Conduction and Convection Heat Transfer
Prof. S.K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology - Kharagpur

Lecture - 07
Problems on 1D Steady State Heat Conduction in Plane Wall

Well, we come back again to this session. Before I start for any problem, just I tell that in last class, an important mistake, the student has pointed out.

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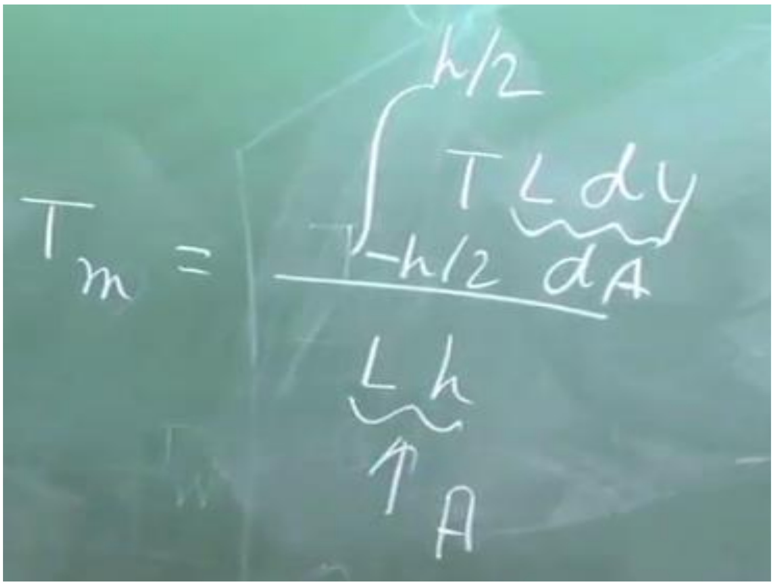

There was some mistake that when I define this, that it was TLdy minus h by 2, that means it is cross sectionally average. This is nothing but dA, and here it will be L into h. That means this is A. In earlier session, I just omitted probably this L. So, this will be the area average dimensionally it is T, Okay.

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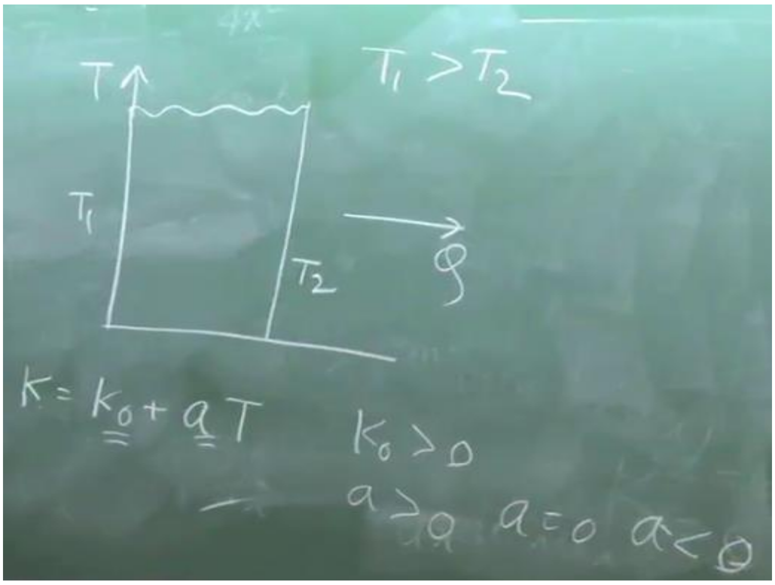

Now, we come to the second problem. The first one was discussed, that this problem is a simple problem, we have to critically analyse. Let us have a plane wall, this surface is, it is T. This is surface is kept as T1, and this surface is kept as T2 and T1 greater than T2, so the heat flows in this direction. Now the problem is this, thermal conductivity of this plane area is given by K is equal to K0 plus a into T, where a is a constant. K0 is a constant which is greater than 0.

Obviously K0 is the thermal conductivity, and a may be greater than 0, a maybe 0, a may be less than 0. a may have any value greater than less than 0 and also 0. Now accordingly that means when a is 0, a greater than 0, less than 0, we will have to plot the temperature distribution. That means the qualitatively the temperature distribution. Now when a is 0, K is K0 constant thermal conductivity, so temperature distribution is linear.

But when a greater than 0, and a less than 0, where from we will start, we will start from the steady state heat conduction equation for constant area. That means it is always greater, I will tell you that you start with this.

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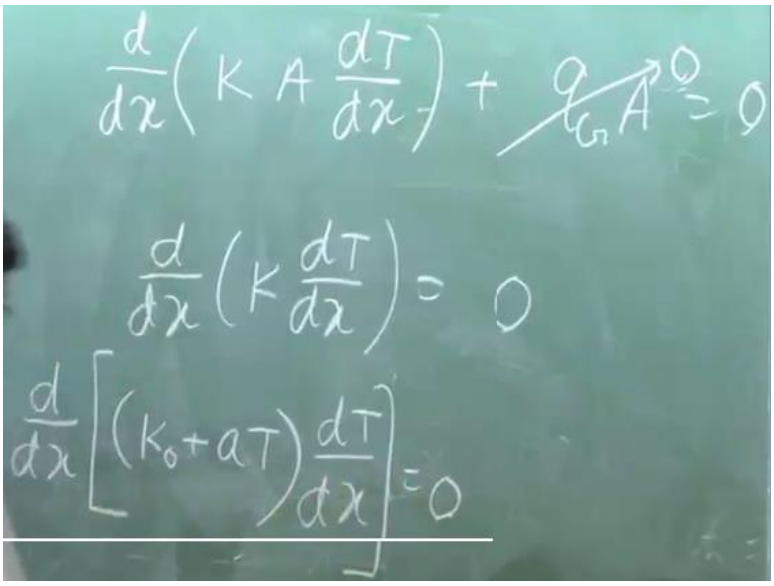

Now you always start with this KA dT/dx plus q G into A zero. You should always start from the most generalised form of the steady state one dimensional heat conduction. While there is no heat generation, this is zero. When there is no variation of area, then area will come out and then d/dx of K into dT/dx is equal to zero. And when K is constant d square, dT/dx square, dx of dT/dx zero means dT/dx is constant.

So, state wise we will do. Now here, since it is a plane area without heat generation I can start from here. So, I can find out, that if K is constant that a is zero, then it is a linear profile. It is like this, so this is a temperature scale T2, this is in scale temperature T1.

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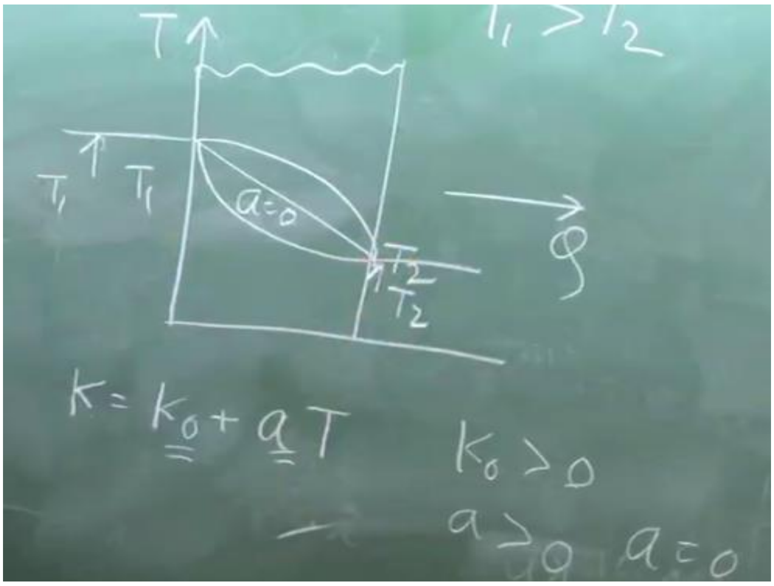

So, this is a linear one when a equals to zero, but when a less than or greater than zero, a very component at first reaction is that it is a nonlinear, but with a negative slope, obviously temperature will decrease. But the problem was what it the nature of this profile. That means whether the slope will increase or the slope will decrease, because we can have two types of nonlinear thing, one is this, another is this. Which one is for a greater than zero, less than zero.

To do this we have to find out the variation of dT/dx, by putting the value of K, as a function of temperature, so therefore we will do that, d/dx of, K is K0 plus aT into dT/dx is equal to zero. So, now if you solve it we will get a K0 plus aT into d square T/dx square. Then K0 plus aT differentiation is aT/dx. That means plus a dT/dx whole square is equal to zero. It is as simple as anything at school level thing.

So, we can find out d square T/dx square is equal to minus a dT/dx whole square divided by K0 plus aT. Now you see ultimately the problem is transferred to a mathematical problem. So, up to this the concept of heat transfer is involved, after that it is mathematics.

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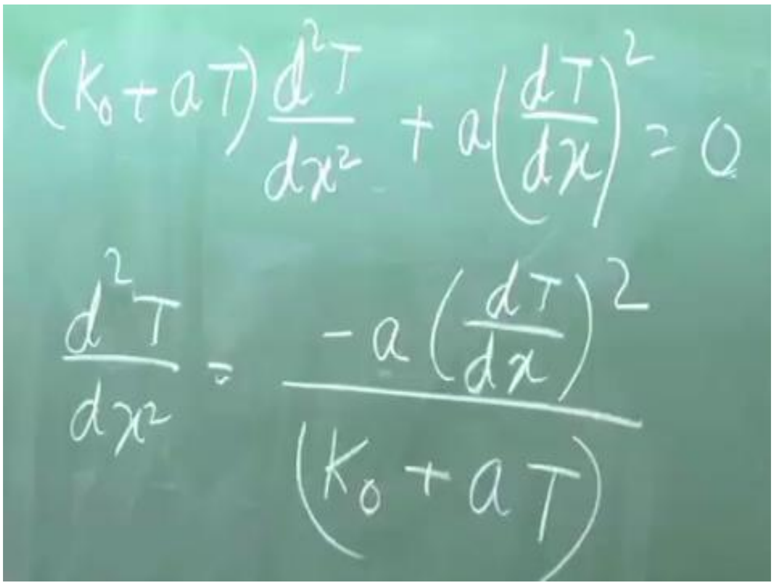

Now we can decide the sign of d square T/ dx square with a with the logic. dT/dx square is always positive, the denominator is always positive, whatever may be the value of a, why, please denominator is always positive whatever may be the value of a, positive or negative, why? This is not from mathematics, everything will not evolve from mathematics, that is why I am telling.

If you feel that mathematics is all then wrong. It is the physical concept, physical understanding is very important, because mathematics has been evolved from the physical understanding, why? This is because thermal conductivity, which is to be positive, so it is scalar property thermal conductivity, so K0 plus, so therefore d square T/dx square is exclusively with the, that means when a is greater than 0, it is negative and a is less than 0, it is positive.

Negative means d square T/dx square is decreasing. Okay, that means this curve dt/dx is decreasing and this is for a greater than 0. When a less than 0, d square T/dx square is positive means, dt/dx is increasing, so a less than 0 and a greater than 0, d square T/dx square is negative, that means the slope is decreasing., clear, okay. I am sorry, this is greater than this. this case, this slope is decreasing that means the curvature is negative.

Sorry, this convex. This concave side is that slope is decreasing that means if you draw a slope here and if you draw a slope here. That with x, this curvature is positive, a less than 0 that means the slope is increasing. Whereas here the slope is decreasing that means the convex side where the curvature is negative. D square T/dx square is less than 0, okay, alright. That is all, there is nothing more to say with this problem.

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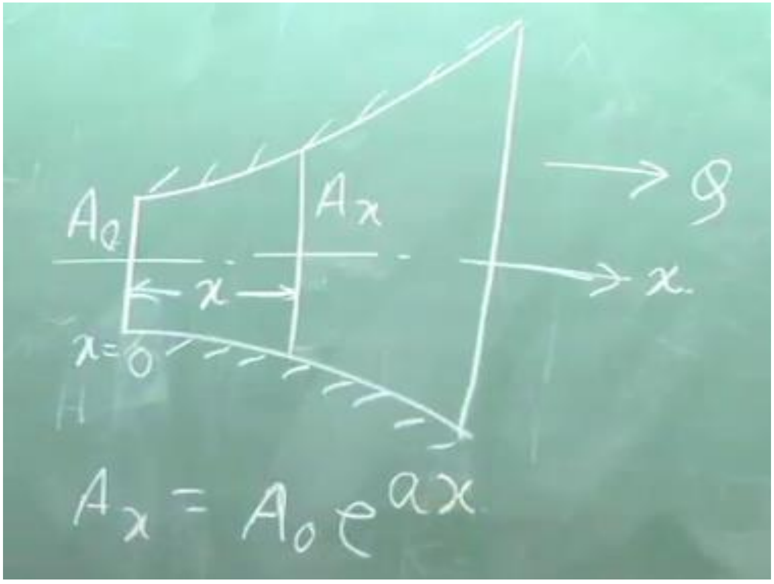

There is a rod like this the same thing which we did an application that I told the lateral surfaces are insulated. Now, x, this is x = 0 and the area is A0. Heat is flowing this direction because of the difference of temperature is mid, constant temperature at this phase and some constant temperature at this phase, temperature at this phase is more than this phase, which is not prescribed in the problem.

What is prescribed in the problem is that Ax at a distance x, is a function of x, that means this is Ax is A0 into the power Ax. This satisfies that that x = 0, the area is A0 and then increases exponential. Using this expression determine the temperature distribution, okay. Considering the presence of volumetric heat generation rate, okay.

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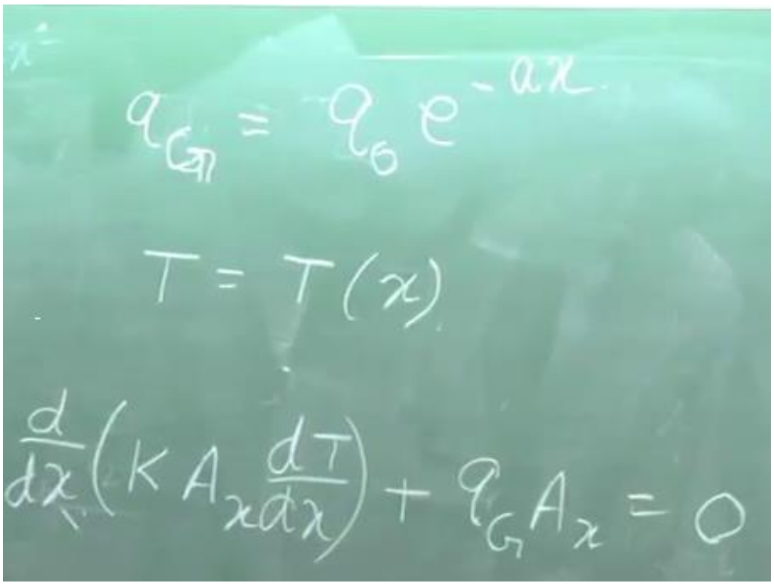

Now, first without the heat generation rate, well and then with the heat generation rate. So, heat generation rate is this, qG is equal to q0 e to the power. There is a volumetric heat generation rate, which is a function of x only. It is a one-dimensional problem, so as I told that cross sectionally average temperature or a temperature uniform about the cross section, whatever you take this is T is a function of x.

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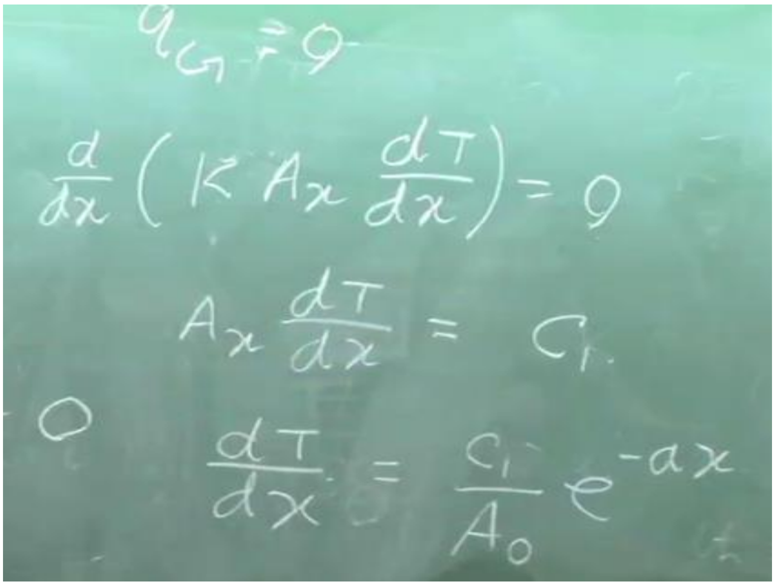

Now, again the problem will start with the same equation as I told. When qG is that means there is no volumetric heat generation, then it is very simple then d/dx of K Ax dT/dx is 0. When qG is 0, that means there is no volumetric heat generation, the d/dx of KAx of dT/dx is 0. Now in this problem, it has been told that the thermal conductivity is constant that means simply K comes out, that means d/dx of Ax dT/dx 0.

That means Ax dt/dx first integration yields a constant which I call as C1, tell as C1, written as C1, okay. Then it is simply the integration dt/dx, C1 by Ax, Ax is A0 e to the power ax, that means C1 by A0 e to the power minus ax.

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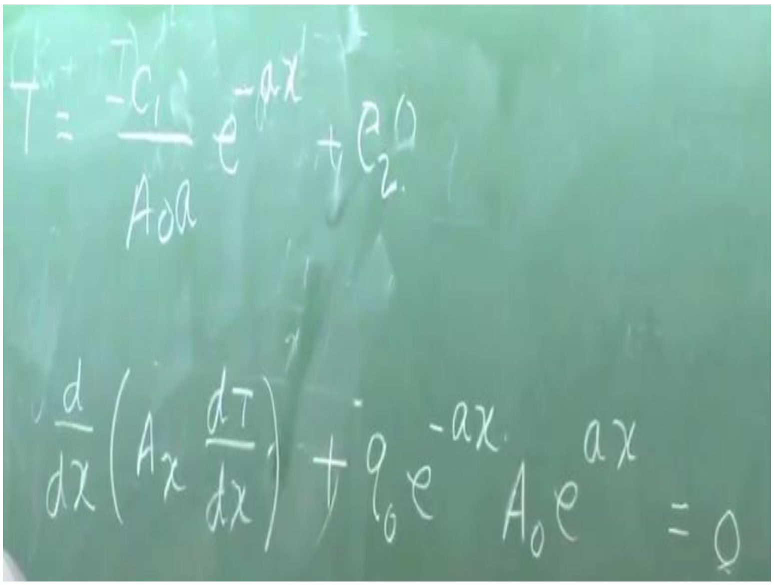

Well, then you can integrate T is minus C1A by A0 A to the power minus Ax plus C2. Now, this you can find out both the C1 and C2 provided you know the boundary. This is a general expression for T. In terms of two parametry, hello please, A will be integration, I am sorry, A will be denominator, very good, very good into the power minus Ax, so it is a denominator. It is an integration, if it was differentiation, it was there, okay, minus 1 by a, very good, so it will be in the denominator.

So, this is the expression in terms of two parametric constant C1, C2, which will be found out if I know this temperature, T1 and T2 and X= 0 and that X=L, if we have prescribe this length as L, then in terms of T1, T2 and L, this C1, C2 will come, so this is the first part. Okay. Now, next one is the heat generation. What is the heat generation, q0 into the power minus Ax.

So, this thing if you take care of, then you right here this, that means in that case, it will be d/dx of by taking the thermal conductivity of course, last qG into Ax, q0, qX is q0 into the power minus Ax into Ax. Ax A0 into the power Ax. q0 into the power minus Ax and Ax A0 into the power Ax, qGK will be there, so now if K is constant, then K will come out, K will go here, does not matter.

This divided by this, e power minus Ax into the power e to the power ax will cancel off, that means let me write this d/dx of ax dT/dX, now ax is also a0 into the power of ax dT/dX. For constant thermal conductivity is q0a0/K. There is nothing great in it because it is simple mathematics, if you do it very carefully it will be alright, but your concept is okay. By chance if you do some mistake I think mathematically is not a serious thing.

For exam purpose, some mark you may lose, but so long you can write the correct governing equation, you get maximum credit, because that is the main part, then of course you have to be very careful about the mathematics. Now the only thing is that you have to find out T as a function of x purely a problem of simple integration. Now a heat flux distribution, they have asked.

What is the heat flux distribution, actually this is the heat flux, d/dx minus Qx, Qx is minus KAx dT/dx, Qx is minus KAx dT/dx, that means I can write d/dx of minus Qx, Qx is minus KAx dT/dx plus Q0A0 is 0.

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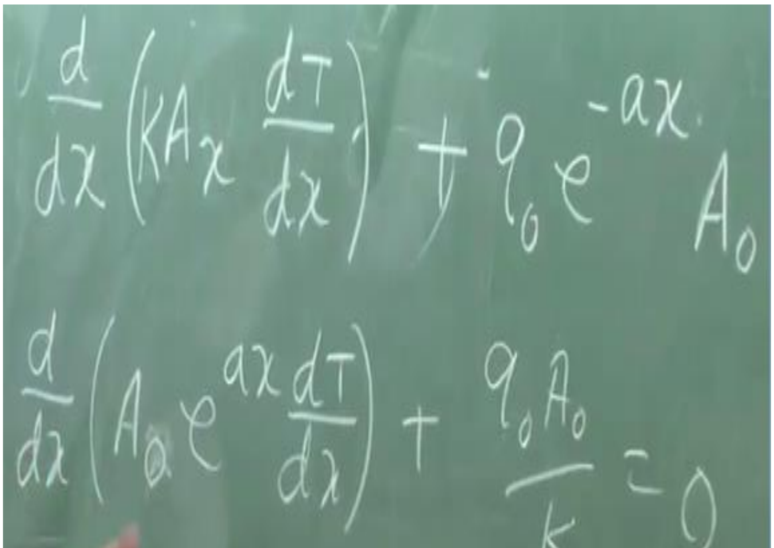

That means from this step I can write, if I have to find out the distribution of heat flux. That means distribution of heat transfer, Qx not the heat flux distribution of total heat transfer over the area Qx, then we get dQx/dx is Q0A0, which means that Q0A0 is constant, that means heat flux is linearly increasing, that means Qx is Q0A0x plus some constant C, whose value is the value of Q at x = 0. If you put x = 0, then you get C, okay, so this is a linear distribution of heat flux.

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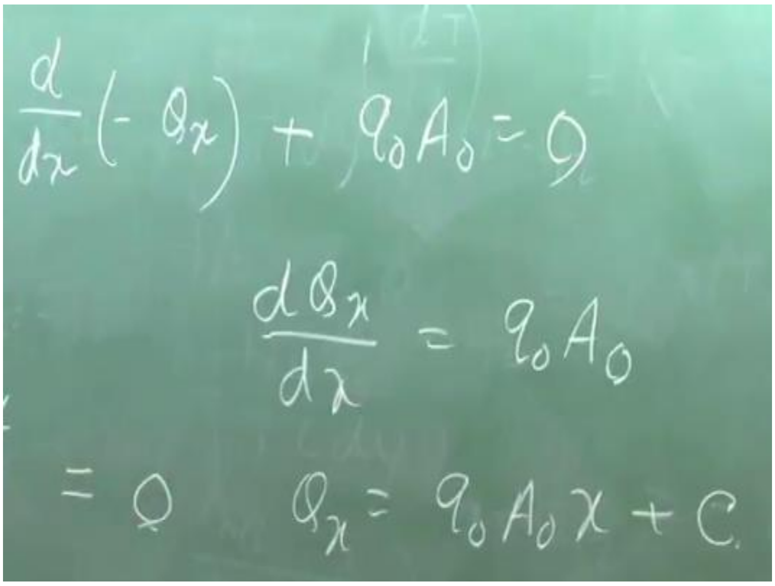

So, therefore, we showed that you write this general expression that d/dx of Qx plus QGAx is 0, that is your equation for variable area, variable thermal conductivity and Qx is KAx dT/dx minus, so this is minus actually, minus Qx. So, if you have to find out the temperature distribution, you write in this fashion.

If you have to find out the heat flux distribution, you write d/dx minus Qx, that means minus dQx/dx plus QGAx is 0, which is the first line after derivation, Qx plus del x. Qx plus del x is Qx plus QG2x times del x. Del x is cancelled because this is valid for any element of any del x, so therefore, del x is vanished and we get this expression minus dQx/dx plus QGAx equal to 0, okay, clear.

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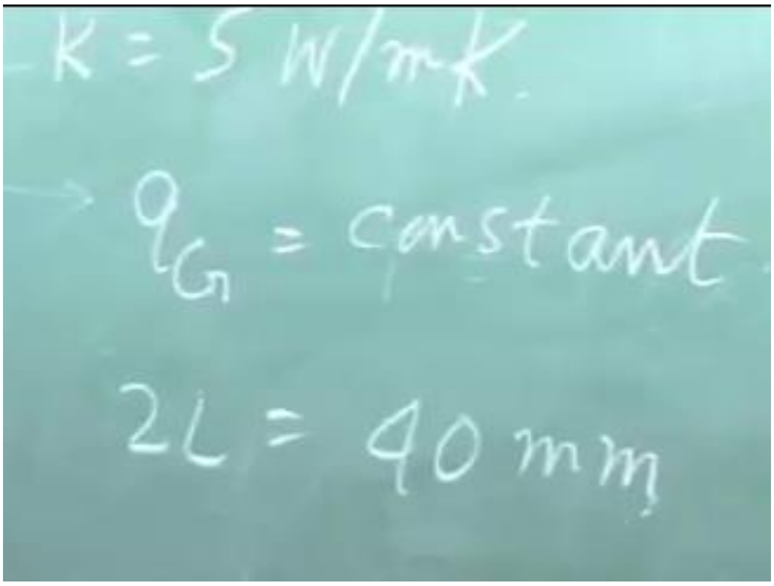

Now, next problem is we have a plane wall of thickness 40, that means a plane wall. I am doing in an exaggerated or amplified version, that L = 40 mm. Plane wall 40 mm and thermal conductivity K is 5 watt/mK, very good experiences uniform volumetric heat generation rate Q. That means here in my nomenclature, QG is constant, so there is no variation of QG, very good. The problem is simple.

While convectory heat transfer occurs at both its surface, x = minus L and plus L that means, the problem is given like this. I am translating it in this fashion QL and there is a figure for this problem, they are taking this. This is x = 0 that means this is x = minus L and x = plus L that means 2L = 40 mm, so they have prescribed that. That is why they are telling that while convection heat transfer occurs at both the surfaces, x = minus L and x = plus L.

Both surfaces, that means the convection heat transfer. Heat is transferred in both the direction, the way I have done that both surfaces the convection is there. Convection through the fluid, convection from the surface. That means, surface is being cooled by convection, I am not bothered whether it is natural or forced convection, some convection is there or with which it takes the heat from the surfaces because surfaces becomes hot due to the internal heat generation.

I am bothered for this type of problem, the knowledge which I have up to the date, that whether heat transfer coefficient is given or not. No, heat transfer coefficient is not given.

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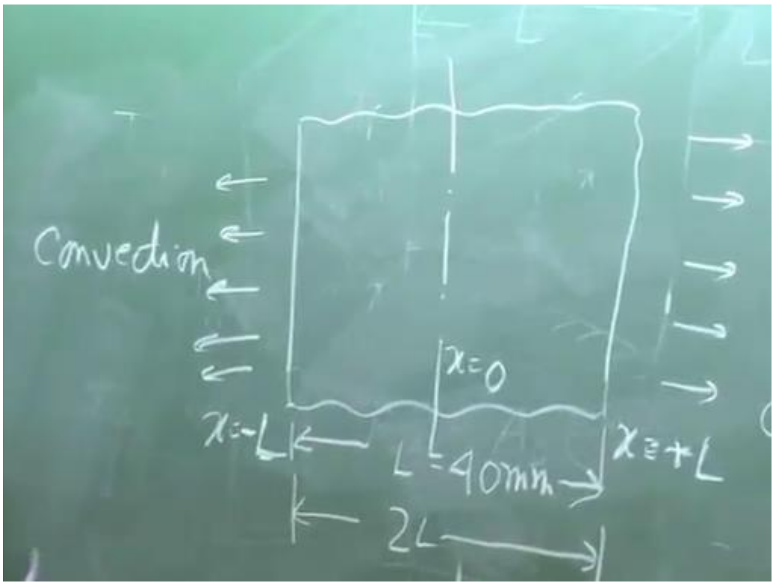

Instead, what is given the temperature within the solid is A plus Bx plus Cx square where A is 82.0 degree Celsius, B is minus 210 degree Celsius per meter and C is minus 2 into the 10 to the power 4 degree Celsius per meter square, x is in meter. Origin of the x coordinate is the meet point that I have already shown.

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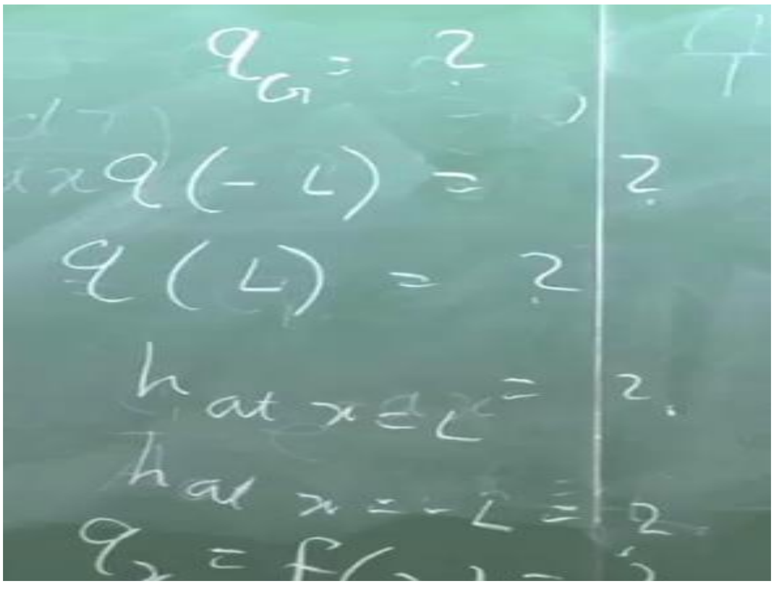

Now, what they ask, what is the value of volumetric heat generation rate QG. Determine the heat flux Q the heat flux at minus L and heat flux at plus L. What are the convection coefficient at x = L and what is the convection coefficient at x = minus, all these things have to be found out. An often an expression for the heat flux distribution, Qx, Qx as a function of x, so many things are asked for, this is because the problem is very simple.

Now, you tell me, first we have to find out the QG, what equation we will start with. This is a plane area, no variation, thermal conductivity is also constant, they have not told the variation of thermal conductivity. Thermal conductivity is also constant, so what we will do. We will start from which equation, please tell me. So, therefore, we will start from the simple equation d square T/dx square plus QG/K.

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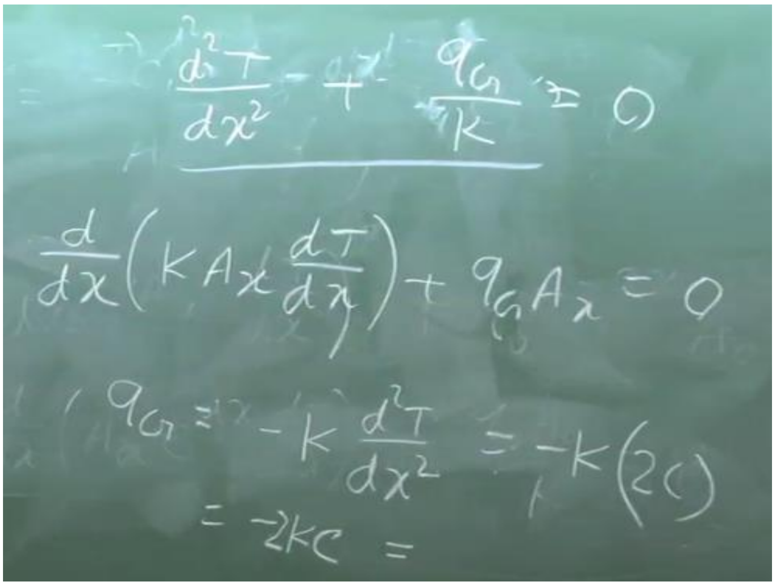

This is plane area, so we can develop this if we remember only one formula, or we can derive it from the fundamental that d/dx of KAx dT/dx plus QGAx 0, then everything is done from a simple heat balance or energy balance from a variable area, then for constant area and constant thermal conductivity, this becomes this, which is a special form can be obtained directly from the general heat conduction equation.

So, if you start from this, then, QG = minus K d square T/dx square. It is as simple as this. What is d square T/dx square tell me. I may make a mistake here. Now dT/dx is B plus 2Cx, so therefore d square T/dx square is 2C, okay. So, that means minus K into 2K into C. The value of K is given, what per mK, the value of C is given 2 into 10 to the power 4 degree Celsius per meter square, so if you substitute this, this will come.

I am giving you answer, Q is what. They have also not found out the answer, 2 into 10 to the power 4, 5 you just change into K into C10 and 20, 20 into 10 to the power 4, this is minus, plus 20 into 10 to the power 4 was permitted, so that you can get the answer, okay. Alright any problem, very simple.

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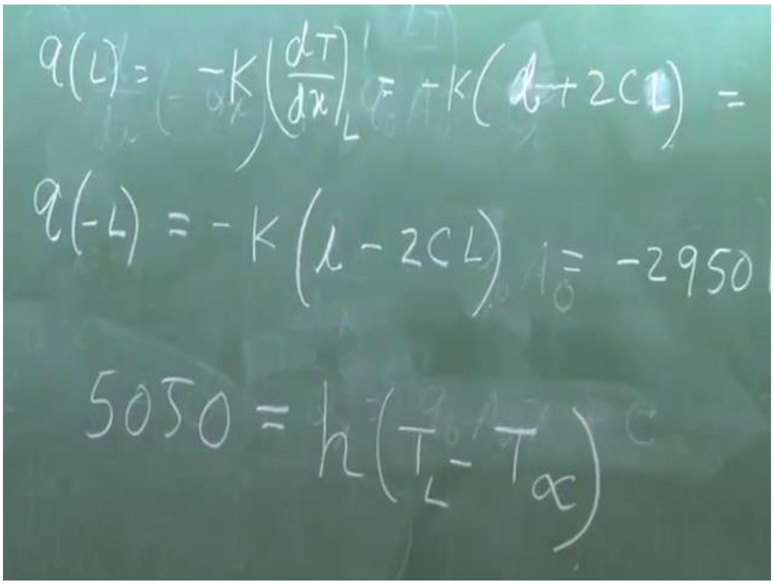

Now the next part is how to find out the heat flux q minus L and q plus L. How to find out, very simple. That q at L is minus K dT/dx at L = minus K, dT/dx is what is dT/dx b plus 2CL. Similarly, you can find out q at minus L = minus K b minus 2CL. Because x is plus L and x is minus L and this value if you put the value of K and if you put the value of b, K is 5 and b minus 21 and the value of L, L is 20 here, plus L, minus L, 2L is 40 mm.

You get the value of the qx at L is 5050 watt per meter square, because it is per unit area. Here, normal plane area, so always you express in terms of per unit area, that is heat flux. It is simply multiplied by the area at any section give you the heat flux and this thing is equal to q minus L comes to minus 2950, so if you substitute the value with the sign you get, this proves that the qL is in the positive direction of the x axis.

And q at minus L is in the negative direction, so the heat flowing out is this plus this, which will be equal to the, you can check the value of q, we have obtained that q into the volume, that means cross sectional times the length, so per unit area, this has to be matched. You see that it will be matched, so that this two fluxes added together must be equal to the total heat generation per unit area, that means, you q value is there times the 40 mm is the length, clear?

Now, how to find out h, next is how to find out h and x is equal to L and h at x is equal to minus L.

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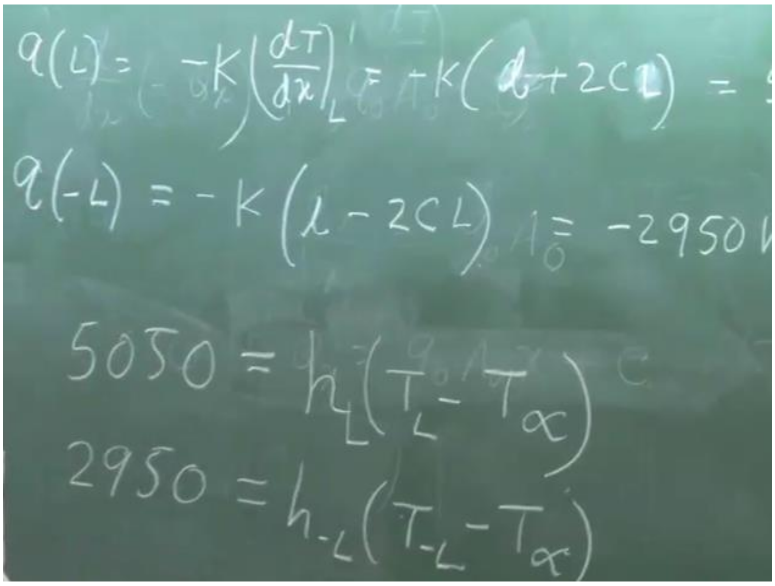

Here what you do, you take at L, so this heat has to be transferred by convection and by definition of heat transfer coefficient, the convection heat transfer can be written as per unit area, heat transfer coefficient into the difference of temperature means, surface temperature minus the air temperature, which we tell as free steam temperature. So, free steam temperature has to be given for the problem.

Otherwise I cannot find it, so free steam temperature is given 20 degree Celsius that means, free steam temperature T infinity is 20 degree Celsius, so therefore, you can find out this h at L, h at minus L you take the scalar value. This is the heat to be transferred and this will be equal to T minus L minus k, okay. Whenever I am finding out with the surface temperature minus the free steam temperature.

The surface temperature is always higher than the free steam temperature. You understand, I am not finding out the directions. I am finding out the quantity of heat that is being transferred by convection, so therefore, I will not do it with the plus/minus sign. In that case, a scalar quantity like h will come as a negative value, h is always a positive, it is a heat transfer coefficient. Do you understand?

So, therefore, the scalar value of the heat transfer is being equated with these as the surface temperature minus. TL and T minus L are already found out, how, we have found out qL, q minus L, so TL and T minus L will be found out by substituting x by L and minus L, this will only change and A, B, C we know, so it is extremely simple, school level thing, so that one can get the value of hL and hC and the values are, what are the values, I tell or that they have done it correct or not, I do not know.

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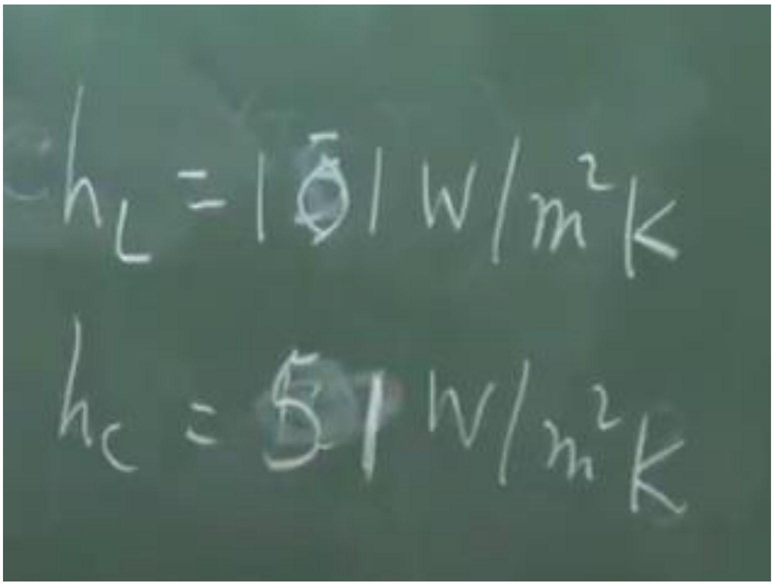

What are the values, you just check. If you do it correctly, so your thing will be correct, 51 watt per meter square K and 101 watt per meter square K. Now, this is the value of hL and h. Well, I have done the wrong thing. This will be 101. They are right and left. So, right is 101 means right plus L and left is minus L 51 and next is the heat flux distribution. Heat flux distribution is very simple. Heat flux is minus K dT/dx.

So, simply multiply dT/dx that B plus 2CxK, K is constant that means heat flux is linear, okay. Clear, any question for this problem, very simple. Okay, these problems are very simple and in examinations also, these type of problem will come. Now, there will be a problem which is a little different from what we have done so far. This problem is like this.

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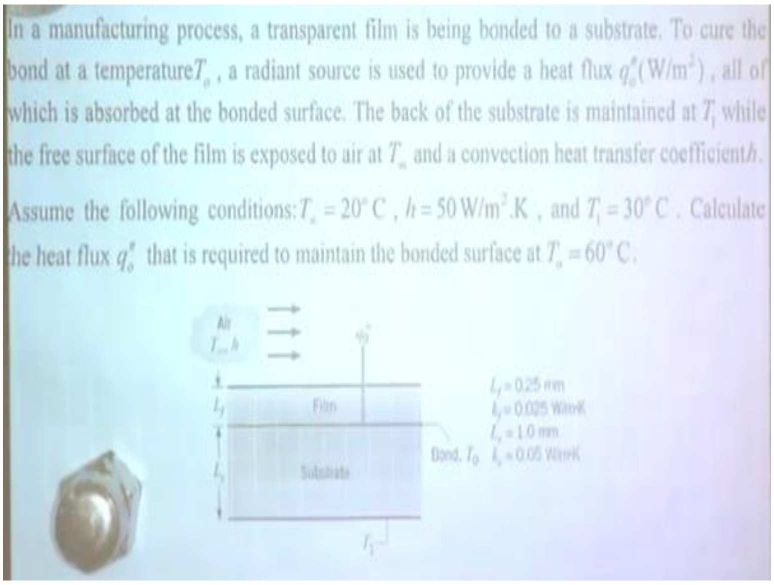

Now, this is a problem try to appreciate. I hope all of you can see. That problem requires a description. Well, in a manufacturing process, a transparent film is being bonded to a substrate. This is a statement of a practical process. This is a transparent film, which has to be bonded on a substrate.

This is not our botheration. Why this transparent film is being bonded to the surface or some metallurgical things or some process operation, which I am not bothered of, we are not bothered of, but this is the problem film and the substrate. To cure the bond at a temperature T0 that means bond surface should be maintained at a temperature T0 and to cure this bond, so that it has maintained at this temperature T0, a radiant source is used to provide a heat flux, Q0 told as watt per meter square, all of which is absorbed at the bonded surface.

That means, this radiant heat comes through the film where the film acts as a transferring medium and all the heat, which is being incident there is being absorbed by the substrate, okay. The back of the substrate is maintained at Ti. That means this is the back of the substrate, which is maintained at a constant temperature Ti. While the free surface of the film is exposed to air.

That means, this surface of the film is exposed to air at free stream. This air may be flowing, may not be flowing. This is shown to show that this air that the infinity temperature provides a convective environment to have convective heat transfer or convection transfer from the film surface. The film surface becomes hot obviously. The heat is absorbed at this bonded surface, film and substrate from where the heat will come out and film will be hot surface, free surface, the heat will come out.

Assume the following conditions, that means the air temperature is 20 degree Celsius, h the convective heat transfer coefficient is 50 watt per meter square K and the back surface temperature Ti is 30 degree Celsius, calculate the heat flux q0" that is the radiant heat flux that is required to maintain the bonded surface at 60 degree Celsius that means bonded surface T0 is 60 degree, Ti is 30 degree.

The infinity is 20 degree, h is 50 watt per meter square, so with all these things I have to find out the radiant heat flux, so problem is well understood by the language. Now, you think in terms of the energy flow that is the heat flow, what happens, this entire heat flux, now this is a plane area problem, so only in terms of heat flux we will tell. So, heat flux is being incident on the surface, film is transparent.

Now, you see when the heat comes here, a part of the heat physically, try to understand, is going through the film by conduction and then from the surface by convection and a part of the heat will go to the substrate by conduction. Convection has not to be taken into account because the temperature boundary condition has been heavy, which means that by taking into convection heat transfer temperature is fixed.

You have seen that, whenever we are dealing prescribed temperature at the surfaces, we are not bothered about convection. Because convection may be there, but because of the convection, somehow, he has gave the temperature T1 and T2, which is given for the problem, so students from pure mathematics point of view we will see that when T1 and T2 is given.

I do not require h, problem is solved, but physically that the convection heat transfer is taken into account to prescribe the temperature at the surface. This concept has to be made very clear. So, therefore, here it comes due to conduction from this temperature at the bonded surface to the temperature at the back, so this is purely a one-dimensional heat transfer through the substrate and film, which is flowing in this direction, through the film and downward to the substrate.

So, if we understand these heat flow scenario, then we can extract the thermal circuit, then heat is divided q0” into two parts, one part is going in series by accepting or overcoming this conduction resistance in the film, then overcoming the convection resistance in the air film over the film surface and another part is going overcoming the conduction resistance in the substrate. This is simply the problem.

Alright, any question, okay. I have not told that Lf and this value. This is also found there, but however, I am drawing it again.

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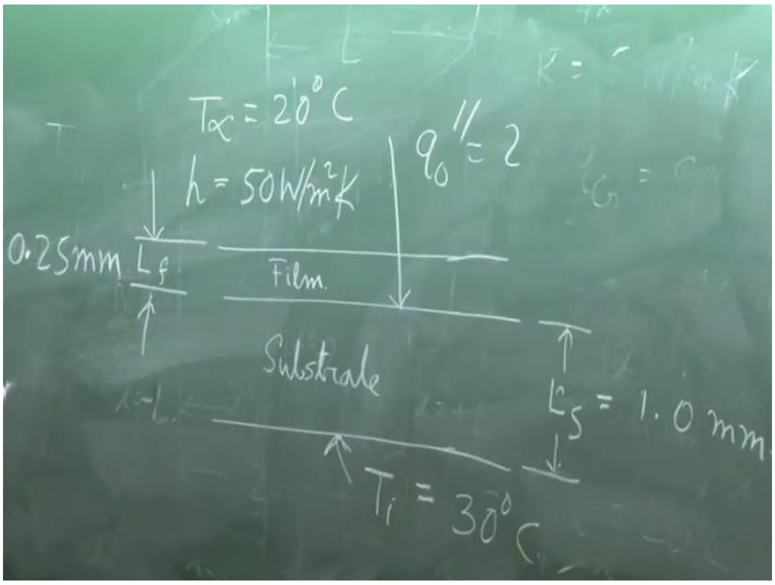

This is film, this is substrate, this is q0”. T infinity is given as, what is T infinity, 20 degree Celsius, h is given as 50 watts per meter square K. This has to be found out. Now, this distance, this is Lf, length of the film where the length of the film is given as 0.25 mm. Now, the substrate length Ls, length of the substrate, which equals to 1.0 mm. This surface is kept at Ti, which is equal to 30 degree Celsius.

What else is require K, thermal conductivity of the film Kf is given as 0.025 watt per meter K and Ks = 0.05 watt per meter K, so these are the values given and we have to solve this problem.

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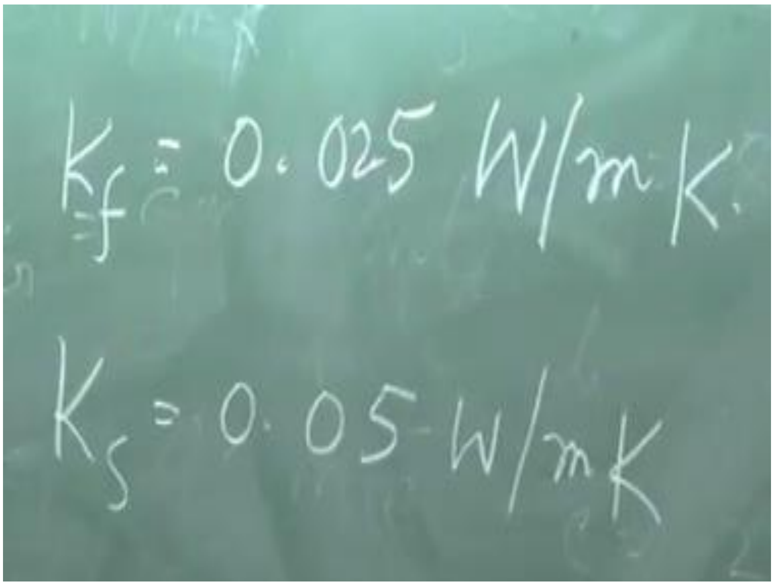

So, as I told you earlier, while showing this problem earlier that this is a problem where we have to conceive the energy flow. It is a steady state problem. Now the q0" a part of it is going through heat, let this is q1 and a part of it is going through this, let this is q2. That means q0" = q1 plus q2. Now, what q1 going there, q1 is by the conduction. With that, you draw the electrical analogous circuit, that will be better. So, let me draw that.

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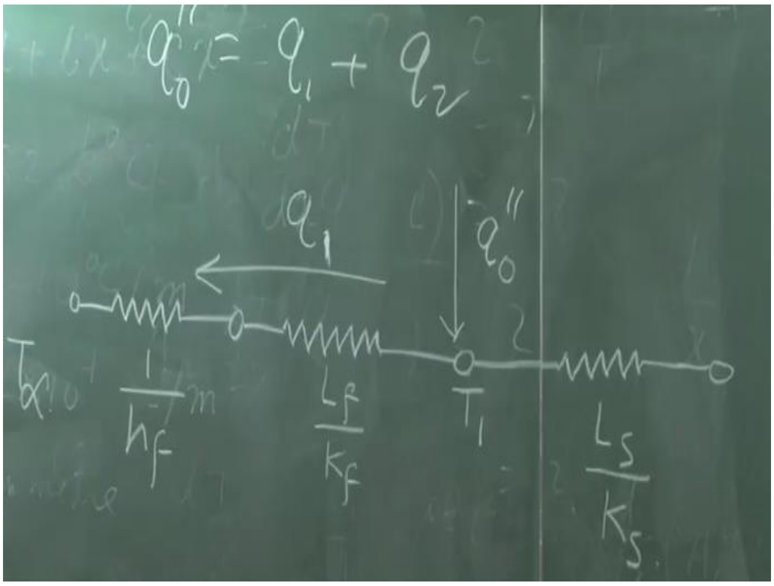

Without writing the equation, if you understand this, then it will be better that now the radiant heat, this is the node at bonded surface, that is T1 and this is the T infinity, so this is q1, which suffers that Lf/Kf as resistance, a I am not writing because we are finding out the heat flux q/a, so therefore air resistance is this, okay and this resistance is 1/hf and here this resistance is Ls/Ks, so this is the electrical network.

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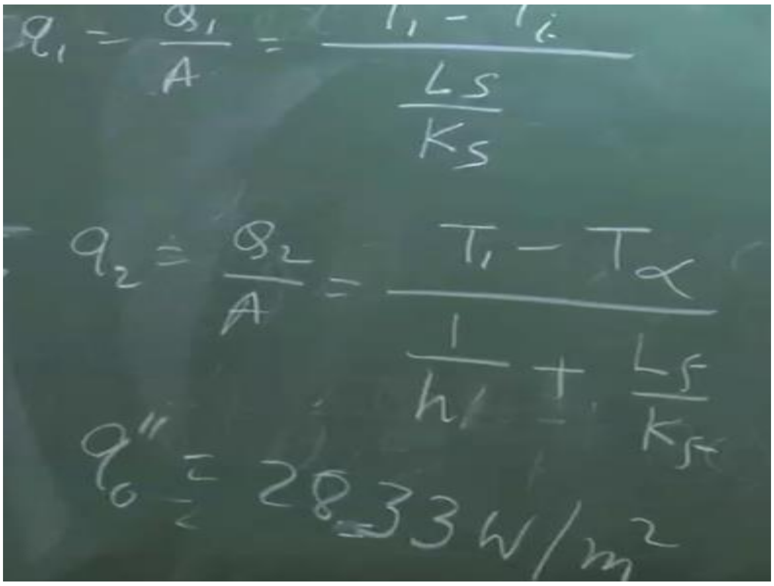

That means, you can generate this this way that q1 = Q1/A, which is equal to T1 minus Ti. This is the Ti node. Divided by Ls/Ks, okay and q2 is Q2/A, which is equal to T1 minus T infinity as did earlier 1/h plus, not hf because only one side h is there, f is not required, by Lf/Kf because it is a series problem. That means, the heat, which is going through the film first crosses the conduction resistance of the film, then the convection resistance and the heat, which is going to the substrate only through the conduction.

Because the end surface temperature is given, so this is the electrical network, so therefore, you can find out both q1 and q2, because all these things are given, Lf, Kf, Ls, Ks, T1, Ti, T infinity and you can find out a value of q. Is there any problem? Q2, this is q2. This is not retained. This is q2 going to the film. I am very happy that you are minutely listening to my lecture that this q1. Everything is given to cure the bond at 60 degree Celsius.

T1 is 60 degree Celsius, otherwise how can you solve. The bond surface temperature is given. The problem reads, we have not seen, to cure the bond at a temperature T0, okay, this T0 is 60 degree Celsius, all of which is absorbed the back surface is maintained at Ti while the free surface is exposed to T infinity. Now, the T0 is 60 degree Celsius. It is given, clear. Any problem, now the answer is, what is the answer?

If you do it, you have to see that the answer is 2833. The answer q0” is 2833 watt per meter square. Now, if the problem is stated this way, that radiant heat is incident on the film surface where the film acts as an opaque material, then what we will do. If film acts as an opaque material, then what we will do, that means this heat is not totally being transported to the surface, then what we will do. Think and try to do it. This is your problem that if the film is acting as not totally transparent medium, as an opaque medium, then what we will do and which data you require. Who has told that, very good.

That means, you have to know what percentage of the heat is going through that, from the surface and in that case, this side the heat will fall and go by convection one part and another part will suffer a series resistance heat transfer, conductive resistance of the film, conduction resistance of the substrate. That means, we have to know the percentage of the heat that is being transmitted.

If the film is totally opaque, things will not work, because heat will not reach to the bonded surface, so therefore in that case heat is incident on it and will be lost there and the percentage, which will go there will go through this, okay.