damping operator. Just like this "K" is known as the stiffness operator. So let us first see that these terms that we have considered they indeed lead to dissipation of energy because as yet we have just introduced theses terms we have not checked whether they really lead to dissipation of energy so let us start with this equation of motion.

And as this is a usual way to obtain energy equation we multiple this whole equation where the velocity and integrate over the domain of the bar. So which means what we do is and this must be zero so multiply by velocity and integrate over the domain of the bar. Now these two terms I will integrate by parts with respect to the space so what I obtain and this term I can write as can be easily checked.

So this first term can be written like this and this term there will be space derivative of this term and similarly now if we use the boundary conditions that we have then these boundary terms they drop out because at x equal to zero the velocity is zero. At x equal to l this is the first precondition so this is zero. Similarly, this is also zero. So what we are left with is this integral here again I can write this term so I can write this also as time derivative.

The time derivative I have taken it out and I will take the other terms on the right hand side so you see on the right hand side we have an integral which is always positive provided dE is positive so if this dE is positive and eta is positive then this integral is positive. Here there is a negative sign so the right hand side is always negative. So which means this is always less than zero provided dE is greater than zero and eta is also greater than zero.

Now this is nothing but the total mechanical energy of the bar. So this is the kinetic energy and this is potential energy. We have already seen these terms. So the rate of change of energy is always negative which means energy always dissipates or drains out of the system. So the mechanical energy is always dissipated by these damping terms that we have considered. So this shows how this the energy dissipates with time.

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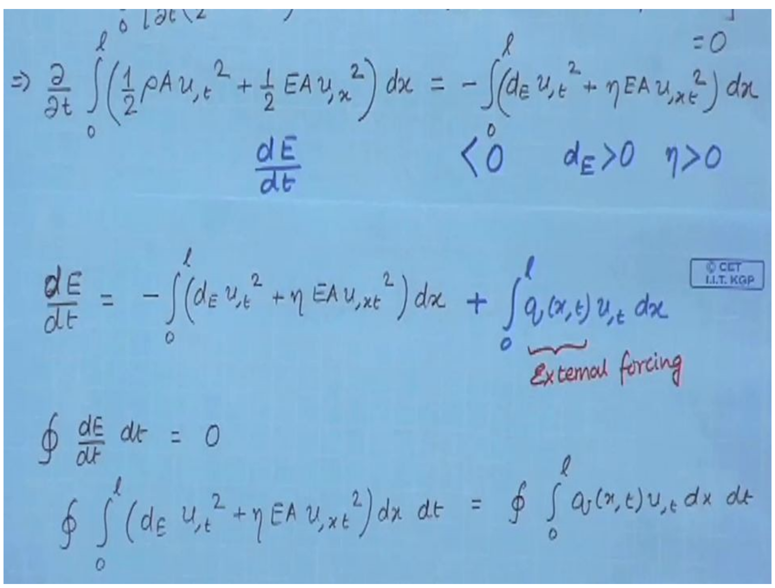

Suppose you have forcing so in that case this energy equation so I will write dE/dT and suppose in addition you have forcing. So this additional term would come because of external forcing. Suppose this forcing is something like this harmonic force that we have considered. So we know already we have discussed this that there is a steady state solution and because of this damping now as we have seen here all the energy of initial disturbances.

Suppose you have an initial disturbance then that energy must dissipate or drain out of the system so what we are left with is the steady state solution. So any initial condition any disturbance created because of an initial condition must dissipate out as we know from here. So if we have the harmonic forcing for example what will remain at sufficiently large times is the particular solution which is generated only because of this external forcing.

So from here you can estimate what happens at steady state so at steady state the motion is periodic therefore if I integrate this energy equation over one period the change of energy must be zero. So therefore I can write here I am assuming that this forcing is periodic or harmonic. If there is a steady state solution, then at steady state the energy change over one period must be zero therefore this is what we obtain.

So what this says is the energy provided by the forcing over one cycle is equal to the energy dissipated by the damping terms over one cycle so from here you can estimate many things for example you can estimate the damping. Suppose you give harmonic input and you record the amount of energy that you are supplying over one period then you will know how much energy is being dissipated.

So you calculate this term and will give an estimate of energy dissipated by the damping mechanism in the system. So what we have concluded that any initial condition or any initial disturbance must die out because of this damping terms and what remains is the steady state solution and that can be used to estimate the energy that is dissipated and that can be used to model the internal dissipation as well as you can estimate the amplitude of motion in certain cases.

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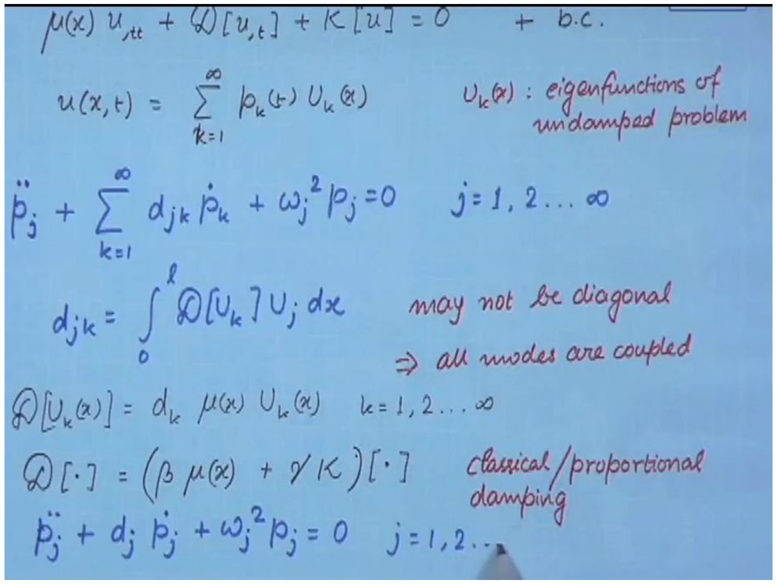

Now let us once again look at this damp system with certain boundary conditions. So this along with certain boundary conditions let us solve, try to find a solution or let us discretize this system as we have done before using the model expansion. So these are the Eigen functions of the undamped Eigen values problems. So when you substitute here then finally take inner product with the j of the Eigen function.

What we obtain these steps we have done many times before so I have directly written the co-efficient of the jth term in the solution so I can carry this out for all j and get all the equation governing the model coordinates. Here this djk this matrix is given by this integral. Now in general there is no guarantee that djk is diagonal. If this is not diagonal, then all the modes are coupled so all the modes are coupled through this damping term.

There is a special situation this damping matrix this will be a completely diagonal matrix. Let us look at that condition so that condition follows very easily from here if D uk is some d k times, mu x times mu k so for all Eigen functions if the operator operating on this Eigen function gives this. In other words, uk is also an Eigen function of the operator D the damping operator. Then the damping matrix is completely diagonal as you can see from here.

So one choice of this damping operator for which this happens is when the damping operator is a linear combination of the inertia operator and the stiffness operator. Such damping operator is called the classical or proportional damping operator. We say that the system is classically or proportionally damped when the damping is linear combination of the inertia and stiffness operators. So in that case our equation completes decoupled.

So we have completely decoupled system of differential equations when the damping operator is classical or proportional.