Conduction and Convection Heat Transfer

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Lecture - 05
1D Steady State Heat Conduction in Plane Wall Without Generation of Thermal Energy

Good Morning to all of you and I welcome you all to this session of Conduction and Convection Heat Transfer. In the last class Prof. Suman Chakraborty deduced in general the heat conduction equation. Now today, I will be discussing One Dimensional Steady State Heat Conduction, we will start today and it will continue for few classes. Now I repeat again in any heat conduction problem not only heat conduction, convection also the conduction affected by flow.

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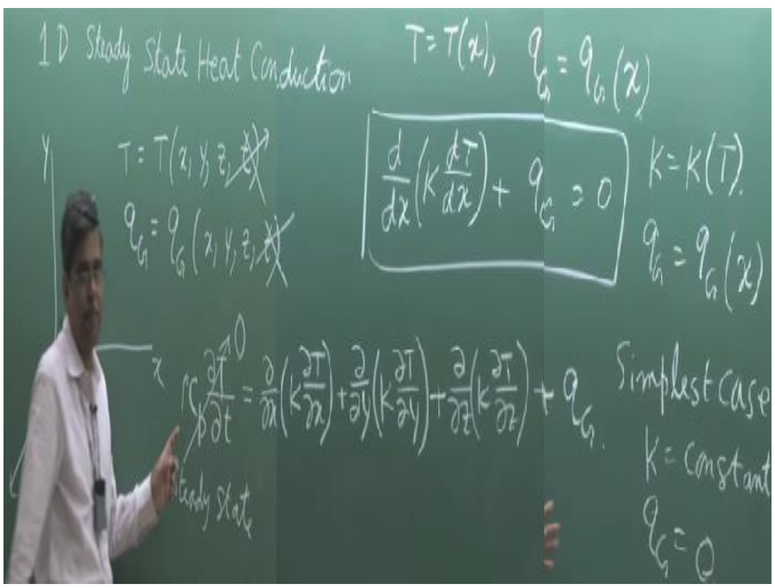

We are mostly interested with the heat flux q which flows through the system and in convection the heat flux which flows from a solid surface to a fluid or from a fluid to a solid surface when fluid is in motion adjacent to the solid. As you know, the Fourier heat conduction law states that heat flux is proportionate to the temperature gradient and precisely q flowing in one direction that is heat flux, flowing in one direction is equal to - thermal conductivity k time tdx, the temperature gradient along that direction.

So therefore, it is very important to know the temperature gradient to find out the heat flux. It is other way it is the temperature gradient which determines the heat flux and to know the temperature gradient we have to know the temperature field that scaler field, temperature field means temperature as a function of the space coordinates. For example, x as I have told x, y, z if it is a Cartesian coordinate.

And on top of that if the problem is unsteady in nature that means if temperature is also time dependent so we have to know the temperature as a function of space coordinate and the time because the heat flux will then change from instant to instant and this expression of temperature in terms of the space coordinates and the time is deduced in consideration of the first law of thermodynamics that is the conservation of energy.

And it is usually expressed in a differential form which was deduced by Prof. Suman Chakraborty in the last class. So therefore, now what will be our job is to use these equations under some special circumstances. We have to appreciate physically that yes, how this special circumstances in practice occurred and how can we make the simplification of the general heat conduction equation for those cases.

So therefore, I start with again the same thing that if we denote a Cartesian coordinate frame like this then we can tell that temperature T in general is a function of the space coordinate x, y, z and time t in a conducting medium. In a heat conducting medium usually solid so as the mode of heat transfer is by pure conduction.

Now along with that this may so happen after our observation in practice that there are several conducting system while conducting heat, they generate energy which colloquial we tell generation of heat but they generate thermal energy because of some chemical reactions or by any other action.

Sometimes we absorb thermal energy so therefore generation of thermal energy is also very important so that we also took care of this generation of thermal energy in deducing the expression that this function in terms of differential form so that we express this generation of thermal energy a small qg because I am using the q a heat flux small q I will be using qg which is the generation of thermal energy per unit volume and which becomes point function like the temperature that means at any point.

We can define the quantity that the generation of thermal energy per unit volume the system which does not generate it okay for that qg will be zero at all point. The system may generate at all points which may be also dependent of the space coordinates or may be uniform or the system may generate at some part of heat the thermal energy all this things will be taken care off in solving the equation these are mathematical problem.

But in general, we write this qG is also a function of x, y, z, and t. So, taking this into consideration that there is a temperature filed continuous temperature field in the space coordinates and also varying with time. Similarly, the system generates thermal energy which is represented by the generation per unit volume at a point which also in general will be function of space coordinates and time.

We applied the conservation of energy principle and deduced the general heat conduction equation which I write now again in the Cartesian coordinates system like this rho Cp del T del t = del del x of K del T del x + del del y of K del T del y + del del z of K del T del z + qc So, this is the general heat conduction equation.

Now if we make simplifications to this, first if we make simplification of steady state mean both the temperature and the heat generation ceases to be a function of t, t is not there so therefore this temporal derivative is zero this term on the left-hand side represent the change of internal energy in the system.

So, this term will be zero so that in the element if you consider it as a control volume or an element in solid that influx of energy and a flux of energy along with the generation of energy they balance each other. If you do not have any generation that whatever is coming in is going out. So that is the principle of steady state as I already told in one of the earlier classes.

Now mathematically this becomes zero then therefore this right-hand side equal to zero is the general heat conduction equation in Cartesian coordinate system with heat generation q thermal energy generation sometimes I may tell heat generation which is usually told widely but this is not strictly correct generation of thermal energy and this nomenclature is generation per unit volume which is a point function and in that case, that means under steady state this will be function of x, y, z.

Now if we further assume that it is a one dimensional that means if we consider that temperature is a function of x only that means this dependence I cut similarly if it has to be a one-dimensional problem of heat conduction is generation of thermal energy has to be a function of x only or sometimes it may be constant that means the polymeric heat generation rate that is with the rate of again I am telling heat generation.

However volumetric generation of thermal energy that means the rate of thermal energy generation per unit time may be constant at each and every point in the medium in that case this will be simply constant even the x dependence will not be there. But in general x dependence will be there in that case we can write this equation with this nomenclature T is a function of x and qG is a function of x we can write instead of partial derivative.

Now we can use the coordinate differential that d/dx of K, dt/dx ordinary differential because T is a function of x + qG is equal to zero and this is the equation for temperature distribution for steady one-dimensional heat conduction. And therefore, if we have to find out the temperature distribution in the h direction then we have to integrate this equation. Now if we have to integrate this equation it is not easy to integrate until or unless we know the variable K.

Now what happen is that in general K is a function of temperature I already told that K being a property of the medium is dependent of temperature even isotropic medium K is dependent only on temperature. So, K may be a temperature dependent and qG in one dimensional steady state will be a function of x and this also may be a function of temperature and temperature also indirectly a function of x so therefore here I write simply qg as a function of x

If I know this thing, then only I can integrate this equation if I know the explicit form of this function. So therefore, what happens afterwards is purely mathematic. The concept of heat transfer as far as steady one-dimensional heat conduction is here when we know this equation then it is the application and the different situations with variable conductivity with temperature with variable thermal energy generation is the problem of mathematic.

Now for a very simple case, simplest case when K is constant that means K is not a function of temperature thermal conductivity is constant in the direction of heat flow and there is no generation of thermal energy that means qg, zero at each and every point then what happens this becomes d square t, dx square equal to zero.

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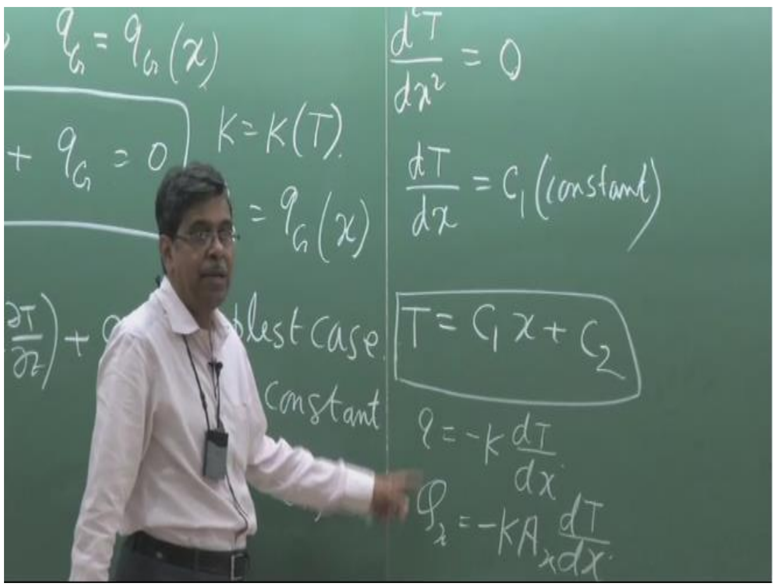

This gives because K comes out of this derivative d, dx and qg, zero which means that temperature gradient dT, dx is constant let us denote it by C1 constant and if we further integrate we get the temperature as a linear function of x where x is this direction along which we consider the temperature varies and temperature does not varies in other two directions y and z in this particular Cartesian frame of reference.

So therefore, this is linear profile now again I tell you that if you are asked that under which condition we get in heat conduction we get a linear temperature profile. This is the case where the steady state one dimensional heat conduction without generation of thermal energy with constant thermal conductivity.

We get a linear temperature distribution but here is catch again I am telling you here, we always assume the area, area is not coming here in this equation that means when we find, we try to find out the heat flux q is equal to - k, dt by dx, at any point heat flux mean this heat flow per unite normal to the direction of heat flow that means normal to this x direction.

At any point if we consider and elemental area normal to this direction of x that is the direction of heat flow qx, heat flux is the per unit area. And if we find out the heat flux sorry here heat flux over an area then we multiply with the area A. I write Ax means that is the area normal to the x coordinate but this x is not required because the heat flux is defined per unit area normal to the direction of heat flow and heat flux is proportional to the temperature gradient along these directions.

If these two information are there I will find from my common sense that since dt by dx is the temperature gradient, heat is flowing along the x direction so therefore now area which will be multiplied here to find out the total heat flux or the total rate of heat transfer across an area A which is nothing but the area normal to the x direction. So therefore, it is not always necessary to write it and this case this area for a one-dimensional problem is constant in the direction of heat flow.

But sometimes in a variable area which is truly not one dimensional we use an integrated approach that I will tell you in little detail afterward or we define an average temperature over a cross section and solve the problem as one dimensional that will be discussed later on. But here at this moment when we drive it from a differential equation and get this d square t, dx square zero we get the, this is from the differential form.

A linear temperature profile under constant thermal conductivity and zero thermal energy generation like this where the normal area remains constant. So, with this I can now tell you that this we realize in practice in a plane wall like this as I already told what are this practical realization these type of thing that means you can consider a plate or a wall, plain wall whose area is a plane surface, cross in the direction of x.

We consider this as x and specify the geometry as the length L of this wall whose dimension in other directions that means this y direction and the z direction are much higher than this length and if we specify the temperature at the surface at constant T1 and this surface constant T2 and with this thing that T1 greater than T2. Then if we solve this equation, we get then the C1 and C2 the two constant will be found out from these two boundary condition which is very simple that at x is equal to zero. T is equal to T1 and at x is equal to L, T is equal to T2.

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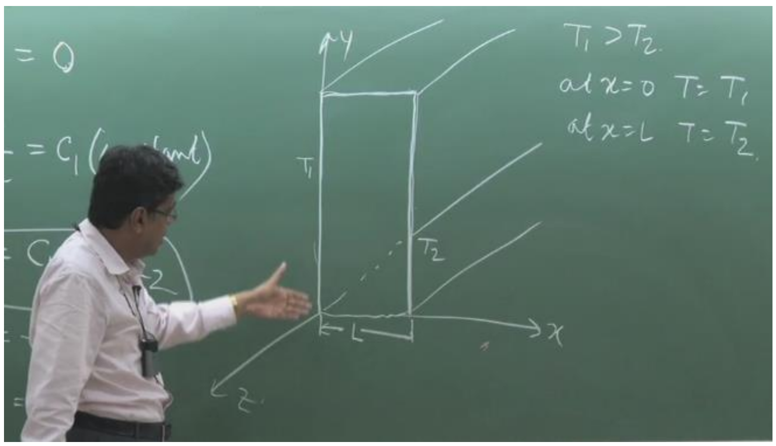

That means if a problem is prescribed like that a long wall with a thickness L having a surface, this surface left hand surface at T1 which is higher than that of the right-hand surface T2. Write the expression for heat flux Q total heat flux or heat transfer Q on the steady straight as we know under steady straight the same heat flows through these whatever coming in is going out what is the Q and what is the temperature distribution.

So, this is the most simple problem, what is the temperature distribution and what is the heat flux. Now temperature distribution will be found by finding out the C1 and C2 by using these boundaries partition and you get an expression if you do so T is equal to T1 - T1-T2 divided by L into x. First boundary condition x is equal to zero, T is equal to T1 you get C2 is Q1 and the second boundary condition if you give, you get the value of C2 which is C1 which indicate the slope which will be T2 - T1 divided by L.

And we write in this fashion that means this is a linear temperature distribution with a negative slope I can express this like this if I represent with some scale the T1 and T2 like this. That means if I take this ordinate as the temperature scale I can show a linear distribution that is with a negative slope which is given by this. Now what is dt by dx? Now dT by dx is definitely T2 - T1, T1 - T2 that means if you take this T2 - T1 divided by L.

And obviously T1 is greater than T2 that is the phenomenological law that heat flow from high to low temperature so therefore dt, dx is negative. That means heat flux is along the negative temperature gradient. Now when I use this expression by introducing A as the area of this slab then Q becomes equal to K into A, into T1 - T2 divided by L.

The simplest form which is being taught at the school level that means with all these assumptions we can write this that means for a long slab or a wall whose thickness L is much small compared to other dimension and if we can give the two surfaces at constant temperature T1 and T2 one is higher than other here T1 is higher than T2 then and with thermal conductivity K constant we get the heat flux equation like this.

Which is independent of x this can be written in a fashion like this T1 - T2 divided by L by KA now this looks like an equation that flux is equal to the potential difference divided by some quantity which includes the geometry and the conduction property that mean the length and area and the thermal conductivity and combined, this define the resistance because flux is equal to the potential difference of that scaler potential which cause the flux divided by the resistant. So, this can be perceived as conduction resistance, Rcond

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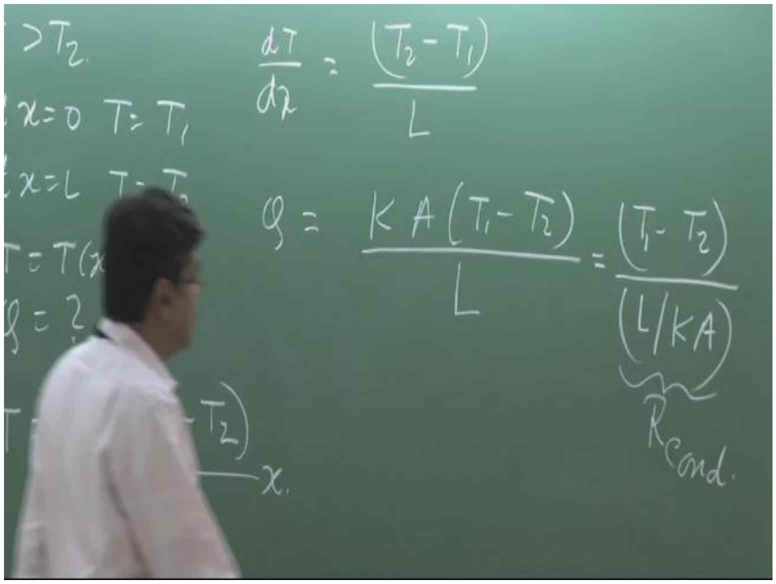

I am writing in short R conduction and this can be expressed shown as an electrical analogous circuit like this as an electrical analogous circuit like this. This potential is T1 this potential T2 and T1 is greater than T2 so therefore a heat flux is flowing like this and this R conduction is equal to L by KA and it is analogous electrical circuit and it is very simple and similar identical to the Ohm's law.

That is mean current flowing through a DC resister where the flux is the directly proportional to the potential difference since the resistance is constant geometry are fixed and thermal conductivity is constant based on which this linear profile deduction is made. Now usually what happens that this type of problem sometimes is prescribed in a different way. Now this type of problems sometime prescribe like this.

Let me draw again this wall similar thing the geometry is same that means its length is less than the dimensions in other directions. But the problem is prescribed not by specifying the temperature at the two surfaces here the problem is specified temperature at the two surfaces then only you can express the temperature profile and the heat transfer.

But here this problem is sometime specified like this a practical problem a slab is exposed on one side to a hot gas and another side cold air which is a very, very popular problem consider the wall of a furnace. It is other way in case of wall of a cold chamber inside is cold outside is hot and wall of a furnace this inside is hot gas and outside is cold air. So, what happens heat is coming from the hot gas to this surface by convection.

Then it flows to the surface through the surface by conduction and again goes to the cold air and the problem is specified by some temperature this is the scale I am just showing you in this direction the hot temperature that means the temperature of the hot gas Th and the cold temperature here, cold air temperature from the same datum, Th and Tc and you have to find out what is the value of Q.

So, in this case what happens if the problem is at steady state that means the heat which comes by convection to this surface from the hot gas is the same one that is conducted through these solid and again is convected from this surface to the cold gas and in this case, we assume that T1 which is not known, T1 is the temperature at this surface and T2 is the temperature at this surface and just by our common sense.

We understand that this T1 will be lower than Th and T2 will be lower than T1 and Tc will be lower than T2 that means Th is greater than T1, is greater than T2, greater then Tc but in this case problem is specified in terms of Th and Tc. Now what is the circumstances that here what happens if the hot gas is there hot gas may be flowing, may not be flowing. In case of a stagnant hot gas there will be a recirculate, there will be a natural circulation, flow like this which I discussed in one of the earlier classes.

This will be discussed in detail when we will take or discuss the convection, heat transfer. Now this is exposed to an (()) (28:15) cold air where this surface is hotter with respect to the cold air. So, there will be a flow natural due to buoyancy flow of air like that and this flow even in absence of any forced flow justify these different from the conduction that’s why I told in a fluid medium when there is a solid adjacent to a fluid, fluid cannot be kept at stationary.

So, there will be flow that may be a forced flow or that may be a natural flow, buoyancy induced flow because of which this heat transfer will be convection heat transfer different from the conduction. Not this conviction heat transfer characteristic is that the temperature Th changes to

T1 within a very short distance from this plan, convection resistance is offered like this. That means if we measure the temperature.

You will see this Th actually goes like this and within a thin film let us consider this as film the temperature drops from Th to T1 and this may not be linear. That depends upon the convection heat transfer equation the convection temperature field from here we get a linear temperature profile T1 to T2 by the similar deduction as we did for this case and then from T2 there is a thin film adhering to the surface within which the temperature change will take place

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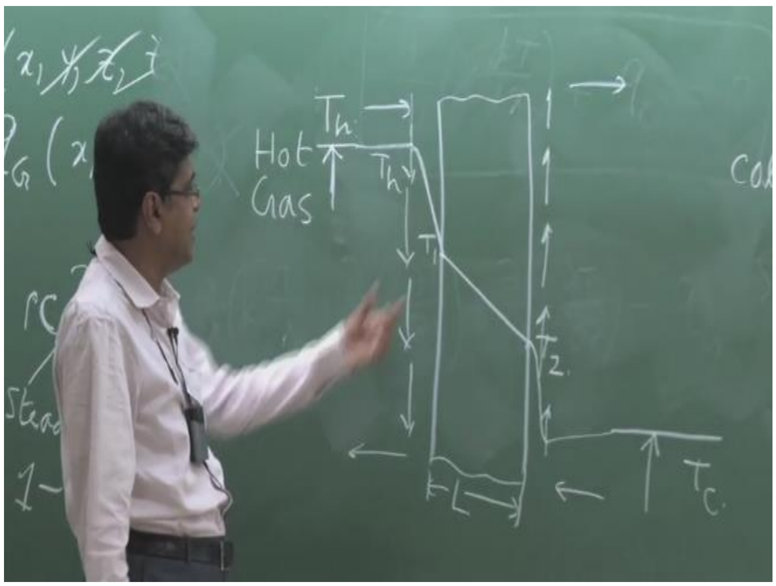

And this is defined as the thermal boundary layer which we will discuss afterward we accept this and at the same time we accept that this q which is coming from hot gas to the cold gas sorry hot gas to this left surface is given by this side q is given by heat transfer coefficient. The problem is also prescribed by a heat transfer coefficient on the hot side hh and the heat transfer coefficient at the cold side hc.

And we consider this heat transfer coefficient is constant over the area in both the sides then we know from convection heat transfer that the q can be defined as Th - T1 as I told you in one of the earlier classes that now we do not have to know anything about the mechanism of convection. How this is being written but whenever we know the heat transfer coefficient h then we can relate the heat flux into area sorry into area.

The total heat transfer is h that means heat transfer coefficient times the difference between the fluid temperature and the surface temperature. Here I am writing the positive value of the heat that means heat is flowing in this direction x so therefore temperature difference I am taking from the hot gas to the left plate times the area of the plate into hh.

Now the heat which is going out from here is again similar way hc into area into T2 - Tc, the same heat. Now if I write this here we write this Q is equal to now in sequence if I write then hh into the area, area is same is a plane are not varying also in the direction of heat flux T1. The same heat flux Q now goes from this surface to this surface flows by conduction which is nothing but T1 - T2 divided by L by KA which already we deduced here.

If the two temperatures the surface temperature T1, T2 conduction is given by this and the same heat Q because steady state, the same heat is there it is going out which is hc into A into T2 - Tc. Now if we write the temperature differences in terms of the heat flux and add it up then what we get and if we write all the equations rearrange it in the fashion as we did here that flux is equal to potential difference divided by the resistance.

Then what we get or other way if I write in this fashion. This Th - this is little rearrangement it is Q into 1 by hh A. Similarly, if I write T1 - T2 it is Q into 1 by L by KA. Oh, Q by hh not by 1 by hh. I am extremely. Sorry, Q into 1 by hh it is all right. Second one is Q into L by KA I am extremely sorry and third one is equal to T2 - TC is equal to Q into 1 by hcA and now if we add this three we can get Th - Tc is equal to Q into 1 by hhA + L by kA + 1 by hcA.

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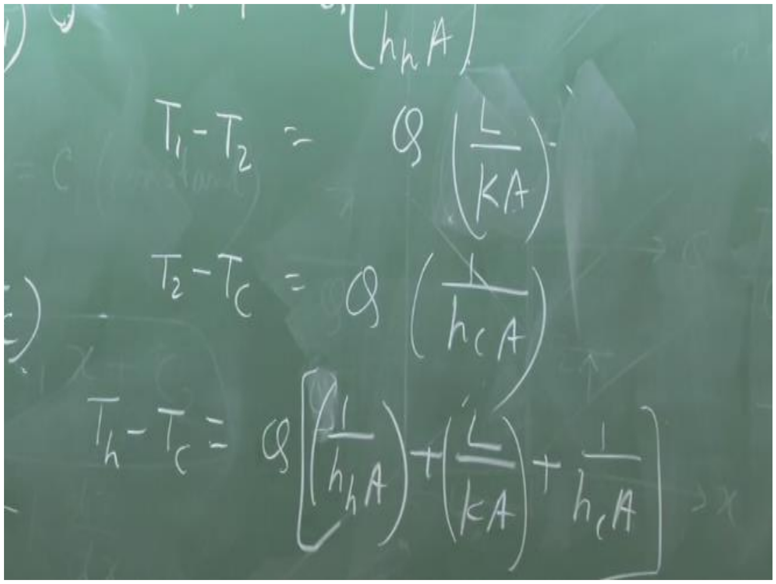

That means we can write this as Q is equal to Th - Tc divided by 1 by hhA + L by KA + 1 by hcA. That means this T1, T2 are eliminated that means if the problem is specified by the hot fluid in one side and cold fluid in one side and the heat is transferred from the hot fluid to cold fluid via the conduction due to the thickness of the wall then I can find out the heat flux knowing only hot fluid temperature and cold fluid temperature provided.

I know the information about the heat transfer coefficient on the hot fluid side and the heat transfer coefficient on the cold fluid side and this equation in the light of the electrical analogous circuit looks like a series that means a series resistances comprising the conduction resistances and conviction resistances so this can be well represented by an electrical analogous circuit like this.

There are three resistances in series and the extreme potentials are Th and Tc, hot gas temperature and cold gas temperature and the same heat that means it is a series resistance problem this is convection resistance, which is equal to 1 by hhA. Obviously heat transfer coefficient is like conductance so reciprocal of heat is the resistance. Similarly, as I already told that conduction resistance is L by KA and here is again convection resistance.

Here we can write R convection hot side and R convection cold side which is 1 by hcA. So therefore, some of the resistances will be in the dominator and in the numerator, is the overall potential drop and if we know each resistance component you can find out this intermediate voltage or intermediate potential or the temperature these are the most simple cases.

Another simple case relating to this purely one-dimensional heat flow through a slab or window type or a wall type of thing is like this instead there is a composites wall. If we have a composite wall for example like this we have a composite wall, this is one wall, this is one wall let us denote it by A and this is another wall let us denote it by B and there is a perfect matching between these two wall.

There is no gap, there is no contact resistance. There is perfect matching of this two wall first of all I will assume that and they are of different thermal conductivity KA and KB which are not equal but the heat transfer problem is steady that means that temperature is a one dimensional because of the geometry and the boundary condition.

Let us consider this face of the composite wall is T1 and this face of the composite wall is kept at T3 not T2 which I will use for the junction interface and T1 is greater than T3 and in this case because of this boundary condition and geometry one dimensional heat flow and if the temperature is independent of time that means we allow the same heat to flow out. Then how to find out the temperature distribution?

Now temperature distribution here we cannot solve the equation, thermal conductivity is same that if you write like this d by dx of K, dt by dx. No qg that means no generation of but this we cannot solve x is equal to zero and let us consider this length is L1 and this length is L2 and here x is equal to L1 + L2 that means left phase is x is equal is zero and the right phase is L1 + L2x and we cannot solve in the entire domain this equation by take K out because K is not constant.

So therefore, we have to divide this to domain like this zero less then equal to x less than is equal to L1 and another one is L1 less then equal to x less then equal to L1 + L2

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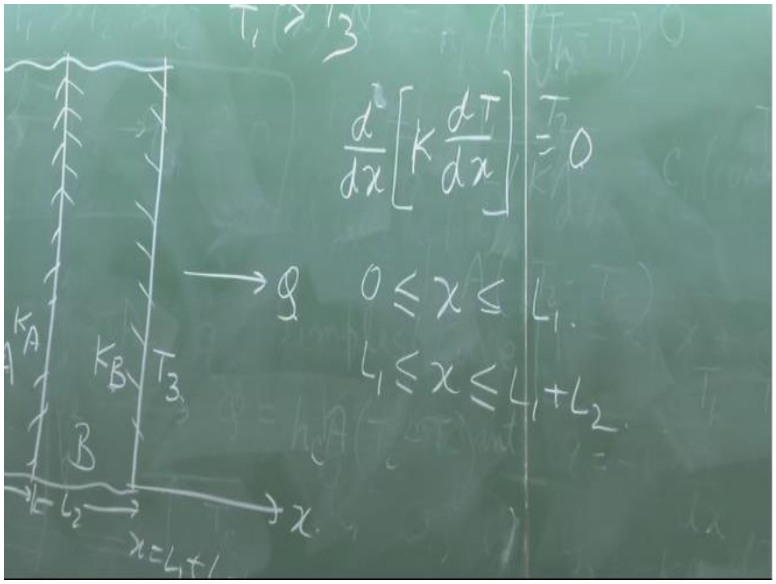

And if you solve the first one with the boundary condition that means T is equal to C1x + C2. This is the general solution for cost and thermal conductivity when we solve in these domain thermal conductivity is constant and that too that equals to Ka. Similarly, in L1 to L1 + L2 it is K constant. So therefore, T is a linear function of x C1 x + C. Now for the first one at x is equal to zero, T is equal to T1 and at x is equal to L1 if we define the interface temperature is T2 which is in between obviously by the second law of thermodynamic, T2.

So, T is equal to T2 and we immediately get an expression T is equal to T1 - T1 - T2 into x by L which is valid for zero less then equal to x less then equal to L1. So, x is equal zero T1 and x is equal to L1, T2 clear. X by L1 any problem please tell if I make inadvertent mistake while doing so please immediately detect it and tell me. So, it is L1.

Similarly, when we solve for this domain that L1 and L1 + L2 then what happens the boundary condition for this domain L1 less then equal to x less then equal to L1 + L2 you get what? at x is equal to L1, T is equal to T2, at x is equal is L1 + L2, T is equal to T3 and you get an equation T is equal to T2 - T3 divided by L2 into x - L1. Automatically it will come but more intelligently one can say that if it shifts the origin here.

And measure x from here which will be actual x because here x is measured from the origin - L1 so it will give the same equation where x is going as x - L1 is the translation of the coordinate.

So therefore, you see the two-linear temperature profile is obtained where we get the different slope and here we do not have any understanding of the qualitative picture of the slope which one is greater or which one is less that depends upon the value of T1, T2, T3 that means T1, T2, T3 for given T1, T3, T2 will be adjusted accordingly.

So that the slope will be determined and the slope will be rather found from this express

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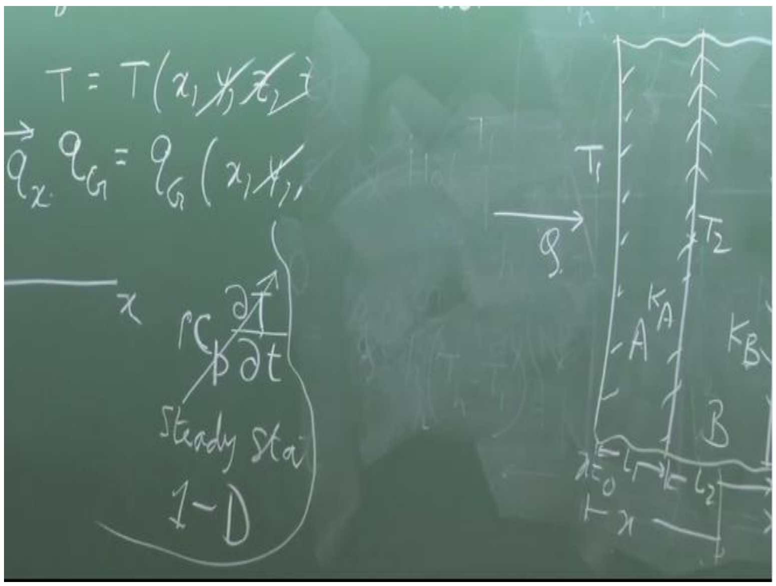

That heat which that means the qualitative picture of the slope that heat which is coming here from this side of this material A to this surface and the same thing flowing here is same Q so therefore one can write Q is equal to - k, dt by dx, KA at A. That means mathematically I write x, L1 - that means in material A that means L1 - means from this side must be equal to - KB into dt by dx at x is equal to L1 + means in material B.

Now therefore we see since the heat continuity will be there heat balance that means K times the dt by dx have to be constant which means there is a discontinuity of the slope and slope is inversely proportional to the respective thermal conductivity. So, though we see from this equation there is a continuity of the function temperature but from here also we also find out the discontinuity in the slope at x is equal to L1.

And the quantitative picture will be clear from this. Now we consider a K that Kb is greater than

Ka then what will happen this temperature gradient will be steeper, this temperature gradient will be flat because KB is higher than K, it is inversely proportional to thermal conductivity. So therefore, we can draw a picture like this I will use a color chalk T1 T2 whereas this is sorry this is relatively flat.

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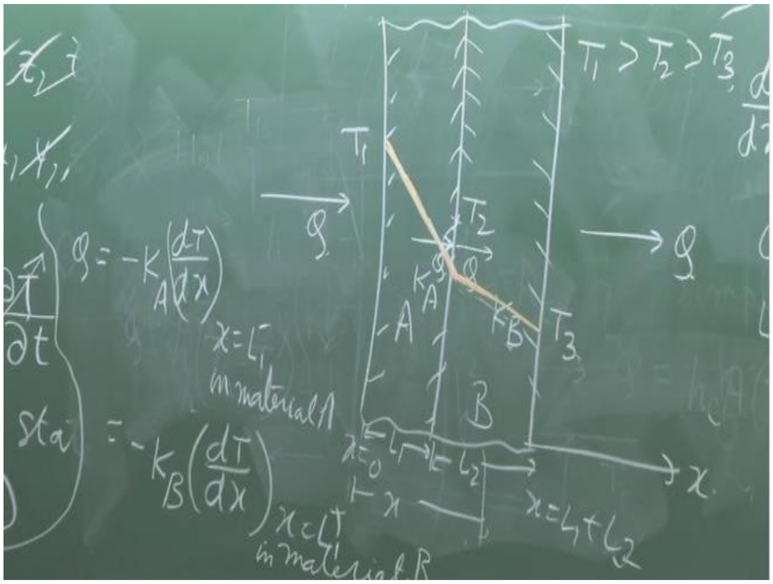

So, this will be steep this will be flat and if we apply our physical sense then also it is true that here since the thermal conductivity is low so this side at the end of this plate the temperature is less then this that means it cannot sense a temperature very close to this but since the thermal conductivity of this plate or this wall is high that means this side down stream side this outlet side or this face the temperature is close to this inlet face.

So therefore, the temperature profile is flat so more is the thermal conductivity, more is the temperature profile flat, less is the thermal conductivity, steeper is the temperature profile with a for a given heat transported. In conduction, for a given amount of heat conducted so temperature profile is always inversely proportional to thermal conductivity. So, this type of composite wall depending on the thermal conductivity.

We can draw the qualitative picture like this and here also if we also prescribe the problem with a hot gas temperature that means it is coupled with the convection boundary and the cold gas temperature then the problem becomes like this if this composite wall also be the same wall is being proposed with wall A with KA, wall B with KB and if the problem is proposed with hot gas temperature Th and the heat transfer coefficient in the hot gas Hh.

And here also cold air with Tc and with the heat transfer coefficient hc then temperature profile can be shown like this from a value of Th there is a thin film here from a value of Th it goes to T1 then there will be a steep change to T2 then there will be little flat relatively flat to T3 because KA is less then KB and then in a thin flame heat goes to Tc. This variation in the thin flame thermal boundary layer due to convection from the fee stream gas temperature to the solid surface temperature here.

Also, this nature at present I do not know until and unless I solve the convection equation, energy equation in convection. So, without that I cannot do so now if this thing is shown the composite wall with convection boundary condition in the electrical analogous circuit then what we do? We show like this. It is also a series resistance problem very simple problem that our nodal point for potential is Th, this is the T1 this is the junction of T2 this is T3 and this is TC.

And the same heat is flowing that means I am writing the electrical analogous circuit for the equation it is rub down which I wrote earlier Q and this R convection hot side which is 1 by hh A then this is R conduction material A this is LA by KAA similar way you can solve that now.

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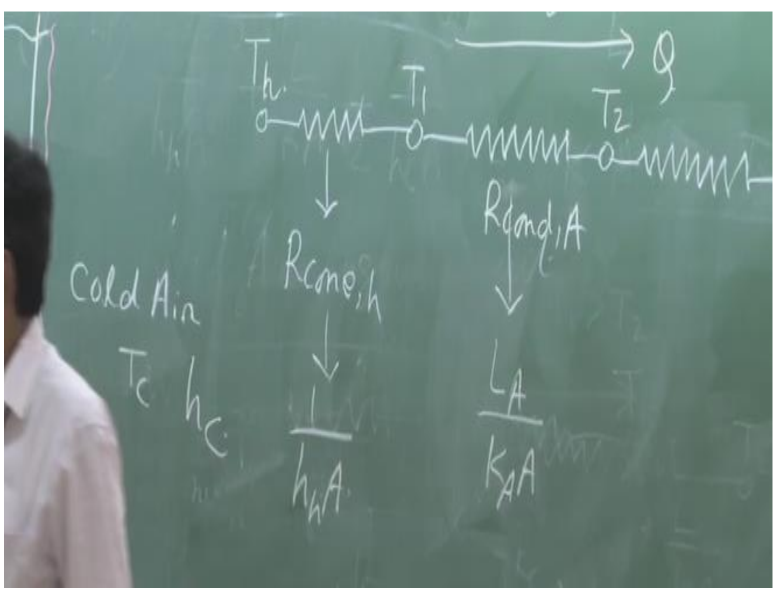

The same heat is going to that at this point there is heat continuity so this will be R conduction B. So, on has to understand these and if you can draw these thing KBA and this is R convection cold and that is 1 by hcA. So, if one can draw this electrical analogous circuit is a combination of resistances then probably may solve and we can write then Q as overall temperature difference if the problem is prescribed in terms of the overall temperature difference.

It is same at the single wall only thing is that instead there are two LA by KAA two conduction resistances, LB by KB A + 1 by hcA. So, La by KAA, LB by KBA these are the two-conduction resistance in series along with the convection. So, this is a series network problem of steady one-dimensional flow. So today I will stop here and will continue tomorrow and tomorrow, we will solve some very interesting problems. So, after continuing for a little time this conduction problem little more than we will solve interesting problems.