Conduction and Convection Heat Transfer
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Lecture-06
1D Steady State Heat Conduction in Plane Wall with Generation of Thermal

Okay. Good Morning to all of you and I welcome you all to this session of conduction and convection heat transfer. Last class, we were discussing about the steady 1-Dimensional heat conduction and we discussed various problems related to plane 1 with boundary conditions, given as temperature at the 2 phases and how to take care of the convection, heat transfer. When the fluid temperature at the two sides are specified.

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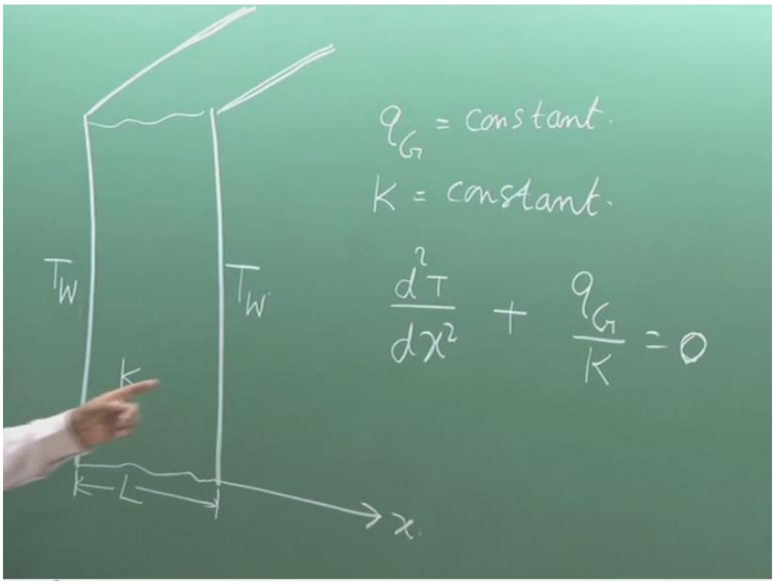

We discuss the analogous electrical circuit specifying the conduction and convection resistance. Today, we will discuss a simple classical problem of conduction, 1-dimensional steady state conduction heat transfer with heat generation. It is a simple but classical problem. The problem description is like this; again, a plane wall whose width is L, while dimension in the other directions; that means this direction perpendicular to the plane of the boat.

These heights are much higher than these dimension, so that we can consider the temperature is a function of 1 coordinate system, x and the problem is specified like this. There is a heat generation or thermal energy generation within this wall, this solid material. In a way, the volumetric heat generation qG is constant, that means with uniform rate of thermal energy generation per unit volume.

Point to point the thermal energy generation per unit volume is constant, that means the total thermal energy generated can be expressed as qG times the total volume. Now the boundary conditions are prescribed like that, both the walls are kept at the temperature Tw. Now we are interested in finding out the temperature distribution and the heat flux. Now here you see both the sides are at the same temperature Tw.

So how the heat transfer will take place? Physically understand the problem, because of the thermal energy generation within the solid, the temperature will go high, so therefore the heat will flow to both these surfaces, Tw is relatively small and we expect that because of the generation of thermal energy, temperature will be much higher than Tw, is a practical problem.

Cooling a wall, cooling a plate, so that Tw is kept small, so therefore heat will automatically flow, when the temperature will rise because of the energy generation. So, it is not necessary that 1 surface has to be at higher temperature, and another for example, without any generation of thermal energy, both surface at same temperature, there is not heat flow. Until and unless you initiate the problem by making temperature within these solid higher than that.

This has been done by the heat generation. Now these problem, a starting point is this, we know in a plane area; and another thing in the problem, thermal conductivity K is constant. Now if we start with our basic equation of 1 dimensional heat conduction with heat generation, this is a starting point that we discussed earlier. Because this our 1-dimensional heat conduction with heat generation.

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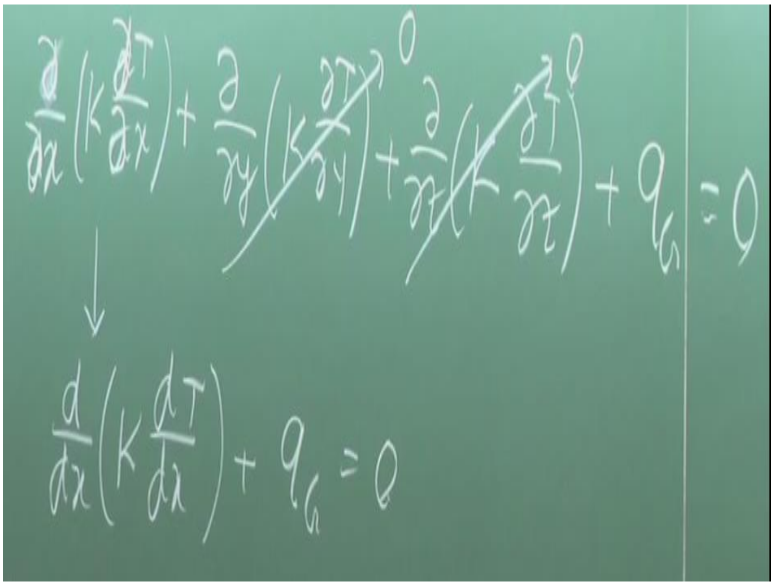

Last class, I have seen that in 1 dimensional; if you again write this, you will get that d/dx(K*dT/dx), sorry; Del/Del x (K*Del T/ Del x) + Del/Del y (K*Del T/ Del y) + Del/ Del J(K*dT/dJ) + qG = 0. Because temporal derivative of temperature with time that is 0 for steady state so for one dimensional this quantity will be 0. again, I am doing this, that only one term in the special derivatives.

For constant K, this now Del Del becomes ordinary differential equations, ordinary differential Nomenclature K* d T/dx + qG = 0 and when thermal conductivity is constant, do it come to this equation. Now we have to solve this equation, now the rest part is the solution of this equation that constant thermal conductivity and do it constant volumetric heat generation rate that is, qG to solve this equation which is extremely simple we have done it in school level.

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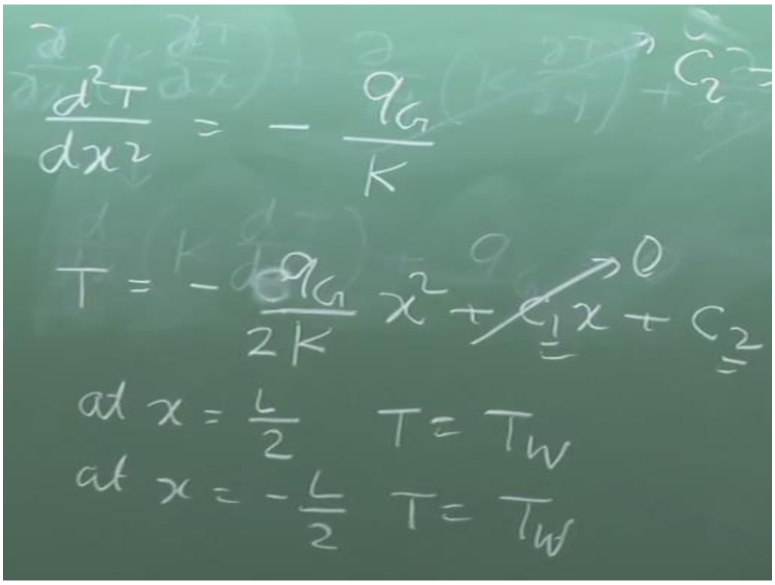

If you write this d square T in this fashion, d square T/dx square = - qG / k by the problem, both qG and k are constant, so therefore if we integrate twice we get this solution like this T= -, a constant known, this is qG/K is constant, taking qG/K constant,2* x square + c1x + c2 at 2 constant. You integrate, first you get qG(x/K), then x square/2+c1x+c2. Now to know the constant c1 and c2, we have to know the boundary conditions.

What are the boundary conditions? Now when you specify the boundary condition, you have to specify the coordinate axis. Now this type of problem usually for convenience, we take the coordinate at the middle. Why? because this is the cemetric problem, the geometry and the boundary condition is such that the both sides it is Tw, that means if you a mid plane, about the mid plane, the problem is symmetry.

Q is uniform throughout, which means it is symmetric about the mid plane, the boundary conditions Tw at the two phases are same; that means the problem is symmetric about this mid plane, so therefore this mid plane or mid axis, as you seen this view, is taken at x=0, so that let phase is that x= -L/2 and write phase x= L/2, but it does not matter you can take x, 0 here, so that this will be x=L.

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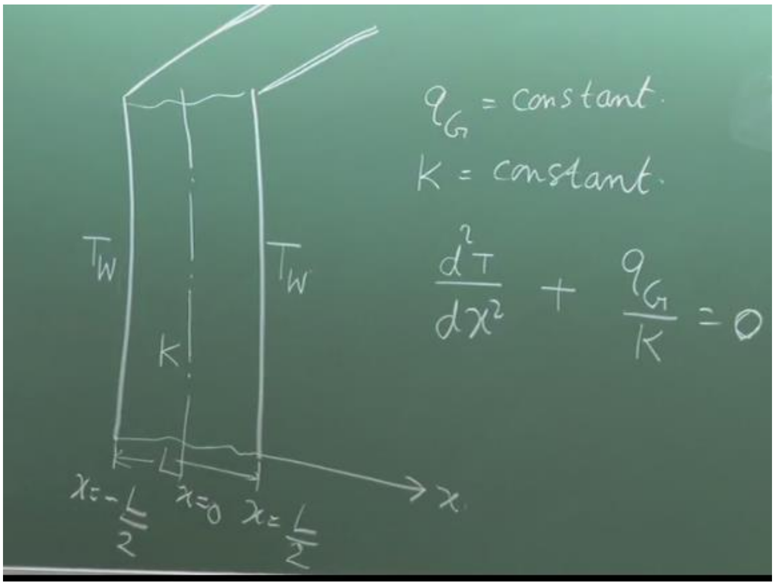

Now if you take the axis at the middle because of the symmetry of the problem, then boundary conditions are that at x=L/2, T=Tw, at the same time at x= -L/2, T=Tw. Now if we use this two-boundary condition, mathematically c1 will be 0, but this c1, 0 is obvious without writing the boundary condition, this is because the problem is symmetry, so that they are cannot be in any term containing x.

Another row you can look at that, when this is a symmetrical problem, that means temperature distribution will also be symmetry about these axis, that means these axis will correspond to either a maximum or minimum temperature, which means the derivative is 0, which is sometime known as symmetry boundary condition at x=0, dT/dx is 0. This we call as symmetry boundary condition or symmetric boundary condition.

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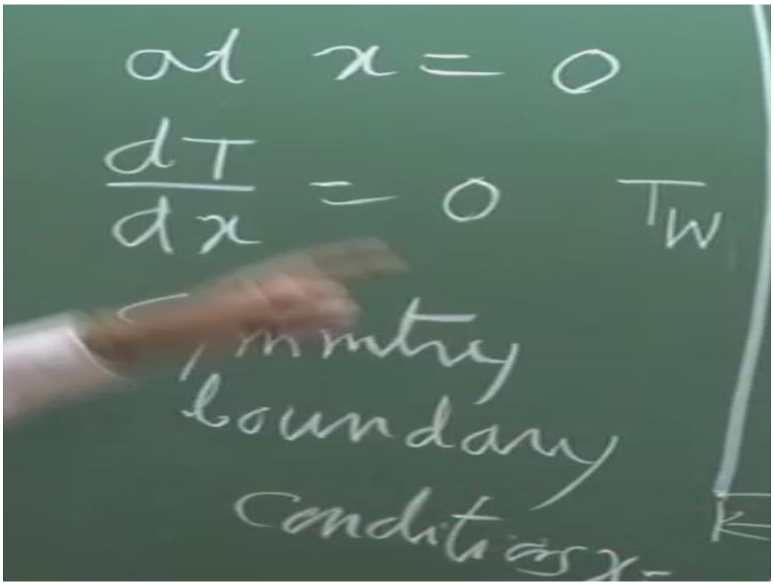

That means if x=0 is the lying of symmetry, then they are the variable in minimum. If you straight put these boundary conditions here, you will find out that c1 is 0, c1 cannot exist. Because the temperature function, if it is symmetric, it cannot have a term x, which will assume a plus value in the right side of the axis and the minus value of the left side of the axis.

These are extremely simple thing, but from many angle that can be considered, so that immediately you make c1, 0. So, what is c2? c2 will be simply (Tw + qG/) L/2, if I use any one of these, that will give the same result that means 8 K, L square, this is c2. Then the final temperature distribution will be T, now if we put c2 here –Tw=qG/8K*L square – qG/ 2K* x square, which we can write in these fashion, qG/8K, taking these common into Lsquare- 4x square.

So, this is precisely the temperature distribution within the rod. Why we are neglected c1 means, we have taken these symmetry boundary condition dT/dx, 0 at x=0. Did you see that, you will see dT/dx is 0, when x will be 0, and the second derivative is negative which means the temperature acting its maximum value because of the internal thermal energy generation at the axis.

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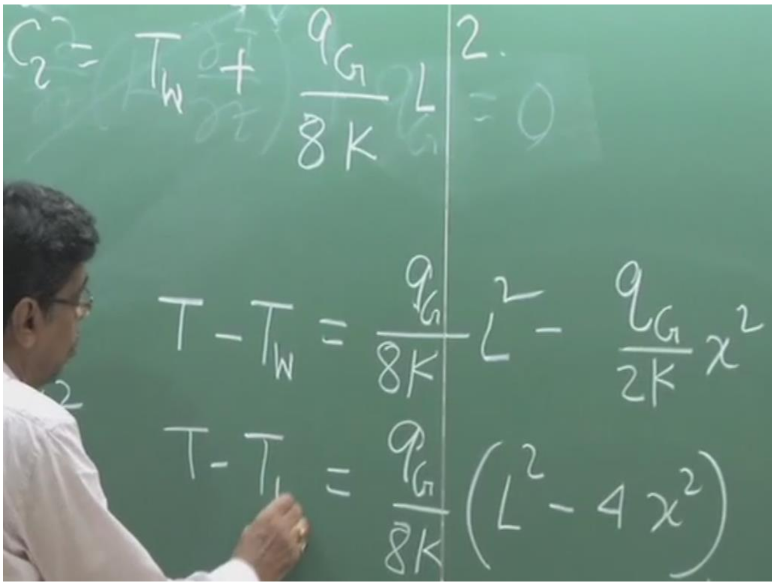

So, if we expressed this at T0, (T0 is the maximum) T=T0 when x=0, that is be maximum temperature, then we can write T0-Tw is qG/8K*L square. So, I can find out the maximum temperature, we occurs at the centre or the mid axis like this and we can express the temperature distribution in a normalised fashion like this, T-Tw/T0-Tw=1-4x square/L square.

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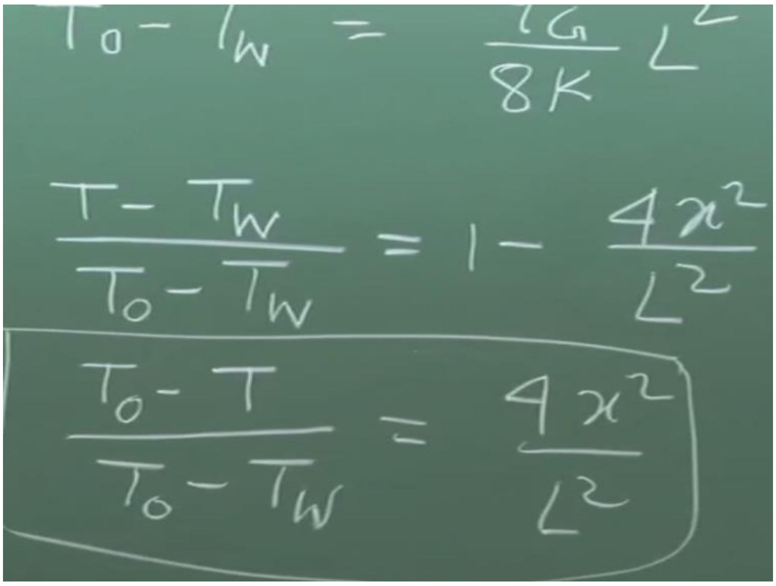

This is one way of expressing this in the normalised fashion or if you add -1 on both these sides, then you can write this thing as T0-T/T0-Tw, -1 I am adding and changing the sign is = 4x square /L square. Because this shows a parabolic variation with the maximum value at the centre, so this can be represented in this figure by (()) (14:25) Tw scale here, like this, which is right T0 value in the same scale.

So, this will be the temperature variation, that means the equation of this is T0-T/T0-Tw=4x square/ L square, that means x=0, it will assume a value of T0, sorry it will be something wrong, T0-T0 at, Yes, x=0, the value is T=Tw at x=L/2, that means T=T0, okay T0 is T, so it is all right. I think it is x=0, T=T0 and x=L/2, T=Tw. This is perfectly all right, that means this is a parabolic distribution.

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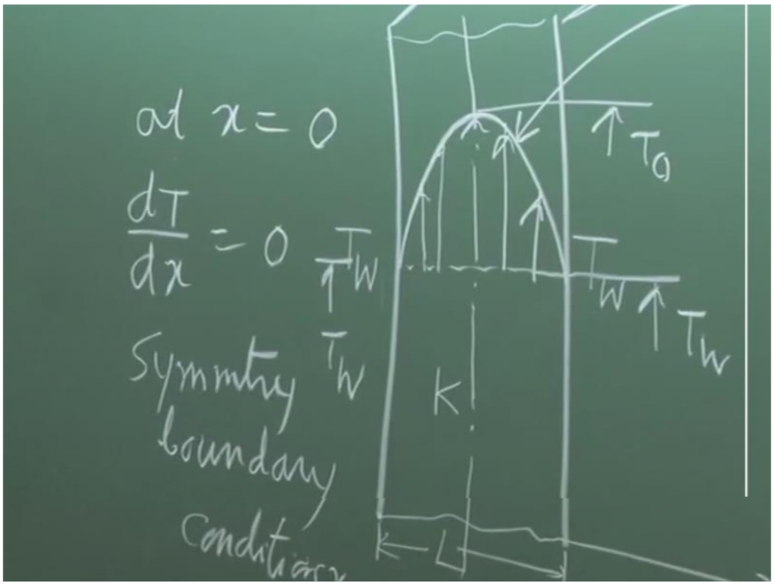

Now we are interested with the heat flux from the surface. What are the heat losses from the surface, we have to know this? Accordingly, we can design the coolant, we can find out what coolant we will use? What type of heat transfer coefficient I have to provide? Whether we will make a natural convection or force convections? So therefore, these parameter is very important Q at x=L/2 and Q at x= -L/2, these two things are required.

Now let us write that Q = -KA*dT/dx, that is why Fourier law of heat conduction. So, at x=L/2, it will be dT/dx at x=L/2, under –L/2, it is dT/dx as x=-L/2. So dT/dx at x=L/2 is what? That you can find out from these dT/dx at x=L/2 is T0-Tw, from this equation, dT/dx is T0-Tw, now this 8x/L square and at x=L/2 that is 8x/ L square and if x=L/2, it is -4/L. This is okay.

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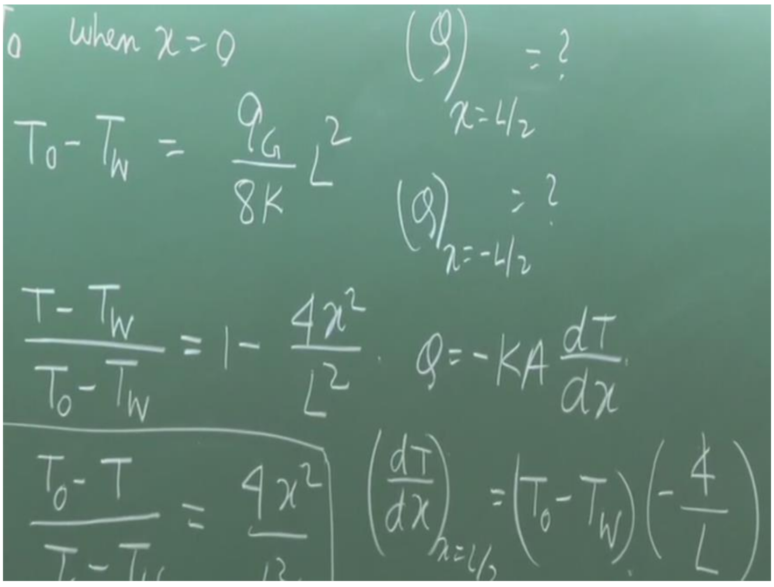

T-Tw/T0-Tw is 1-4x square/ L square, so therefore dT/dx will be T0-Tw (-8x/L square), 8*L/2, that means 4/L. This is okay. That means –(T0-Tw) *4/L. Then what is dT/dx at x=-L/2? Try to find out the same thing but with a positive sign, that means, (T0-Tw) *4/L. Okay. Clear. Now here Qx, Q at x=L/2 is –KA(dT/dx) that means it is positive. That means KA(T0-Tw) *L/4, okay.

Q at x=-L/2 is (is there any mistake, oh sorry! it will be 4/L)- KA(T0-Tw) *4/L, okay. So, therefore we can find out the rate of heat transfer in terms of T0 and Tw by this. Now here the positive sign indicates that the heat is transferred in the positive direction of the x, this is q and here heat is transfer in the negative direction of it. So therefore, we can find out the heat transfer from the two surfaces by finding out this dT/dx. Okay.

Any problem? All right. So, you can also change by energy balance that if you make summation of these two, that is the scalar sum of these two heat transfer that will be ultimately equal to the heat generation, if we write this Tw in terms of this, this is your task that Tw in terms of T0, T0-Tw is qG/8K*L square, you just substitute it and you find out the value of it, that I am not doing, Qx=L/2, that means it is a very good check that Q at x=-L/2.

Now if you just substitute T0-Tw in terms of the heat generation qG/8K*L square, that means there will be, A and this KK will cancel and this will ultimately comes like ALQ. If you do it, this will come like that, that means if you know eliminate this T0-Tw, they check that the total heat loss from the two surfaces, the sign is the direction but if I tick this scalar, some of these true, that means energy balance says, that heat loss from both the surfaces equals to be total heat and energy generation.

Because we are satisfied the same conservation in a differential form, point to point, that means a gross has to be balance. There is no doubt of it. It is a stupid phenomenon that why are you doing sir? but still for satisfaction, we have used the differential equation for point to point. Why not you make it gross balance as a whole; that the heat which is flowing out from the two surfaces equals the heat generation, which means a steady state concept.

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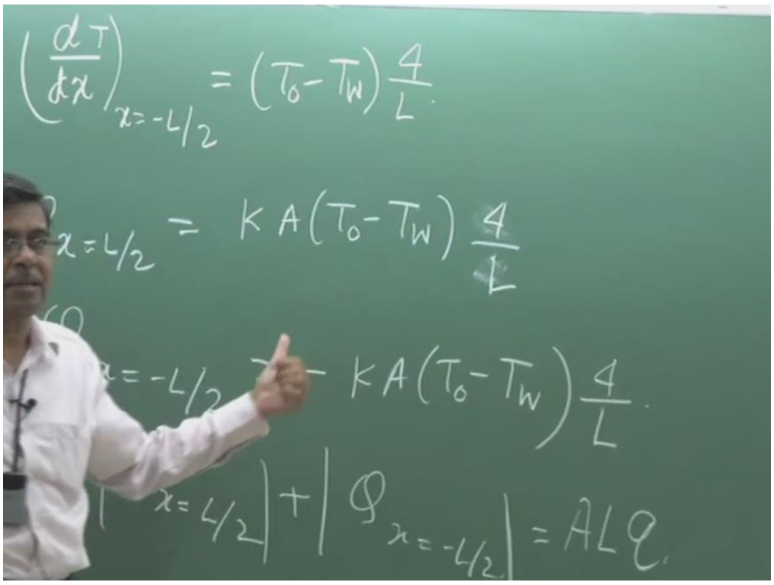

Persists, that means there is no energy accumulation within the system. So, this is a very simple classical example of heat generation. Now, I will solve some important and interesting problems in the class, but before that, I will discuss one thing which has not been told very explicitly in your class already by this time, though I mention it at the beginning but I will tell that, (Let me rub the board).

Before going to the problem, I discuss one thing that I told that 1 dimensional steady state heat transfer truly happens, when the area normal to the heat flux is not varying in the direction of heat flow, but sometime by some approximation, we use 1 dimensional analysis (()) (25:11) let us consider a tapered rod like this, where the boundary condition is such that these phases is kept at some time as T1, these phase is T2 and T1 > T2.

Sometime the problem is specified by the insulating the lateral surfaces, but what is done, that here, if x is in this direction or you can take in the middle axis, the same thing x, then we tell that T is a function of x, and sometime the problem is a 1-dimensional problem, what is the meaning of that? When the area varies like this in the direction of heat flow, in fact, T became also a function of y.

But if the area variation is not that much or the boundary conditions are such, the lateral surface and the end surface says, we can neglect the variation in the y direction, that means cross sectional variation and we can consider almost a uniform temperature, the way we have consider for plane wall and that is only a function of its, where the heat is flowing by virtue of the main potential difference T1 and T2.

We prescribe the problem these way that though truly it is the 2-dimensional heat conduction T is also a function of y. Truly it is a function of x and y, may be a weak function of y, but it is a function of both x and y. We are considering area varies temperature and that area varies temperature as a function of x, whatever may be in both the cases, how do you participate? But it is a function of x, so there it is an area, where temperature or constant temperature neglecting its variation.

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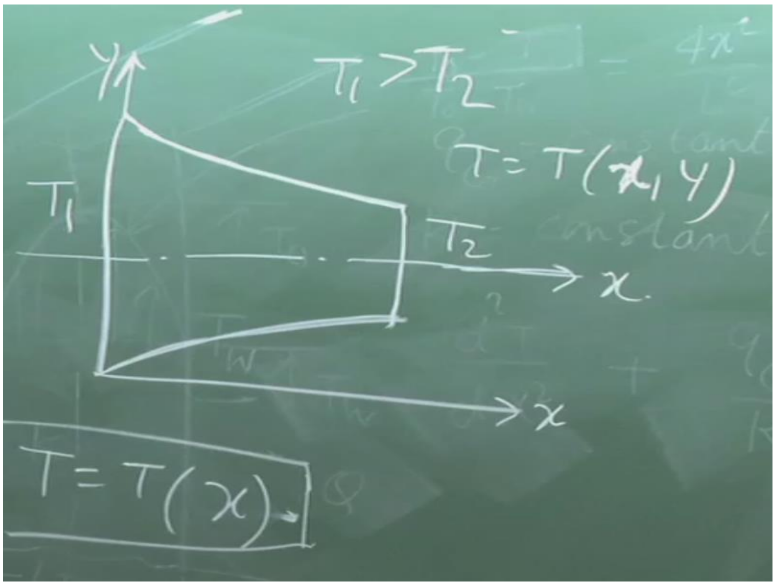

But we cannot neglect the variation of area, so therefore we cannot write the equation d square, dT, x square 0, because we know the general equation in these form, KA(dT/dx) +qG*A=0. This is the basic equation, where from does it come? Your general heat conduction equation is not there A, coming into these, because there we have consider plane area.

Now here if you derive this, I think it will be always better; I suggest you that for steady 1-dimensional heat conduction, better you always derive the basic equation relating the temperature variation by taking a small element. The way we derive the heat conduction equation in 3 dimension. Taking a small element at a distance x, where the area is Ax; now Ax, why I am telling that this A is a function of x, Ax (now I write Ax).

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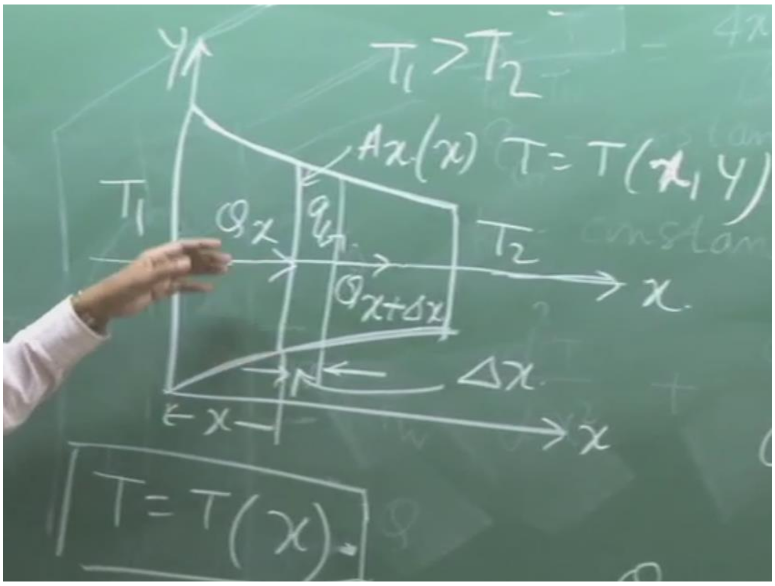

I derive this equation Ax, which is a function of x, it is changing. The heat flux is Qx and we consider the element of thickness; that means this one, delta x, so therefore the heat which is going out Q at x + delta x, Now I write by Fourier heat conduction equation at that section Ax at a distance x. Qx is – Kax (dT/dx), this T is a temperature which is cross sectionally uniform or a cross sectionally average temperature.

We can write the expansion in Taylor series, Qx+ dx is Qx, that is (–KAx (dT/dx)) + (this quantity qx) d/dx(-KA*dT/dx), overall this is delta x, but this is an infinite series, we have to take higher rod at tone, hot. I am writing in delta x, that means delta x square /2, delta x cube/3, and you the Taylors series and since delta x is extremely small in the limit tends to 0, so these terms are been neglected.

So therefore, from energy valence, if we consider at that section, qG is the heat generation. Then definitely I can write, Qx + Del x, what is coming out from that small element is nothing but Qx+, the amount of energy that is generated, that is qG*Ax*delta x, so therefore derivation of general heat conduction equation or for 1 dimensional simple heat conduction equation for your purpose.

The procedure is same, you make an element take the energy balance by describing the heat flux by Fourier's heat conduction equation and in energy balance we are not consider the change in internal energy, because it is at steady state, that means the energy coming in and energy going out must balance with the energy generated, so that no energy may be accumulated or depleted with in the element.

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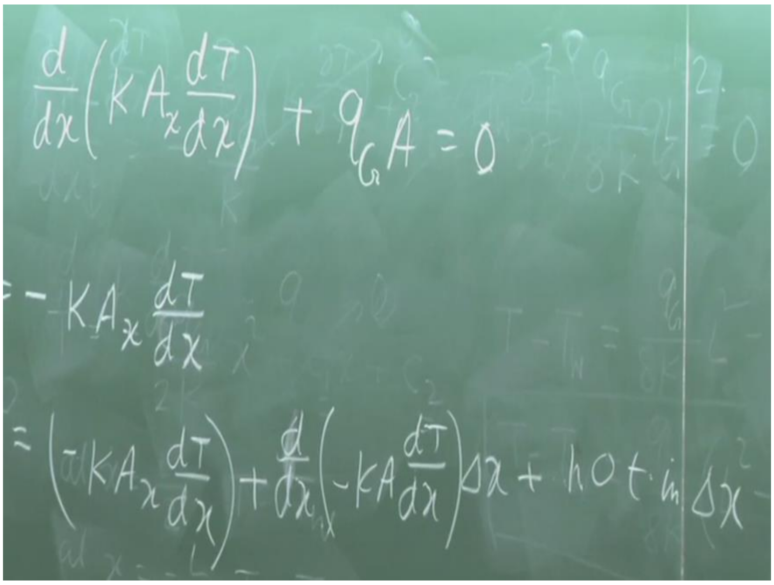

So, if you write this and if you make this, then you get that equation. You write this Qx + dx in the side, so this, this cancels, so this becomes equal to qG*Ax* delta x. So, ultimately by substituting these 2 equate this. So that means our starting point if it will the area varies along with the thermal conductivity is this which in a very special case (I can rub this now) becomes that if qG is 0, that means there is no heat generation.

If qG is 0, then this becomes d/dx (KAx * dT/dx), that means we will see that, without heat generation, these quantity is 0 and if we take K, constant, thermal conductivity does not vary. Then we will come to this conclusion d/dx (Ax*dT/dx) is 0, that means still we cannot say that dT/dx is constant, temperature is linear, because we are considering the variation of area. But if the area variation is not there, that means the plane wall, then it comes out and d square, dT/dx =0, then only we will get a linear temperature variation.

There will be problems where area variation will be there, that means if the area varies, how do you tackle the 1-dimensional heat conduction equation, clear. This can be also appreciated that if we consider the variation will go high, how I am telling that mean temperature comes into consideration? let me do this way, that you can be satisfied these also, that we have already consider a mean temperature. Where from these comes?

Let us consider a section at x, now if we consider that T is a function of x and y, this is an integrated form. Now if I have to find out the value of Qx, that means that at this section, what is the value of Qx? Then what I do, let me consider this is h, the height at that section and we consider a unique width of the rod perpendicular to the both. Now since T is a varying with y, so dT/dx will change from y to y, this is not same along the cross section.

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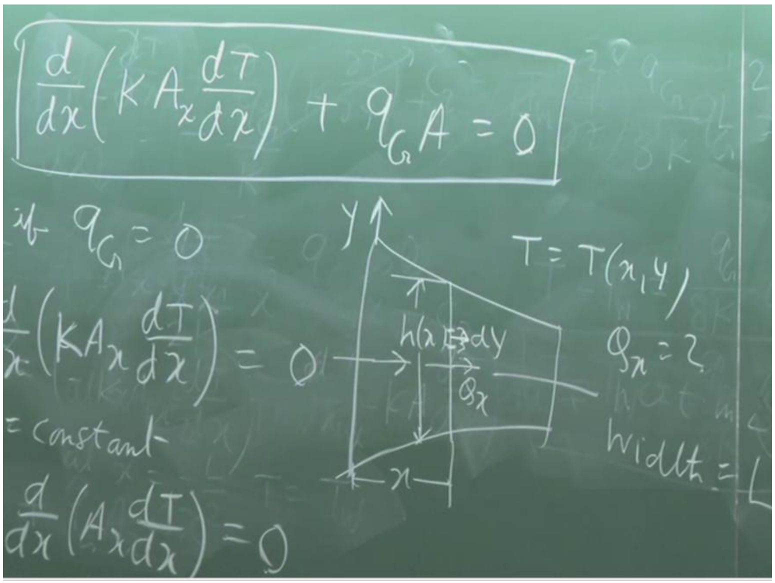

We allow this variation, then what we do, we take an element dy in this height, whose area is dy times this width. Let this width be L, width perpendicular to the both be L. So, if we tell this the heat flux through this elemental area dy*L, not h*L, that is Ax, not that, then we can write, Del Qx= -K*L*dy (Del T/Del x) at that particular y. because T is a function of y also, so Del T/Del x will vary from point to time, I take an element dy.

So that I can integrate that means Qx is nothing but the integration of Del Qx. Now to integrate this, let me take an axis, symmetry axis is x here, so that +h/2 to –h/2. What we do? We do this one, - integration of KL*dy (del T/Del x), -h/2 to h/2. Clear, L is the width in this direction. Now if we consider K, note to be a function of y, only T is a function of y. K may be a function of x, may not be a function of x, rather I take K out, -KL.

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Then what I write –h/2 to h/2(Del T/Del x) * dy. Now if T is a continuous function of both x and y, which is supposed to be there in a physical system without any discontinuity, then I can take differentiation out of this integration, that means Qx= -KL (Del/ Del x) integration of T. because this is Del/Del x of T. T (dy), -h/2 to h/2, that means I can take the differentiation out of the integration and this is nothing but a mean temperature.

If I define now a mean temperature, Tm at any cross sectional average mean temperature, which I can define TL(dy) from –h/2 to h/2 divided by h. So therefore, if I define a mean temperature like this, then Qx becomes =-KLh (Del Tm/ Del x) and in that case, when we have defined mean temperature cross sectionally average, then Del x will not be there. That will be simply ordinary differential an L*h is the Ax, area that that section.

That will when I represent the heat flux in a variable cross section at –KAx (dT/dx). This we consider that T either that cross sectionally average temperature, which is the function of x only or we discard variation over the cross section. So, until and unless you appreciate this, it will be difficult to understand that variation of area, how I will take care of, that means we have to start from this equation.

If you start from this equation, d square T/dx square =0, you are gone, because that is not the governing equation, that does not take care of the variation of the area in direction of it flow. Truly speaking this is not a 2-dimensional heat transfer problem, here T is a function of both x and y. If the area variation is not great or not large and the boundary condition are such, then only we can do that. Okay.

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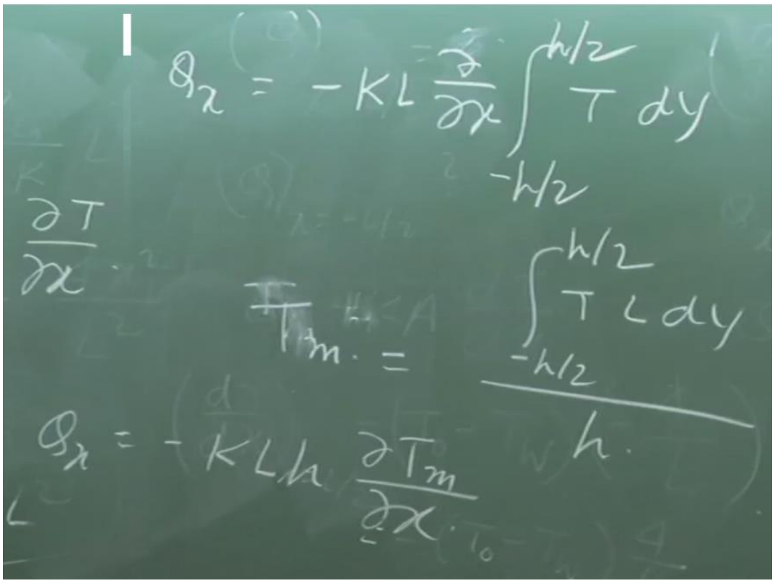

After this now, I will solve some interesting problem. Now the first problem which I will solve is like this please, this is very important, until and unless we solve problems, we will not be capable to develop our ability to analyse problems. Now these problem states that it is steady state heat transfer in a tapered rod like this, there is in some problem which I will not tell you the language, I will not show you the language, you can write in your own language.

This problem is described like this, but for a big problem, I will show you the language. So now this is a tapered rod, the same thing which I did now. It is a straight forward application of a symmetrical tapered rod, where x is this direction. This is x, where it is given. The total Q which is coming here is 6000watt, Q=6000watt. Now a steady state heat transfer, now A(x), here they are writing like that as a function of x.

That means any cross-sectional area this is A(x), which is given by a (1-x), where x is in meter and this in meter square, how we consider a unique depth of the tapered rod, that means depth. If you consider L, that means, this direction, this direction dimension L, then L=1 meter, so x in meter, so therefore this into 1, so this is the expression. The temperature distribution as wizard experimentally is 300(1-2x-x cube) it is given in kelvin (big K).

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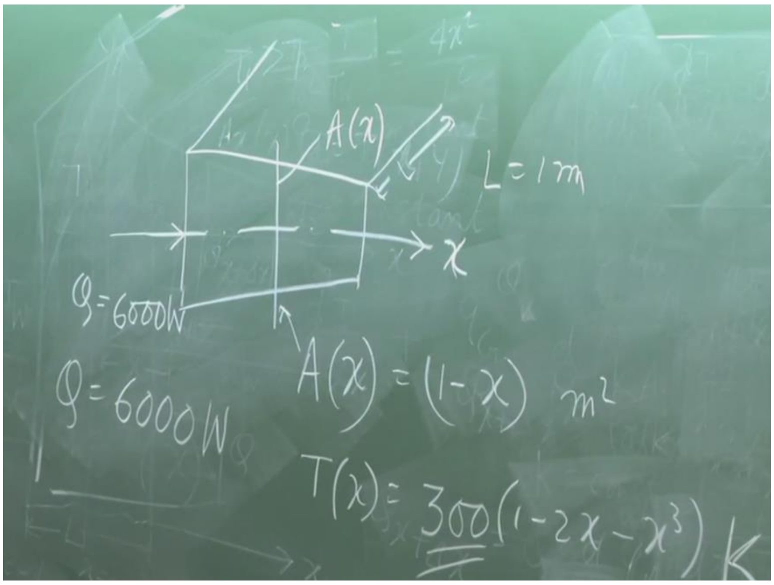

X in meter and this is kelvin, 300 is dimensional constant. This will be kelvin/meter, oh No! kevin /meter cube, because 1-2x-3x square, this is given Tx. Now what we have to find out? We find out the expression for K. This is the simplest problem to start, that means this is a steady state equation, so which equation we will write? We will write d/dx of KA(x), A is the function of x* d/dx=0, that means K A(x), dT/dx is 0.

But we are not interested in the temperature distribution, temperature distribution is known. So, we are not interested in this. So, what we have to do to find out the K? We know the heat flux, steady state, total heat transfer not heat flux. I am sorry. Here the heat flux will not because done. Because it will be the times the area, the total heat transfer will be constant, that means Qx, we have to use this equation is –KAx (dT/dx).

We have to use this equation, because Qx is given, so we write this equation and find the value of K. S what will you do? We will write the value that given 6000watt, that means 6000=-K*A, A is what? (1-x) *dT/dx, that means 300*(2-3x square) dT/dx, -Kx, -2, because this is -, 300*(-2-3x square), -2x, correct. Sorry that (-2-3x square). This is the simplest problem and finally K comes out to be 20/(1-x) *(2+3x square).

This is to start with, just one-man match like that, so warming up with these problem that if a variable area is there, can I take care of this area variation, that means, our starting point that you have to remember that without thermal energy generation, this is the equation, not d square/dT square is 0, taking care of everything. If thermal conductivity is constant, this comes here, so d/dx (KA(x)*dT/dx) =0.

But here temperature distribution is given already, I have to find out K and I have to see what is given in the problem? total heat conduction, which is be same for all section under steady state without heat generation, that means for any section, Qx is –KAx (dT/dx), where Qx is 6000, going through all sections. Clear. This is as simple as any teaching, very simple, primary school level things like that.

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