

Conduction and Convection Heat Transfer
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Lecture - 41
Heat Exchanger - II (Contd.)

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$$Q = m_h C_h (T_{h1} - T_{h2}) = m_c C_c (T_{c2} - T_{c1})$$

$$\epsilon = \frac{1 - \exp\left[-(UA/m_c C_c)\left(1 + \frac{m_c C_c}{m_h C_h}\right)\right]}{\left(1 + \frac{m_c C_c}{m_h C_h}\right)}$$

$$\frac{UA}{(mC)_{\min}} = NTU$$

$$NTU = \frac{UA}{m C_{\min}}$$

In terms of this quantity. Clear? Now here $m \dot{c} C_c$ is the minimum heat capacity. Now this term minus $u A / m \dot{c} \text{ minimum}$ is known as number of transfer unit NTU which is a dimensionless term overall heat transfer coefficient, surface area of the heat exchanger divided by $m \dot{c} \text{ minimum}$. Now if you can ask me the question you should ask this question to me. Without gossiping or chatting with your friend at the back sir, if the hot fluid is the minimum fluid what could you have done?

Now if the hot fluid is the minimum fluid first I tell you this expression will be same just $m \dot{c} c$ and $m \dot{h} h$ will be interchange with each other so that in a generic formula I can write in terms $m \dot{c} \text{ minimum}$ and $m \dot{c} c$, generic formula. So, this will interchange if $m \dot{h} c h$ is the minimum then it will be $m \dot{h} c h$ NTU is minus $u A / m \dot{c} \text{ minimum}$. This will be NTU and this will be one plus $m \dot{C} \text{ minimum} / m \dot{c} c$.

Similarly, it will be one plus $m \dot{c}_{\min} / m \dot{c}$ but how to do it just interchanged. Here I start again I started with cold fluid as a minimum fluid hot fluid as the minimum fluid I will $m \dot{c}_h c_h$. This is not all I will first take secondly, I will not T_h to that is the second catch here T_{h2} will not be substituted from this equation T_{c2} will be substituted and in that case T_{c1} will not we added subtracted T_{h1} will be added and subtracted.

This is having a rhythm; Good boys can understand this thing better. Immediately by intuition here, we will take $m \dot{c}_h c_h$ as common then it will be one plus $m \dot{c}_h c_h / m \dot{c}_c C$. Here T_{h2} will remain as it is T_{c2} will be replaced in terms of other temperatures from here T_{h2} T_{h1} and T_{c1} and after that I will add and subtract T_{h1} and in that case, I will use the definition of Epsilon as $T_{h1} - T_{h2}$ divided by $T_{h1} - T_{c1}$.

Because the definition of Epsilon is the change of temperature for the minimum fluid and the change of temperature maximum change of temperature available in heat exchange. Now if you write in a very generic fashion, a generic expression is like this if I make a very shortcut compact form by defining a parameter –now if I define it like this $m \dot{c}$ as the capital C

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The image shows a chalkboard with the following handwritten equations:

$$m \dot{c} = C$$

$$m \dot{c}_c = C_c$$

$$m \dot{c}_h = C_h$$

$$C = C_{\min} / c$$

$$\epsilon = \frac{1 - \exp[-NTU(1+C)]}{(1+C)}$$

$$|\epsilon = \frac{1 - \exp[-NTU(1-C)]}{1 - C \exp[-NTU(1-C)]}$$

That means the capacity rate or simply the capacity and $m \dot{c}$ $m \dot{c}_c$ C_c is the C_c and $m \dot{c}_h$ c_h is equal to C_h and if I define C as the ratio of C_{\min} / c which may be either this by this that means this by this or this by this depending upon the case. In that case the expression will be

very simple it looks like simple expression remain same $1 - \exp(-NTU)$ where NTU is $U A / C_{min}$. Sorry, not minus, sorry.

NTU is a scale quantity dimensionless $m \dot{C}_{min}$ that means C_{min} with this nomenclature where NTU , is NTU into $1 + C$ and therefore here it is $1 + C$ this is the expression. Where capital C is the ratio of the total capacity that means $m \dot{c}$ specifically ratio, ratio of the total capacity. What is the ratio minimum to the other one? That means C_{min} by C so this is the expression and this expression is for parallel flow.

Counter flow expression will be again a tricky and that I leave to you to do the counter flow expression in both the cases that is left and an exercise and the expression will be ϵ is equal to $1 - \exp(-NTU)$ here it will be $1 - C$ and here it will be $1 - C$ into again $\exp(-NTU)$ into $1 - C$. Now, you see that in condensation and boiling I told one of the fluids will be parallel in case of condensation the hot fluid is a parallel line.

In case of boiling the cold fluid is parallel line and parallel line means temperature does not change and I told qualitatively that counter flow and the parallel flow remains same but now mathematically you see that for a condensation and boiling problem this one of the fluid specific heat tends to infinity. The fluid which is boiling, and the fluid which is condensing C infinity. I showed you earlier in the figure which means that C is equal to C_{min} / C that means

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$$C = \frac{C_{min}}{C_{max}}$$

$$C \rightarrow \infty$$

$$Q = C_{min} (T_{h1} - T_{c1})$$

$$\epsilon = 1 - \exp(-NTU)$$

$$T_{h2} = T_{h1} - \frac{C_{min} (T_{h1} - T_{c1})}{1 - C}$$

This higher C tends to infinity because which tends to infinity that will always be higher no question of calculation. That means an intelligent boy will tell that for boiling condensation the ratio which you denote as C will be always zero and if you put C zero in both the expression will yield the same value Epsilon is one minus exponential minus NTU this is the argument. If you put C zero because the ratio is always zero.

Because the higher heat capacity rate whatever we got the product of $m \cdot C$ is tending to infinity. Which is not a matter of hot fluid or cold fluid, which one is higher and which one is lower. But here one thing is very much open that if I do not know the temperatures explicitly then what we can do when we know the NTU because area you know overall heat transfer coefficient we know and the mass flow rate and specific heat.

We know, we can straight away find out the effectiveness and when the effectiveness is known, I can find out the temperature. So therefore this is a case where the rating will be done explicitly no iterations rating means the size is given we have to find out the heat transfer that means all four temperatures are not known explicitly. So, we can find out the Epsilon.

And whenever we find out the Epsilon. T_{h1} and T_{c1} all right I will work then from the definition of Epsilon I can find out the temperature difference of the minimum fluid that means heat load will come in the temperature difference of the other fluid will also come. So therefore,

where rating heat load and cooling load is not done but size is given NTU Epsilon method is an explicit method to find out effectiveness, very quickly temperatures are found.

But where the temperatures are given sizing is to be made LMTD is useful because from here knowing Epsilon NTU as a function of Epsilon will be little difficult you understand this may require tedious calculations or iterations. So, I will take another maximum half an hour to solve two problems to make this thing clear, but before that I will tell you one thing that before solving a counter flow exchanger.

Now we will solve two problems on Heat Exchanger which will demonstrate our understanding of the discussion that which have been met just now in the earlier class.

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A counterflow heat exchanger of heat transfer area $A=12.5\text{m}^2$ is to cool oil [$c_{ph}=2000\text{J}/(\text{kg}^\circ\text{C})$] with water [$c_{pc}=4170\text{J}/(\text{kg}^\circ\text{C})$]. The oil enters at $T_{h,in}=100^\circ\text{C}$ and $m_h=2\text{kg/s}$, while the water enters at $T_{c,in}=20^\circ\text{C}$ and $m_c=0.48\text{kg/s}$. The overall heat transfer coefficient is $U_m=400\text{W}/(\text{m}^2\cdot^\circ\text{C})$. Calculate the exit temperature of water $T_{c,out}$ and the total heat transfer rate Q using both the LMTD and the ϵ -NTU method.

Now the first example is the counter flow heat exchanger of heat transfer area 12.5m square immediately you should strike that it is not a sizing problem that means size is given, that means a rating problem is to cool oil whose specific it is 2 kilo joule per kg degree Celsius. 2000 joule is taken from a book they have not written in the term of Kilo Joule. With water with specific it is 4.170J/kg degree Celsius.

Now oil is the hot fluid and water is the cold fluid. The oil enters at 100 degree Celsius that means in our nomenclature T_{h1} is 100 degree Celsius. Here it is written $T_{h\text{ dot in}}$. And the flow

rate of the hot fluid that is oil is to 2kg/s, while the cold fluid enters that is T_{c1} in which is in our nomenclature T_{c1} is 20 degree Celsius and the cold fluid flow rate m_c is 0.48kg/s. The overall heat transfer coefficient is given.

This will be given for both the cases rating and sizing problem. 400W/ (m² degree Celsius). Calculate the exit temperature of water and the total heat transfer rate Q using both LMTD and the Epsilon NTU method. Now an idiot will start with the any of Epsilon-- LMTD method but it is told that we have to do it, if nothing is told, so I can start with LMTD. Now you see LMTD how difficult it is.

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Handwritten equations on a chalkboard:

$$2 \times 2 \times (100 - T_{h2}) = 0.48 \times 417 (T_{c2} - 20)$$

$$Q = U A LMTD'$$

$$LMTD' = \frac{\Delta T_2' - \Delta T_1'}{\ln \left(\frac{\Delta T_2'}{\Delta T_1'} \right)}$$

$$\Delta T_2 = T_{h2} - T_{c2}$$

$$\Delta T_1 = T_{h1} - T_{c2}$$

Additional notes on the right side of the board:

$$T_{c2} = 60^\circ C$$

$$T_{h2}, T_{c2}$$

$$T_{c2} = 85.5$$

Now what is $m \dot{h}$, you tell me $m \dot{h}$ is 2kg, what is that, where it is written 2kg/s? 2 into- I am writing in terms of Kilo Joule it will be cancelled then T_{h1} is what, T_{h1} the oil enters 100 - T_{h2} equals to what? Water enters at 20 degree Celsius 0.48. I have to find out Q. Good that is the mistake. I was discussing with people that I could not rectify though my teachers told me several times.

What is it flow rate? Flow rate is 0.48 into 4.17 ($T_{c2} - 20$). Now LMTD is $\Delta T_2 - \Delta T_1$ divided by $\ln \Delta T_2$ by ΔT_1 . Now what is ΔT_2 ? $T_{h2} - T_{c2}$ nothing is known. Counter flow $T_{h2} - T_{c1}$ and ΔT_1 is $T_{h1} - T_{c2}$. Now I cannot find out the LMTD because I

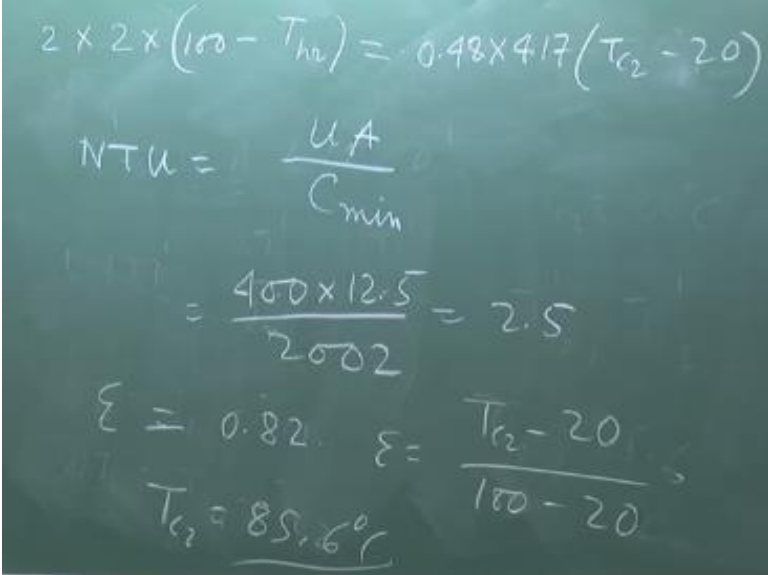
don't know this T_{h2} T_{c2} . So what to do? We assume T_{c2} or T_{h2} . Let us assume T_{c2} is equal to 60 degree Celsius.

It will be heated to a temperature, I assume something it is less than 100 definitely which maybe more than T_{h2} it cannot be more than T_{h1} , let us assume 60 degree Celsius. Then I find out this and I find out T_{h2} , so T_{h2} and T_{c2} is found out. And I give that one superscript that is the first iteration. Accordingly, I find out LMTD with the first approximation of this. And when LMTD is found out this two are known-- I now found out Q as the first approximation.

When Q is found out I can use this with the unknown T_{h2} or this with the unknown T_{c2} and try to match this T_{h2} , T_{c2} to found out with the earlier value, if they are differing again we put that value and we change this as superscript 2 next iteration and go and this loop will go until and unless these two matches. And finally depending upon your guess value if 1 gives a guess value by the magic of P. C. Sorcar, 85.6 degree Celsius.

Then you did not have to do any iteration, that means you will get a value of T_{h2} such and LMTD such and accordingly Q which will exactly match this value. But it is very difficult only the magician can do. So, some iteration will be there. But this problem if you solve by NTU Epsilon method, so therefore we see this is a tedious method, but what is NTU LMTD method?

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$$2 \times 2 \times (100 - T_{h2}) = 0.48 \times 417 (T_{c2} - 20)$$
$$NTU = \frac{UA}{C_{min}}$$
$$= \frac{400 \times 12.5}{2002} = 2.5$$
$$\epsilon = 0.82 \quad \epsilon = \frac{T_{c2} - 20}{100 - 20}$$
$$T_{c2} = 85.6^\circ C$$

NTU is known, what is NTU? uA/C_{\min} . What is C ? $2 \times 2 = 4$ and this is 0.48×4 that means which one is minimum? That one, right hand side is minimum. So if you do that minimum then you find out C_{\min} —sorry NTU is uA/C_{\min} then we find out NTU as Au , u is what? What is u ? What is A ? 12.5 And what is C_{\min} ? I just see that multiplication is difficult for me 2002 what means 2, because this Joule unit has to be there.

Because this unit has to be consistent, here I can write Kilo Joule because both the side it will be balanced. Be careful with this unit. NTU is a non-dimensional. u is what per meter square K or meter square degree Celsius. A is meter square, so here C_{\min} has to be in Joule. Do not be that smart to put it here and finally come with a wrong result I will be giving 80% marks, if you are happy than it's alright. So therefore, NTU comes out to be what? Where is NTU? 2.5.

That means if you clear it NTU then Epsilon for a counter flow heat exchanger expression there if you put NTU it is an explicit calculation by a small calculator you can do it and Epsilon comes straightforward value if Epsilon comes 0.82. Now when Epsilon is 0.82 Epsilon will be the temperature difference of the minimum fluid $T_{c2} - 20$ divided by $T_{h1} - T_{c1}$ that means maximum temperature $100 - 20$. And this gives T_{c2} is equal to 85.6. Explicitly done. So you see that a rating problem is explicitly done by this.

Anybody has feel that LMTD method is not at all valid or you cannot do it for the rating problem? No, we can do it. But it is not explicit. And in today's world both are almost same. By putting into calculator just making a small program and do it nothing is there. But it is explicit and there it is implicit by iteration that is the only difference. Now the next problem, problem 2, this is an interesting problem.

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Water is heated from 30°C to 90°C in a counterflow double-pipe heat exchanger. Water flows at the rate of 1.2 kg/s. The heating is accomplished by a geothermal fluid which enters the heat exchanger at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin walled and has a diameter of 15 mm. If the overall heat transfer coefficient of the heat exchanger is $600 \text{ W}/(\text{m}^2 \text{ } ^\circ\text{C})$, determine the length of the heat exchanger required for the purpose. The specific heats of water and geothermal fluid are 4.18 and $4.31 \text{ kJ}/(\text{kg } ^\circ\text{C})$ respectively. Use both (a) the LMTD method and (b) the ϵ -NTU method.

Water is heated to 30 to 90. Two temperatures are given. In a counter flow double-pipe heat exchanger. Water flows at the rate of 1.2 kg/s that means total load is given. The heating is accomplished by a geothermal fluid which enters the exchanger at 160°C at a mass flow rate of 2 kg/s, that means everything is explicit we know all the temperatures, the heat transfer everything is given, because by heat balance, we can find out the exact temperature of the geothermal fluid.

Specifics are not given – no specifics are given at the end. The inner tube is thin walled and has a diameter of 15 mm. Thin wall, this is extra language that means we can neglect this wall resistance. It is not required because overall heat transfer coefficient is given. If the overall heat transfer coefficient of the exchanger is 600 Watt per meter square degree Celsius, determine the length of the heat exchanger required, -- Oh! That is why it has given thin.

Because area is to be find out then area divided by pie the is the length that's why the thinness is required otherwise it will be difficult. The specific heats of water and geothermal fluid as give use both the LMTD and Epsilon. Here, this is the reverse LMTD is very simple. Epsilon NTU is done. Why? Because the temperatures are known. Let me write, very simple. Tell me, you dictate will write.

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$$\begin{aligned}
 2 \times 4.3 \times (160 - T_{h2}) &= 1.2 \times 4.18 \times (90 - 30) \\
 T_{h2} &= 125.10^\circ\text{C} \\
 \Delta T_2 &= 125.10 - 30 = 95.10^\circ\text{C} \\
 \Delta T_1 &= 160 - 90 = 70^\circ\text{C} \\
 \text{LMTD} &= 81.91^\circ\text{C} \quad L = \frac{A}{\pi D} \\
 A &= \frac{Q}{U(\text{LMTD})} \quad A = 6.12\text{m}^2 \quad = 129.87\text{m}
 \end{aligned}$$

Water is 1.2 or the hot fluid first, hot fluid is; what is the flow rate? 2 into You tell me, you dictate me I will write, I will not see. First heat balance, $2 \times 4.3 \times (160 - T_{h2}) = 1.2 \times 4.18 \times (90 - 30)$. What is the value of T_{h2} ? Nobody can tell without calculating it. T_{h2} is 120.10°C . Then ΔT_2 125 -- sorry $125.10 - 130$ is equal to 95.10 degree Celsius this is meaningless but still I am doing. ΔT_1 is $T_{h1} = 160 - T_{c2}$ that is 90 that is equal to 70 degrees Celsius.

With this value the LMTD will come as 81.91 degree Celsius. This is the main temperature, 81.91 degree Celsius. Then A is Q/u LMTD in the dominator. So, Q is this one either this one or this one you put u is given, what is the value of u ? u is 600 Watt per meter square degree Celsius and LMTD is 81.91 degree Celsius which gives A if you put the value which gives A as 6.12m^2 square. And L is equal to A by πD and D is given as diameter is 15mm .

We have to put into meter that means 0.015 m and A is m^2 square finally it is thin walled so diameter outer and inner is same and finally we get $L = 129.87$ meter. So therefore, is just the reverse. LMTD is straightforward. Clear? Because it is sizing problem. But to do it with Epsilon NTU what you have to do. Because here Epsilon is known but NTU has to be found out. Here Epsilon is known NTU has to be found out. In Epsilon NTU method, what is Epsilon?

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$$\epsilon = \frac{1 - \exp[-NTU(1-C)]}{1 - C \exp[-NTU(1-C)]}$$

$$\epsilon = \frac{90 - 30}{160 - 30} = 0.461$$

$$NTU = f(\epsilon)$$

$$NTU = \frac{1}{C-1} \ln \left(\frac{\epsilon - 1}{\epsilon C - 1} \right)$$

Minimum fluid is which one? What is Epsilon? Minimum fluid is $m \cdot c \cdot \Delta T$, this is the minimum fluid, so Epsilon will be $90 - 30$ divided by $160 - 30$. So, Epsilon will be $90 - 30$. This is the minimum. So this is the Epsilon So Epsilon becomes 0.461. Now, problem is that Epsilon is known, you have to find out surface area and finally length. That means I have to express NTU is a function of Epsilon, for that little more labour you have to out.

We express Epsilon as a function of NTU, that function if you just write other way NTU is equal 1 by $C-1$ into \ln just you do it, Epsilon minus 1 divided by Epsilon $C-1$? What is C ? C mean by C . We express Epsilon as a function of NTU. Here I write by rubbing this, if we recall just now I told. Epsilon in a counter flow is $1 - \exp[-NTU(1-C)]$ into $1 - C$ that is the ratio into $1 - C$ again exponential minus NTU into $1 - C$.

So you see, this expression this capital C is C mean by C . You have to first from here express NTU, this is tough but it can be done, sometime this is not possible to express NTU as a function of Epsilon explicitly than the problem face, that means then we have to go for iteration. But luckily with a little labour work one can rearrange this function to do that NTU is this. Now when Epsilon is known 0.46 you just put it and ratio C_{min} by C_{max} or C .

You know and you straightforward find out NTU and find out area, area will come exactly the same 6.12-meter square and L will be 129.87. Now you have been able to appreciate that LMTD

method is simple and explicit when we go for sizing, rating is given. But when the size is given NTU Epsilon method is explicit and simple because we have an explicit relationship of NTU with Epsilon where from Epsilon we can find out the rating which was, which is unknown for this type of problem. So, with this I will finish my lecture on Heat Exchanger. Okay.