Conduction and Convection Heat Transfer
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Lecture - 09
1D Steady State Heat Conduction in Cylindrical Geometry

Good morning and welcome you all to this session of conduction and convection heat transfer. So, last class if you recapitulate, we have discussed the steady one-dimensional heat conduction through cylindrical geometry. We discussed the problem of a cylindrical wall, which in practice is conceived when the heat is transferred through the wall of a pipe, cylindrical pipe.

And we derived the expression for steady state temperature, then heat flux, everything.

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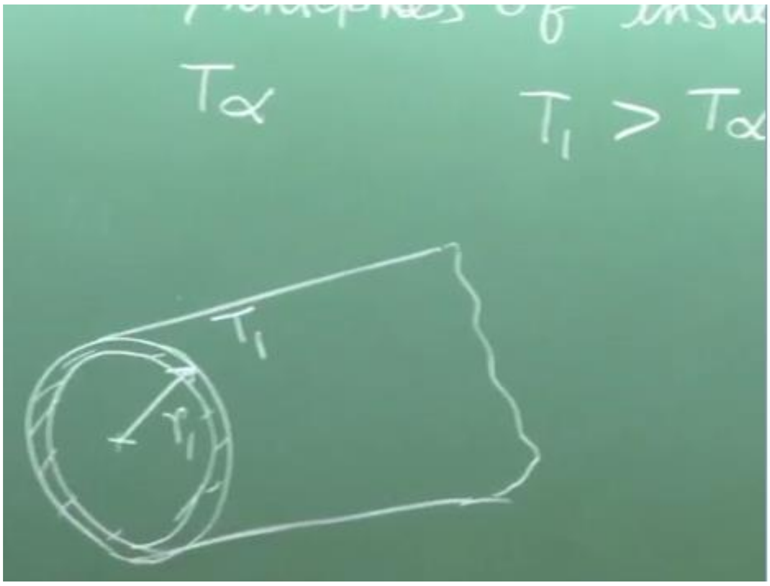

Now today as a corollary of that we will discuss an aspect which probably, I told you in the last class is the critical, critical radius or thickness of insulation, what is the meaning of it. Critical radius or thickness whatever you call, of insulation, what is the meaning of it. Let us consider a problem on the back drop of our knowledge which we had in the last class, that we have a pipe, cylindrical pipe, like this, with external, that is outer radius is r1.

The cylinder may have a thickness, which may be very small or large, but I am not bothered about this thickness. This is because the problem is prescribed like this we have a outer radius r1, at a temperature of T1. That means the problem is prescribed like this. That means somehow heat at the T1 is greater than the surrounding temperature T infinity, which means in practice.

We can consider like this, a hot fluid is going through the pipe, and from hot fluid by convection the heat is coming to the inner wall having some temperature by convection heat transfer, as we discussed earlier, causing the convection resistance, then it flows through the thickness of the pipe, which may be small or which may be large. If thickness is very small, the resistance is very small, temperature drop is less.

All these things we know, then ultimately it comes that the outer surface, whose temperature is T1. Now this problem have not discussed, if supposed like this we have an available outer surface of a cylindrical pipe, whose temperature is T1 and heat is lost from the outer surface to the ambience T infinity. And a convection coefficient is prescribed, what is ambience, here as h, heat transfer coefficient, try to recognise the problem from physical and practical point of view.

Then we will compile it to our own language of solution and we will solve it. So, practical problem is like this, we want to reduce the heat loss. Now if the hot fluid is going through it where we want to preserve the heat, we do not want that heat should be lost. Then what we will desire, that this outer surface will become hot. If this pipe is a conducting material, made of conducting material.

Then we will put insulation that is our common knowledge at the outer surface to reduce the heat loss. As we have seen in practice, now the question comes, that if we put insulation over the outer surface.

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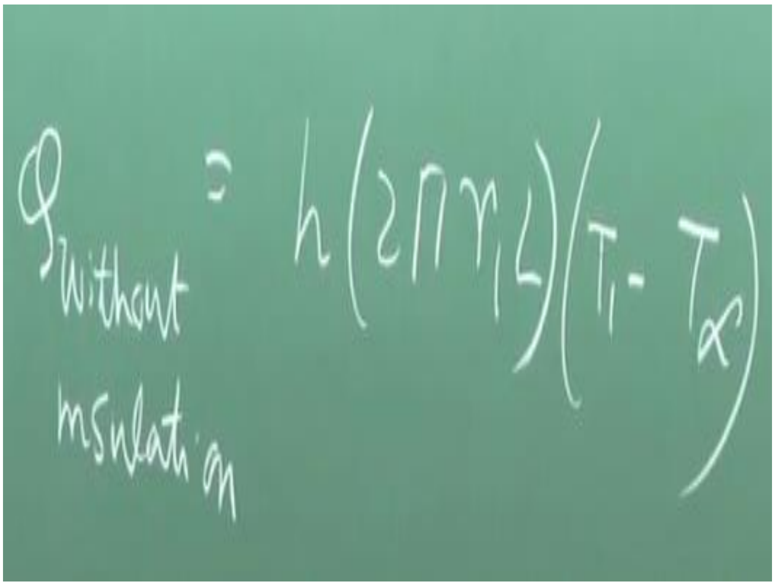

Now if we do not put the insulation what is the heat loss from the outer surface Q is equal to, without insulation, without insulation, without insulation, equals to h into area 2 pi r1 into L into T1 minus T2. Now I want to reduce this heat loss, common sense says that you add insulating material. That is material of lower thermal conductivity, okay.

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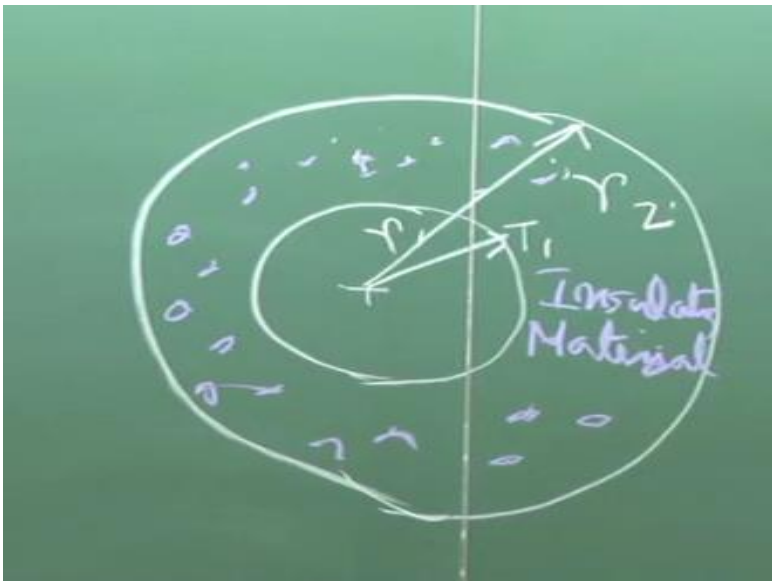

Now if you do so, let us draw this here again, front view that is r theta plane, let us have this insulation. I am only drawing the outer surface. The problem is forced with outer surface at T1. And now you put insulating, this is insulating material, this will look good, this is insulating material. And let us describe this radius r1 and let us consider the insulation is given up to a radius r2.

Then I am drawing this outer surface, and the outer surface of the cylinder and this is the outer surface of the insulating material, which is in the form of another concentric cylindrical surface over it, okay.

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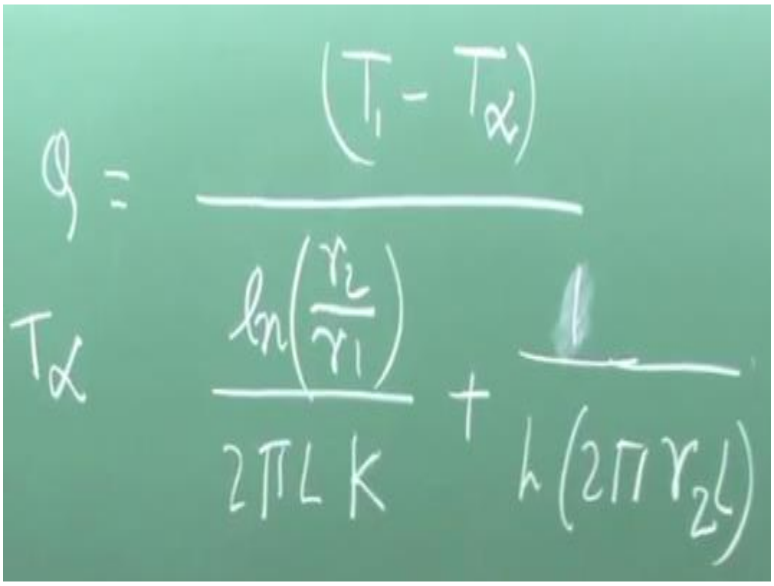

Now if you write the heat transfer equation Q, as you know that now this becomes a series problem as you have already done, from T1 it goes via conduction through the path, r2 minus r1. Then from here by convection to the surrounding T infinity with a heat transfer coefficient h. Now I can write the rate of heat transfer connecting the extreme potential that T1 by which the problem is prescribed.

And the ambient temperature T1 minus T, and in the denominator, there will be two resistance in series. They are conduction resistance, first one, that is ln r2 by r1 divided by twice pi LK plus another resistance, which is the convective resistance from the outer surface to the air, convective resistance when we will discuss convection you will know in detail what this convective resistance is meant for.

When you will understand the mechanism of convection. But so far, we know the convection heat transfer is determined as h to area into the temperature difference, so therefore this is one by this is the convective resistance. That means one by h into r, that is now here this r is the outer radius, that means the radius of the insulation, twice pi r2 into l. So, this is simply the expression.

Now you see that what is the, now if you consider a problem that r1 is fixed, the pipe radius is fixed. We have an insulating material of giving thermal conductivity, length of the pipe is fixed. The problem is, again I am telling prescribed with the given T1 that is the temperature of the outer surface of the cylinder and the ambient temperature is given. So, therefore the problem becomes at how I will vary r2 the radius of the insulation, to minimize Qr to have as less Q as possible.

That means in other way we like to know what is the influence of r2 on Q. Now if you write the mathematical expression, now student keen on mathematics only, they can, he can immediately tell that if we increase r2, if we increase r2, then this increases, this part, that is conduction resistance, R conduction. Obvious physically, because you are increasing the path of heat flow, path of flow of heat conduction.

So, more you give the conduction resistance thickness of the path, increase the length of the path, then your conduction resistance is increased. But what happens to the convection resistance, convection resistance is decreased, convection resistance is decreased. That means there is, there are two countering parameters. That means with the increasing R one way we are increasing the conduction resistance, which will reduce the heat transfer.

But on the other way it increases the, sorry decreases the convection resistance to increase the heat transfer. That is to physically realize, in which way as we go on increasing this R this surface area is going to increase. It does not happened on a plane wall, if one is told that can you decrease by putting he will, it is a monotonically decreasing function, Q will always decrease, or di distance is always increase.

If you go on adding the insulating material, because surface area is independent of the length or direction along which heat flows. But here if we increase the length of the heat flow path, surface area also increases, so conductive transfer from the surface, outer surface increases. So, physically also we see there are two countering parameters which are observed mathematics, which means Q is not a monotonic function of r2, decreasing or increasing.

That means Q has either a maximum or a minimum. So, check it all is school level thing, whatever I am teaching, so Q is having either a maximum or a minimum.

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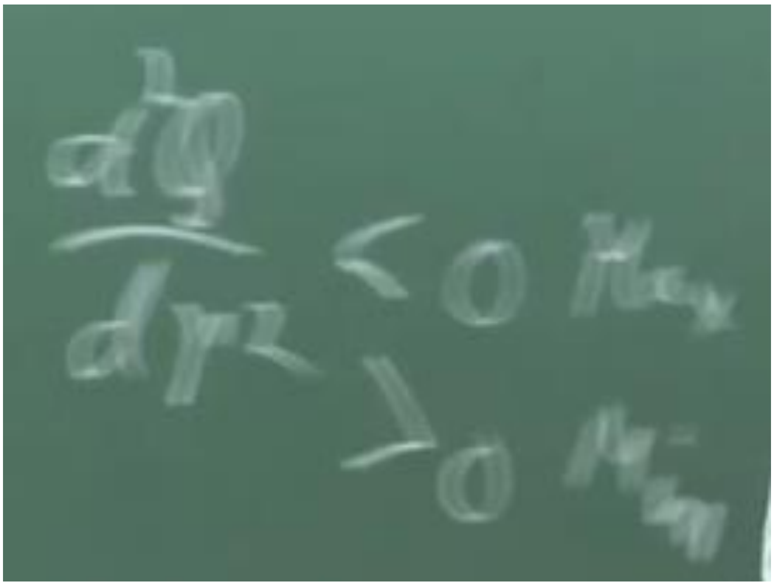

Now how to find out, we find out dQ/di and set it to zero. So, therefore we find out the value of r2, for a given r1 KL, where dQ/dr is zero. So, we set dQ/dr zero for extreme condition, then we will check for minimum or maximum of the function depending upon dQ, d square Q dr square less than zero it is maximum and greater than zero it has a minimum.

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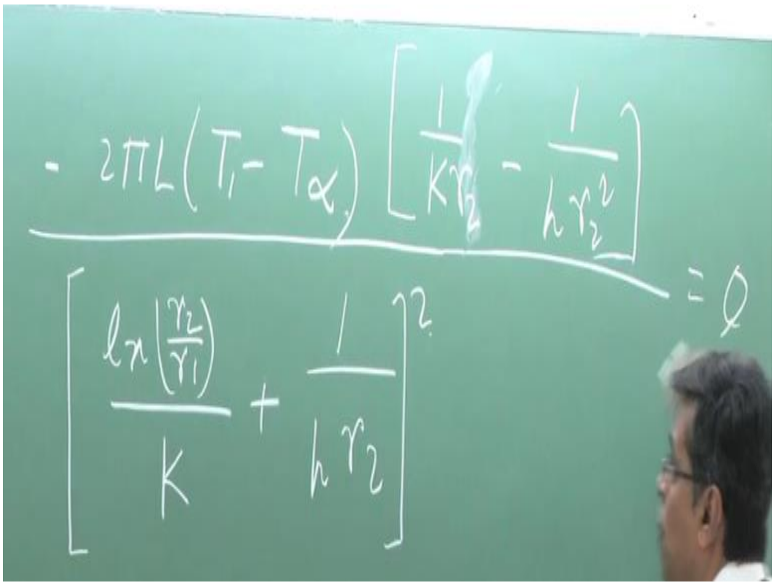

So, this is our routine task at school level. So, if you now differentiate this with respect to r2, let us take this, this is constant that is differentiate with respect to this as a whole then differentiation of one by X minus X square. That means, minus let us consider the twice pi L is constant, twice pi L goes here. Please check it if I do any mistake you tell me. And then ln r2 by r1 divided by K plus 1 by hr2 whole square into here itself.

I will write like this into now, we have to differentiate this expression in terms of r2. That will be first term is 1 by K, now differentiation of ln r2 by r1 with respect to r2 is 1 by r2, Kr2 minus 1 by hr2 square. Now this is set to zero. The mathematics that certain stage becomes only logic not much of operations, when all operations are done then logic remains what is the logic. Logic is that this cannot be zero, T1 cannot be T infinity.

The problem is not defined, okay. This cannot be infinity, problem is not forced, r2 cannot be infinity h cannot be zero.

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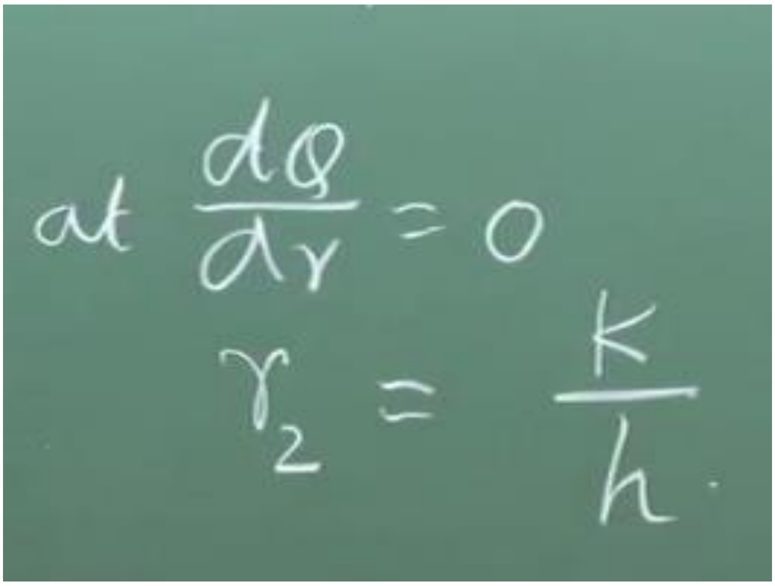

So, for the problem to be defined the only alternative is that this quantity in the brackets will be zero, which tell you that at dQ/ dr zero r2, at dQ/dr zero r2 is equal to K by h. That means K by h value is the value where Q is an extremum, Q attains an extremum value, and if you makes a second derivative and you take that r2 is greater than r1, you can prove that is left to you, that d square Q dr2 square is always negative.

That is a task for you that d square Q dr2 square for this function is always negative. If r2 is greater than r1, so therefore mathematically it can be told, that this function has a maxima at a value of r2 is K by h, okay.

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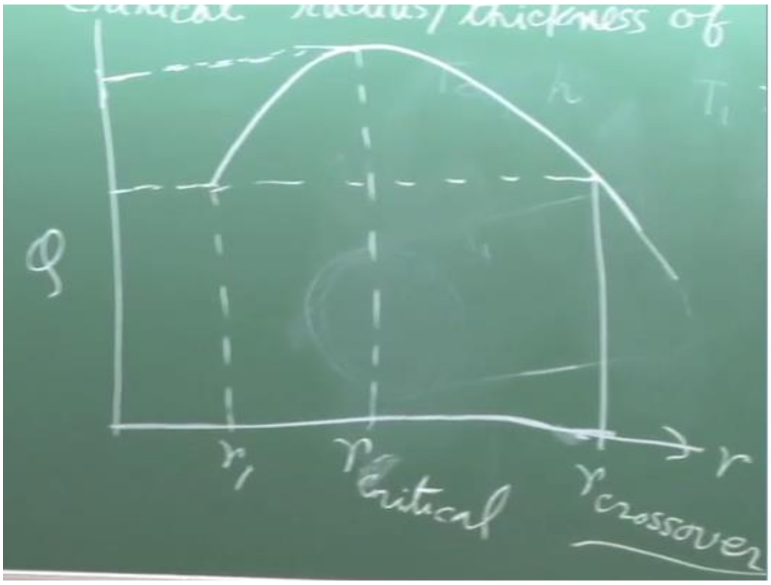

Then immediately we can make a draw a figure like this at r2 is K by h, it has a maximum. That means if I now draw a figure, that heat transfer rate Q verses r2, sorry, it has a, then it decreases. This is r1, this is this value K by h and this is known as r critical, or sometimes rc is equal to K by h. So, this is r critical, this is r critical. This axis is r2 rather r you can tell, r critical.

That means at critical radius this is the, now this is the heat, this is the heat loss from the bare pipe, this is the heat loss from the when the insulation radius is r critical. That means if I go on increasing the radius, adding insulating material to increases radius up to r critical, we are rather not serving the purpose of insulation, we are serving the opposite purpose. We are increasing the heat loss.

That means we have to ensure that r critical thick, our thickness, actual thickness of insulation in practice must be lower than the critical thickness or critical radius rather thickness what is used in the heat transfer. So, thickness is r2 minus r1, it is colloquially used but people tell thickness, but use the nomenclature radius. So, therefore I will tell the radius. So, it should be higher than the critical radius of insulation.

So, it is very interesting and very simple, that there must be a radius where the heat transfer is same as that of the bare pipe, because that is the maximum then it decreases and come. And probably it is not given in any undergraduate heat transfer book, that radius is known as crossover radius, r crossover radius. That means the radius after at which the heat transfer, for any insulating material of any given K, depends upon this value K, K and h, becomes same as that of bare pipe.

So, therefore actual insulation radius should be higher than this crossover radius, clear, hello you, clear. But now the question is that, this also came in our mind when we were students, why teacher is not so much bothered about crossover radius, why the book does not tell anything about that, only gives the expression for critical thickness of insulation K by h, K by h then stops there.

But it is not K by h which is very important to me, because I know that K by h it is maximum. Then after K by h it falls, I am interested to that value where again it comes to the heat transfer from the bare pipe, and I must know that what is that value, so that my insulation radius should be more than that, because if I know only this value and I am satisfied at my work and tell sir, my insulation resistance is this much, a radius is this much.

Okay, this is higher than r critical, but still I get a value higher than the heat loss from the bare pipe. So, therefore I do not gain, it is understood or not. This is answered, this you will not get in any book, I tell you, be attentive in the class, this answer is like this. This is a conceptual information for engineers. Engineers do not go so point to point mathematical information, what is that information for engineers.

Now they calculate two things that what may be the maximum value of r critical, in practice.

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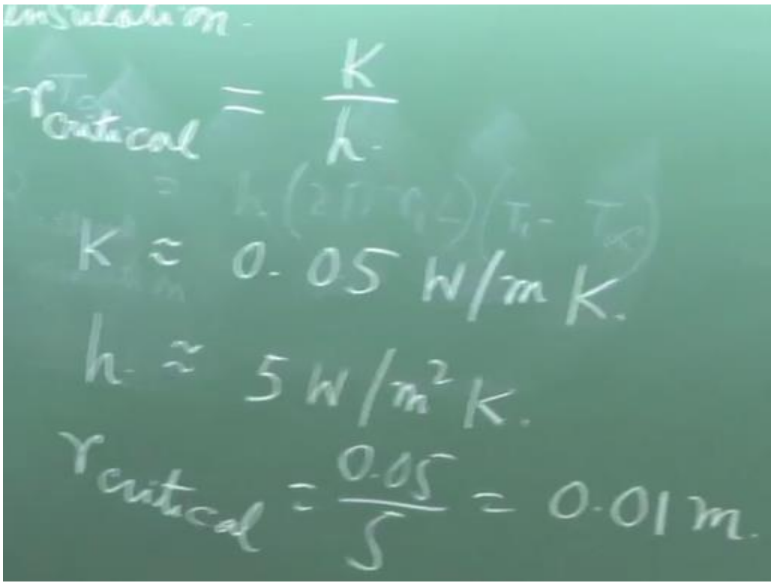

So, in practice the insulating material usually has a range of thermal conductivity like this. It just a scaling type of thing, approximate value, watt per meter K. And the lowest value of h which can be achieved, at a quotient air, having only one significant flow having natural convection is in the order of 5 watt per meter square K. So, if I use this minimum value, because if h is lower, critical radius of insulation is higher.

Then I get a value of r critical, 0.05 by 5. It comes about 0.01 meter. So, why we are only interested with r critical, not crossover, crossover radius is an academic information. We solve problems, but in practice it has got not much value. Now therefore, we see that if the critical radius of insulation is more than 1 centimetre, then it solves the purpose. So, we only know the maximum value.

By scaling analysis, that what can be the maximum value. Afterwards we will see when we will do the scaling analysis, we have already done it in fluid mechanics class, how do you scale the things, you can understand the order of magnitude analysis. A parameter can attain maximum these values. So, therefore if I ensure that my critical radius of insulation much more than 1 centimetre.

I am happy that it is definitely gone more than the crossover radius, clear. So, therefore it is the critical radius of insulation, which is more important. And it is directly dependent on K obviously, if K is very high, it will be conducting. So, insulating material K should be low, so if K is lower we require less radius. Similarly, if h is lower you require higher radius. And if h is higher you require, h is high you require lower radius, okay, clear, okay.

So, this is the concept of, we will discuss the cylinder, cylindrical geometry, internal thermal energy generation.

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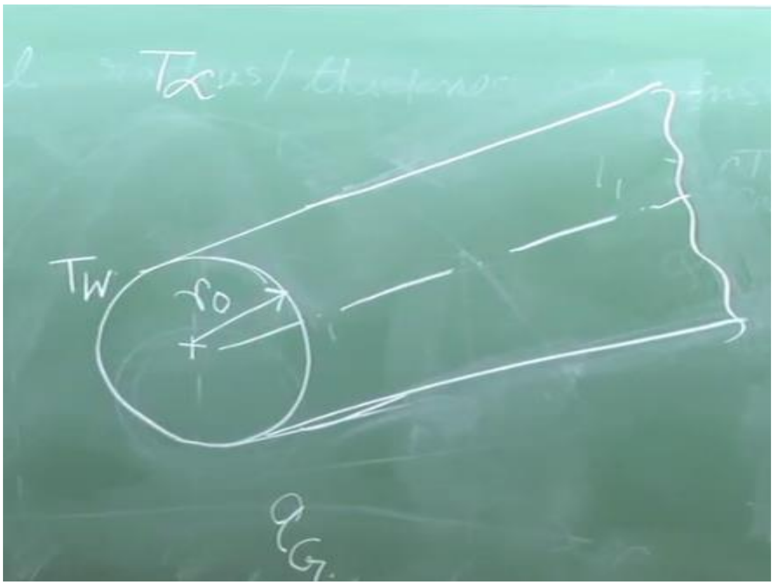

Let us consider a problem like this, we have a cylindrical rod, solid rod of radius r0, thermal energy is generated, which is specified by, the thermal energy generated per unit volume at a point, which may be a function of r. It is a one-dimension problem, so qG may be a function of r that is the radius of this solid cylinder, r may be constant. That means the volumetric heat generation rate is same at each and every point.

And this is the cylindrical element, and this is kept in a fluid or in an ambience, for example air, T infinity. Now energy is generated and this surface is kept at a temperature Tw, now our job is to find out the temperature distribution, within the element and what is the rate of heat transfer all these things. This particular problem originated from the fact that the nuclear power plant, the fuel rod.

That is the nuclear rod, nuclear fuel is used where the energy is generated within it by virtue of a chemical reaction, huge exothermic chemical reaction, to boil the water in which these rods are immersed. But the rate at which the energy is generated, it is very difficult to control the temperature of the water. So, cooling is extremely important, of course there are many more things.

I do not want to discuss these things here to moderators are used all these things are there to control the rate of reactions and all these things. But this originates from a problem like this, that there may be a solid cylindrical element where heat is generated one of that example, one of the example this the nuclear rod in a nuclear power plant. So, this is, my drawing is very bad, this looks like a divergent, sorry, this looks like a, sorry, let me draw it, this looks like a divergent, okay.

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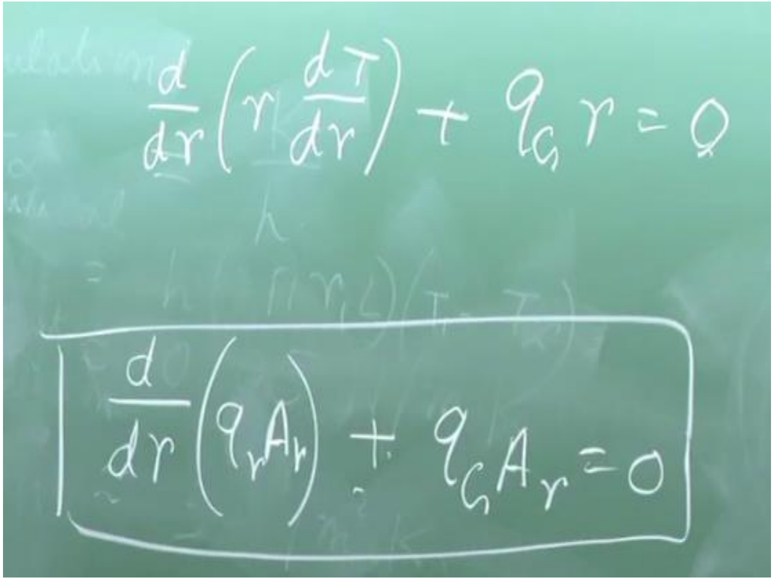

Now therefore we now compile this to our software language, that there is a cylindrical element, where from we will start. We will start from this, that we know d/dr of r dT/dr plus qG into r is equal to where from we get it, sir where from you have got it, you may have got it from the, you may get it from the general heat conduction equation temperature distribution.

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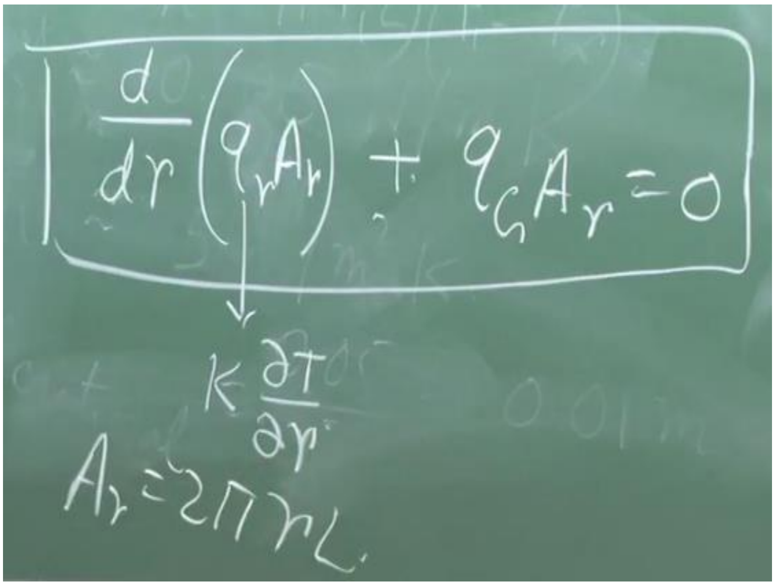

But I will recommend you, I am telling again and again, for a steady one-dimensional heat conduction you generate it form that. This may be cylindrical, this may be plane wall, very important for you people, that this is like this, d/dx or here I will write d/dr of Qr, okay, plus, Qr means, sorry I am extremely sorry, I am extremely sorry, qr Ar plus qG Ar zero, because this is very easy to remember.

I am telling you again, what is the nomenclature, this q is the heat flux for unit area, perpendicular to the direction of heat flow, here r is the direction of heat flow. If it is x direction in the plane geometry, it is d/dx of qx into Ax, Ar is the area normal to the r direction, similarly Ax is the normal to x direction, plus the energy generated per unit volume into the area, it may be Ax or Ar is equal to zero.

Where from you get it, you take a small elemental volume in plane surfaces is a rectangular small element, in a cylindrical surfaces, it is small cylindrical ring and you make an energy balance, that heat coming in, heat going out must balance the energy generated within it, because nothing can be accumulated, nothing can be absorbed, because this is at steady state. So, if you have this thing in your mind, so you can start from here, clear.

Now qr is K delT/delr or delT/delx, Ar accordingly in a plane area, with constant cross-sectional area A will come out, otherwise A may vary in a converging diverging passages. And Ar in case of twice pi rL, so if you put that this is L of heat conduction this is this and you consider K to be constant, then you get this equation, for cylindrical co-ordinate system. Hello why you are doing like this, this is very simple, you do not get it?

This was done in the class, emphatically you are not getting it, K is missing everything is missing, first expression K is missing, first expression where K is there. It is qG by K, okay, Kr, here K will be there, qG by K, okay. K is missing, I am sorry. This is okay, you tell me, sir K is missing at immediately. Here K will be there, qG by K, okay. K is missing, I am sorry. This is okay, tell me, sir K is missing that it will be qG by Kr that is why the mistake is there.

If you just try to remember from the equation, you see, if I do this mistake then it will be there for you. That is why I am telling why you will mug up the equation like this, very good, qG by K. It will be detected when you go on deriving this thing we will see that something is not coming, some K is missing. Where it is missing? The main equation. But better you start from here d/dr of qr into Ar, delta r will be cancelled qrAr is 0, so you get the equation, clear.

Now you differentiate it. Now, if I consider qG is constant or uniform.

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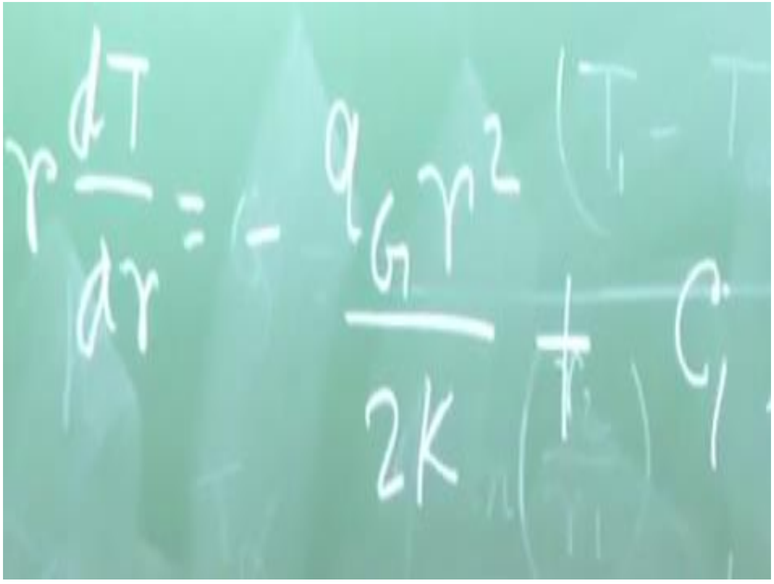

Now you differentiate it. r dT/dr is equal to minus qG r square by 2K plus C1, okay. Immediately you tell, sir your K is missing, okay. Otherwise there will be problem, why do you keep mum, sir K is missing. I will be very happy, okay. Then next part is that this will be C1 by r, so next one T will be minus qG, again r, r, r square by 4K. There are two constants, you require two boundary conditions. What are the two boundary conditions?

One is very simple at r is equal to R0, the problem is prescribed at R0, T is equal to Tw.
This boundary condition comes from the practical information that we have to keep this surface at a given temperature C1 lnr, so I have not integrated, good, so I will get 0, so that is an advantage of a teacher that he does not zero, but I am sorry, in the examination, but I will not give you zero.

I tell you even if you write C1 by r, why do you know, at least you are able to write the first step from the energy conservation, then I will consider that hurriedly you forgot to integrate, I will deduct the marks, but not I will give you zero, but however do not do that, sorry. C1 lnr plus C2. How do you get two constants, quick, quick. What is C1? Just without any boundary condition, what is C1? Zero because it will give a discontinuity at the center.

The problem is not physically discontinuous at the center. It cannot define the function, so C1 has to be zero, but the problem requests a mathematician. A mathematician will not leave you. Engineers are very smart. They tell many things, okay I understand, but it matches correctly, mathematician will tell, no you write the number and condition. Your mathematics teacher will tell that there are two constants you write, second order differential equation, two constants, two boundary conditions,

So, another boundary condition, I told you in the class earlier known as symmetry boundary condition. The problem is symmetry, why? Because the temperature is uniform azimuthally, qG is constant, uniform heat generation, not the one-half heat generation is higher another half lower, it is not like that, so therefore problem is entirely uniform azimuthally, means uniform with respect to the axis r is equal to 0, which means an symmetry boundary condition that at r = 0, the derivative is 0.

Function has a value, which is either maximum or minimum that means function is continuous with a 0 derivative exhibiting a maximum or minimum. That is mathematical statement. Physically, a smart boy will tell C1 cannot exist because lnr ka place nahi hai here, because this has to be 0, so now what will be the value C2 will T0 plus qG r0 square by 4K that means the expression T is equal to what?

T2 is Tw, T minus Tw rather you write like this is equal to qG by 4K r0 square, am I correct. C2 is Tw plus qGr0 square by 4K. It will be in the numerator, r 0 square will come on, it will be in the numerator, so qGr0 square by 4K minus qGr square by 4K. Now it is okay, so this is the distribution. Now where is the maximum temperature, it is very simple that r is equal to 0 is the maximum temperature.

You do not have to make this dT/dr zero, but dT/dr zero will tell you the same thing that r is equal to 0. Maximum value of r is r0, so therefore at r = 0, so this is maximum temperature that means T0 is the maximum temperature.

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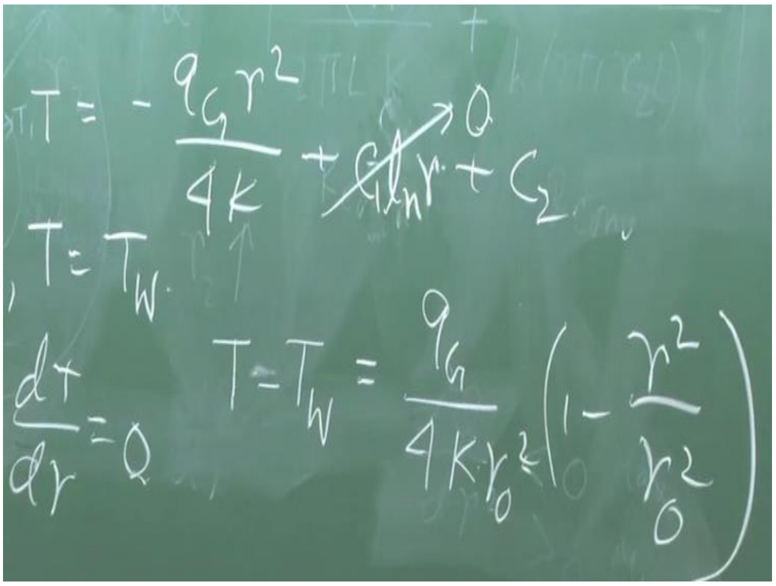

T0 minus Tw is qG r 0 square by 4K, so there is a convention that we write the non-dimensional form of the temperature distribution or dimensionless wall like this 1 minus r square, that means this is a parabola, so therefore, this we can show like this.

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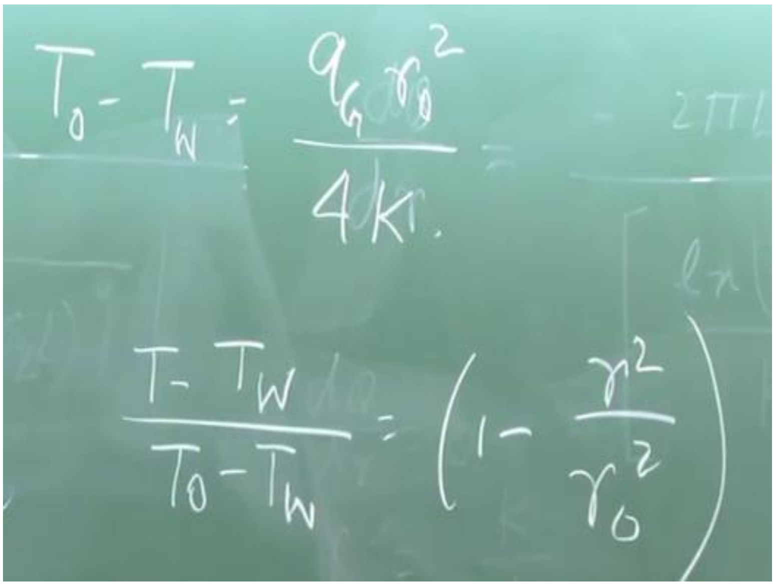

Here, this is the temperature scale, this is Tw and this is T, T as a function of r, so that means it is a parabolic distribution symmetry about r is equal to 0, very simple. You can find out dQ/dr at the surface.

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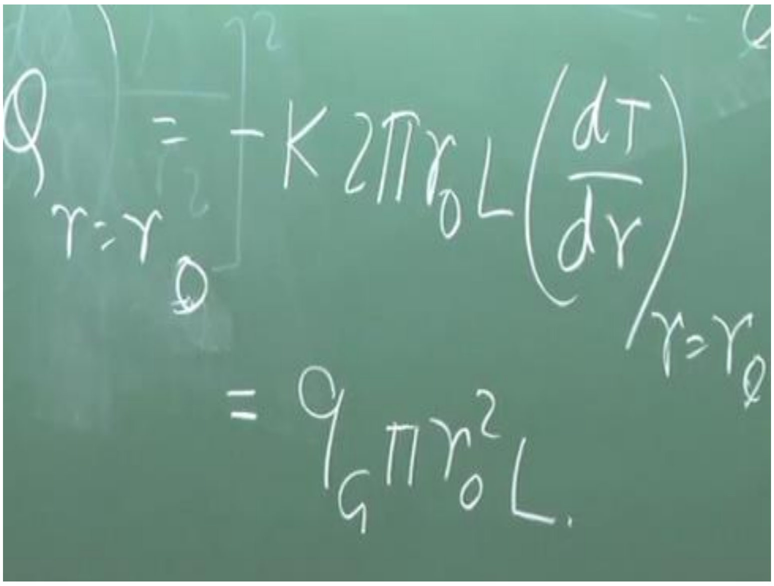

We are interested what is the dQ/dr at r = r0, what is that? It is minus K area twice pi r0 L into dT/dr at r = r0. Heat flux is not same at all radius, because heat generation is there, thermal energy generation, at steady state it is so. So, therefore if you ask me the general expression of, oh god, I am doing so much mistake today. Q at r is equal to r0, so Q at r is equal to r you can find out as an expression.

Then you can put r = r0 or you can write instead the area into dT/dr, r = 0. Now here if you find out the temperature gradient at r = 0 from this expression, you can find this as qG into pi r0 square L, okay, clear. You can find out the value Q at any r, that also you can find out that is minus K twice pi r L into dT/dr, which is qGr0 square by 4K into minus two r by r0 square, r0 square and r0 square will cancel.

One can find out Q this is minus K at any r, twice pi r L, at any radius r into dT/dr. What is dT/dr? dT/dr is twice r qG by 2K minus qG by 2K, r0 square and r0 square will cancel, 2Kr. That means minus pi, minus will cancel, K pi r square qG into L.

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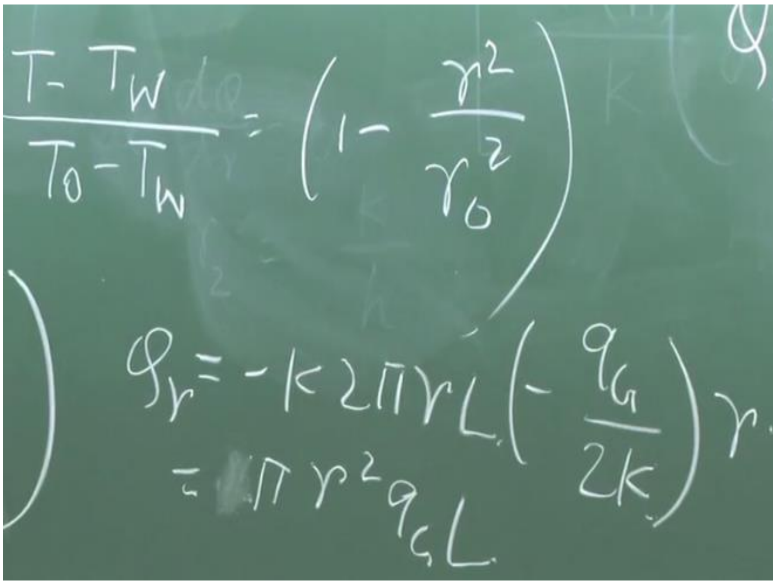

K will also be cancelled, so therefore this is a common expression for Q at any radius and it shows that it is a function of radius, obviously. Q at all r will not be the same because in a disgeneration, it is not a steady state with no generation, so all concepts will be matched. If you do like that that is also possible step for what you can find out that r = r0 if you are told to find out at the surface.

But you can find the general expression r = r0 square, which means that the entire thermal energy, which is generated is coming out from the surface. That is the steady state requirement. That is the requirement of the steady state heat transfer, okay, alright. Now after this, we will solve very simple problem in the class. Now we will solve a simple problem just to apply our equation.

The simple problem, let us consider this one: A tube wall of inner and outer radii ri and r0 that means.

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This is extremely simple problem. I do not know whether there can be any problem simpler than this, ri and r0 whose temperatures are maintained at Ti and To, o is outer, surface temperature. No ambience, nothing. You can see, can you this thing? Now cylinder conductivity, thermal conductivity is temperature dependent, so long in the class Prof. Som has talked about constant thermal conductivity.

But now the problem is a conductivity dependent thing, but does not matter, it is mathematics. K is K0 into 1 plus at, where a is a constant. K0 and a are constant, often an expression what the heat transfer per unit length. How to find out the expression for heat transfer per unit length. There are two ways, I am telling you. I told you earlier also.

You can find out the temperature distribution and you can find out from the temperature distribution of the heat flux, but I think better it will be better if you stop there, the heat transfer distribution, not temperature. For example, what is that?

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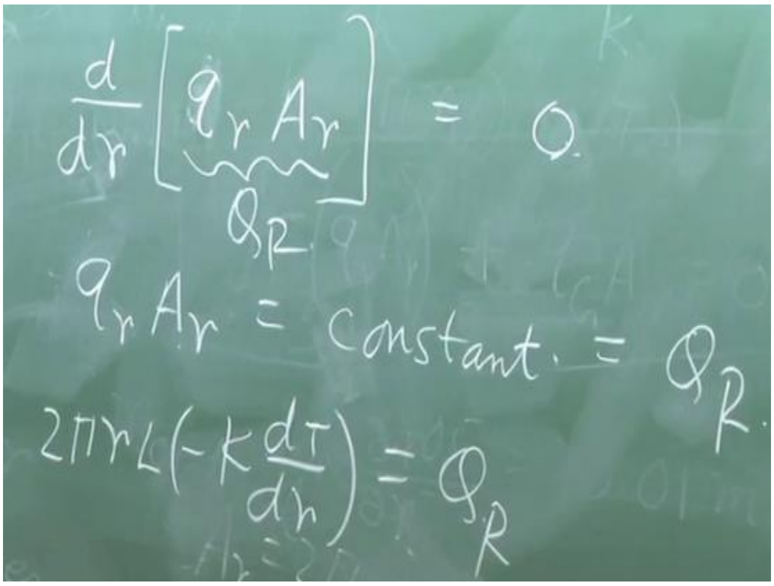

Again, I will go through in a simple unique way d/dr of qrAr, sorry, extremely sorry. d/dr of qrAr is what, there is no generation, 0. I will stop here. I will not go for temperature distribution. Stop here means, I have to put of course temperature because it has to be found out Ti, To, then d/dr, no I will not do like this, not temperature. What I will do, then qrAr is equal to constant and that constant equal to qr, that is the rate of heat transfer.

Total rate of heat transfer. Now what is qr? Ar is two pi rL and qr is minus K dT/dr and that is equal to constant and that constant I will use as QR, which is constant, total rate of heat transfer. Here total rate of heat transfer from any Ar is constant because this equation tells that. This is QR, this is nothing but QR. So, therefore QR is minus twice, now if we use that, if you are told to find out the expression of heat transfer QR.

Then it is greater to use that an integrate. Then, how to integrate?

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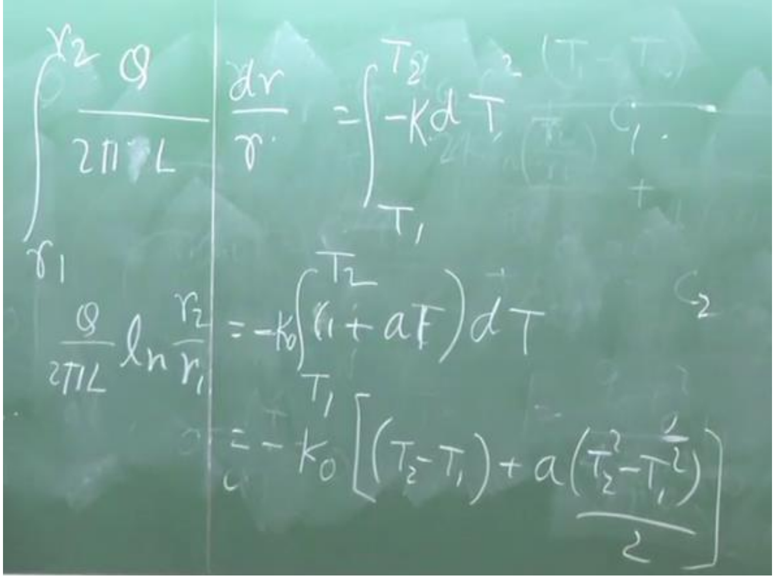

Q by twice pi rL dr/r. Now I am writing QR as Q, because this is constant. This is not dependent on r, Q by twice piL dr/r is minus dT. Since Q is constant, it is better I can integrate simple integration by taking, K is there, I am sorry, minus K dT, okay. Q is constant, so therefore Q by 2 pi L ln r2/r1 is equal to now K is K0 1 plus AT, so minus K0 will come here, minus K0 1 plus aT.

Here this aT is, this book nomenclature is this. It is function of temperature T, small t is for time, a plus T. So, this is minus K0 into T2 minus T1 plus a T2 square minus T1 square by 2. Okay, this is alright. So, you can find out the heat flux per unit length as a function of T2, T1, but if I define a thermal conductivity at the mean temperature T1 plus T2 by 2, then what will be the thermal conductivity at the mean temperature?

Km is equal to K0 1 plus a into Tm where Tm is equal to T1 plus T2 by 2. Why I am doing that? Because this can be written as extremely simple.

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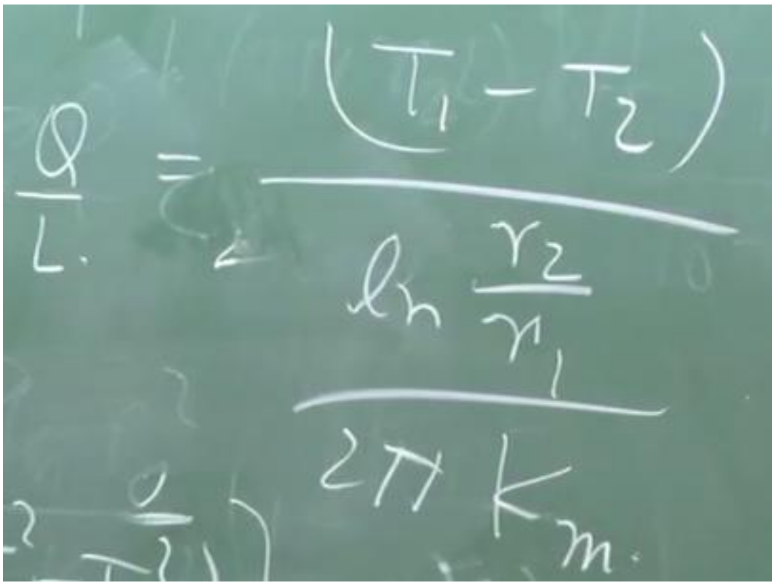

This can be, the right-hand side can be written as minus Km into T2 minus T1 that is T1 minus T2 or you can write the right-hand side now. Again, I write Q by twice pi L ln r2/r1 is equal to, I take here minus sign, K0 into T1 minus T2, not K0, Km. Why I am writing this mathematically? Then, I can tell that if we define, if this type of linear temperature dependence is there.

If we can define thermal conductivity at mean temperature, then it is similar to a constant thermal conductivity case that at the mean temperature. Because for constant thermal conductivity, this is the expression. That means Q is equal to the same expression, the potential difference divided by ln r2/r1 and twice pi Q/L rather twice pi Km, as if it is a constant thermal conductivity, but at the mean temperature.

This is nothing, this is simple class 9 level school mathematics, that means we just take care of the thermal conductivity as a function of temperature, so I think today we cannot solve any other problem. Next class, I will solve few more problems. Tomorrow, I tell you I will solve few more problems interesting with critical radius of insulation. Come prepared or see that interesting problems on this.

And then we will switch over to extended surfaces or fields. How to increase the heat transfer by adding extended surface. Any questions? Please any discussion please or if you have got any suggestion about the teaching also, you can tell me, sir, you are very fast, please control your pace or you are very slow. Everything is alright. Any question, any other. No question, okay thank you.