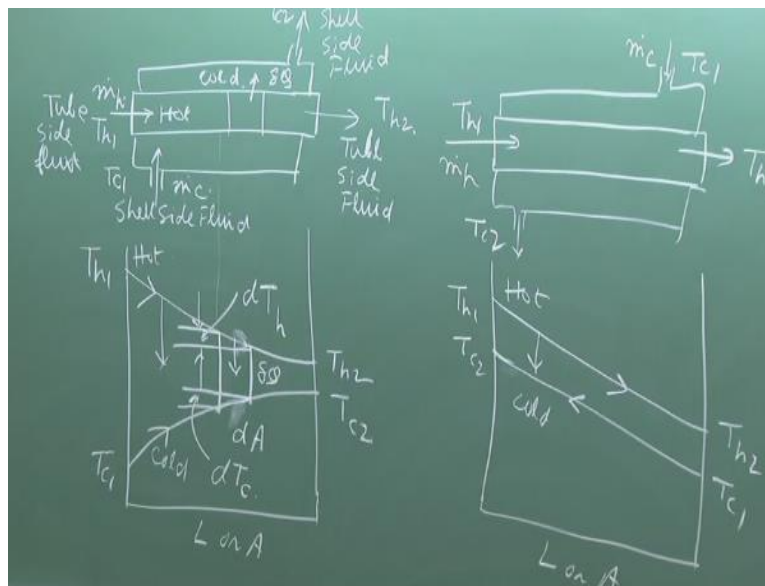


Conduction and Convection Heat Transfer
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Lecture – 40
Heat Exchanger - II

Good afternoon. We were discussing the heat exchanger in the last two classes. Well, we will continue the discussion. What we were discussing about the parallel and counter flow heat exchanger simple shell and tube type. Let us start with that again.

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Let us have a shell and tube type, like this simple shell and tube double pipe. This is the tube side fluid in and tube side fluid out. This is the shell side fluid in and out. Let us also draw side by side, the parallel counter flow arrangement this one the parallel flow. The both shell side fluid and the tube side fluid flows in the same direction but this is the –I am not writing the tube and this thing again. This is a counter flow type shell in.

Now, the temperature diagram along the length of the tube if you see length or area whatever way you think, area is better because ultimately you will deal with the area for heat exchange L or A . Here what happens, the hot side fluid if we consider hot side tube or shell does not matter. The hot fluid not hot side, hot fluid as a decrease in temperature and the cold fluid increase the

temperature in the same direction. Let us consider this is the hot and cold.

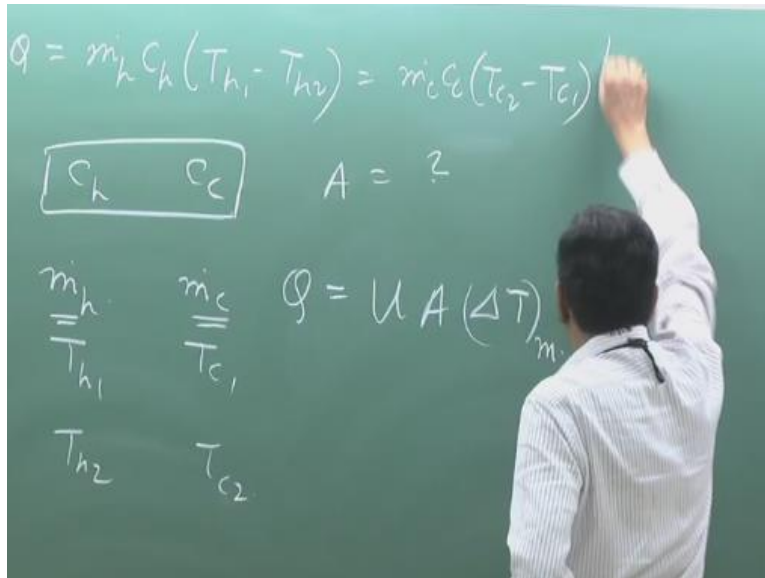
Let this temperature T_h is the subscript for hot, 1 is the inlet here parallel flow T_{h2} is the hot outlet. Similarly, cold inlet is T_{c1} and cold outlet is T_{c2} then in this diagram we can show T_{h1} , T_{h2} , T_{c1} , T_{c2} . This is the picture. The assumption is that the fluid temperature is uniform at cross section and heat is being, this is the hot fluid sorry this arrow I should show here. It will be difficult there. I will draw something else. This is the cold fluid.

This is the hot fluid diagram temperature. This is the cold one and at any section the heat is flowing like this from hot to cold. This is the temperature. Here the situation is this. The hot fluid flows like this from T_{h1} to T_{h2} in a direction. Let, the similar way the hot fluid flow through the tube and the cold fluid flows in the opposite direction for which this is known as counter flow, hot and cold.

Here also the heat transfer takes place from hot to cold because of this temperature difference. Now, this type of problem, now this type of arrangement. One type of problem is like this. First of all, let us see that the energy balance or heat balance is like this if I consider $\dot{m}_h$ as the mass flow rate of the hot fluid. And $\dot{m}_c$ is the mass flow rate of the cold fluid in both the cases $\dot{m}_h$ and $\dot{m}_c$ with nomenclature the total heat.

That is transferred between the hot and cold $\dot{m}_h$ and if c stand for the specific heat at constant pressure is T_{h1} minus T_{h2} that is the heat which is being given by the hot fluid equal to $\dot{m}_c c_p (T_{c2} - T_{c1})$.

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Now, this is the first line of energy balance. Now, one type of problem is this that the conditions are like this. The property, that C_h and C_c are usually known thermodynamic properties, specific heat. Now, the operating parameters or conditions are back flow rate of the hot fluid. Its inlet temperature, its outlet temperature, flow rate of the cold fluid. Its inlet temperature and outlet temperature.

Now, out of this six quantities we have one equation only relating to the six quantity that means five are independent and one is dependent. Usually, the mass flow rates of both hot and cold are given in some class of problems along with three temperatures means implicitly I know the fourth temperature. That means all the six quantities are given. This type of problem means that the rating of the exchanger.

That means the operating conditions as far as the heating and cooling load. That means how much heat has to be transferred from the hot fluid to be cooled or how much heat has to be gained by the cold fluid to be hot is known. Because the mass flow rate and the temperatures are all known, three temperatures are known along the mass flow rate. I can find out the fourth one. So, that this is known as in this terminology in the field of heat exchanger.

The heat exchanger is rated for its heating and cooling load. What is to be found out for this type of problem? We are interested to find out what surface area is required. If we consider a simple

double pipe heat exchanger either parallel or counter. What is the surface area required? This is known as sizing problem means the heat transfer from the heat load side is rated. That means the total load is given. How much heat has to be transferred? is given.

We want to know the size. Size means the surface area required for that heat transfer. This we can find out definitely if we write this equation Q is equal to $u A \Delta T_m$ just I discussed in the last class that we know overall heat transfer coefficient. The concept of overall heat transfer coefficient was told earlier which is the sum of the heat transfer coefficient of fluid sides both the side fluids plus the conduction resistance.

And if we see the resistance that some of the resistances are the equivalent resistance. If we assume that overall heat transfer coefficient is constant along the flow then we can write the rate of heat transfer in this fashion $U A \Delta T$ some mean temperature difference where A is the area based on which overall heat transfer coefficient is defined. That means basically this is the definition of overall heat transfer coefficient but it is other way in practice.

The overall heat transfer coefficient is giving so that practical problem we have to find out the area based on which u is giving when we know the Q . But the only difficulty here compared to simple convection problem as we have done in the class that this temperature difference sometimes we have seen the flat plate, full stream all are constant temperature along the flow. But here this ΔT responsible for heat transfer changes from one section to other section.

Here, it is inlet and outlet because both are flowing in the same direction here from section. That mean there should be some mean ΔT , mean of the 2-extreme difference of ΔT . This is one ΔT , this is another terminal. The mean of terminal ΔT in such a way that could define, this equation. So, to find out that ΔT_m we have to do small mathematical exercise. Let us consider the parallel flow case and an elemental area in this exchanger dA where we have a change in hot fluid temperature which is denoted by dT_h .

And also, this is a decrease in the temperature and at the same time there is an increase in the cold fluid temperature through that dA area here in the exchanger through which the change is d

T_c and here there is an amount of heat transfer δQ which is represented here we can represent here also δQ . That means a small elemental area $\delta T A$ δQ heat is transferred from the hot to cold by which the hot fluid suffers a differential decrease in temperature dT_h .

And the cold fluid suffers a differential increase in temperature dT_c . Then we can write that δQ is equal to $m \dot{h} c_h$ in to minus dT_h .

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$$\begin{aligned} \delta Q &= m_h c_h (-dT_h) = m_c c_c (dT_c) \\ dT_h - dT_c &= -\delta Q \left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c} \right) \\ d(T_h - T_c) &= -\delta Q \left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c} \right) \\ \delta Q &= U dA (T_h - T_c) \\ \frac{d(T_h - T_c)}{(T_h - T_c)} &= -U \left(\frac{1}{m_h c_h} + \frac{1}{m_c c_c} \right) dA \end{aligned}$$

Because T_h has decreased is equal to $m \dot{c} C_c dT_c$. This decrease and increase of temperature with reference to the direction of on particular prescribed direction and here the prescribed direction in parallel fluid in the direction of flow because both the fluids are flowing in the same direction. Then we can consider that change in hot fluid temperature it is decrease and it is an increase and it is interesting to see that if you integrate this equation you get this one.

Because if you integrate this minus dT_h in the direction of the flow it will be T_{h2} minus T_{h1} is the da so minus of that will give this. And dT_c will be $d c_2$ minus $d c_1$ that means simply a differential form of this equation we are writing here. Now, we make a very simple algebraic at a primary school level algebraic manipulation that dT_h minus dT_c equals to what? dT_h minus dT_c is δQ , dT_h is minus dT_c is also minus that means minus δQ $1/m \dot{h} c_h$ plus $1/m \dot{c} C_c$. dT_h minus dT_c clear.

This can be written as $d(T_h - T_c)$ this is algebraic step, minus $\frac{dQ}{m_h C_h + m_c C_c}$ with the nomenclature I have told. Now from the definition of heat transfer by this equation, overall heat transfer coefficient we can write dQ at that section is $u dA (T_h - T_c)$ because T_h and T_c are the hot fluid and cold fluid temperature at that section. Infinite small section of infinite small area dA . So, this has to be valid.

So, therefore I can substitute dQ here and I can write $d(T_h - T_c)$ very simple, divided by $T_h - T_c$ is equal to minus $u dA$ I will write here $m_h C_h + m_c C_c$ into dA . You may or may not take the note this is given in any text. It will also be good if you just go on seeing you can find out inadvertent if I by chance I make it is not always necessary to take the note but you can also take the note.

Because it is given in any book you see the philosophy. So, this is all right. Now if you integrate the both the sides with respect to dA . So left side represents the temperature difference it is the way you express. Between this inlet and outlet with respect to this parallel flow heat exchanger then the left-hand side becomes

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The image shows a handwritten derivation on a chalkboard. At the top, the differential equation is written as $\ln \left[\frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})} \right] = -u \left(\frac{1}{m_h C_h} + \frac{1}{m_c C_c} \right) A$. To the right, the heat transfer equation is given as $Q = uA \frac{\Delta T_2 - \Delta T_1}{\ln \left(\frac{\Delta T_2}{\Delta T_1} \right)}$. Below this, the LMTD is defined as $LMTD = \frac{\Delta T_2 - \Delta T_1}{\ln \left(\frac{\Delta T_2}{\Delta T_1} \right)}$, with a note "(Log Mean Temperature difference)". The main equation is then written as $Q = uA \left(\frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln \left[\frac{(T_{h2} - T_{c2})}{(T_{h1} - T_{c1})} \right]} \right)$. At the bottom right, the simplified equation is given as $Q = uA(LMTD)$. In the bottom left corner, there are small notes: $\Delta T_2 = T_{h2} - T_{c1}$ and $\Delta T_1 = T_{h1} - T_{c2}$.

In $T_h - T_c$ it will be $T_{h2} - T_{c2}$ divided by $T_{h1} - T_{c1}$. Any problem you can ask me. And this side will be minus $u \frac{1}{m_h C_h + m_c C_c} \times A$. So, this is

clearly thing. Now I will substitute $1/m \dot{h} c_h$ and $1/m \dot{C} c_c$ from here there are two equations if you consider Q into picture Q is this and Q is this.

That means this quality gives two and ultimately, I can substitute $1/m \dot{h} C_h$ plus $1/m$ what is $1/m \dot{h} c_h$. It is $Q T_{h1} - T_{h2}/Q$ and $m \dot{1}/m c C_c$ is $T_{c2} - T_{c1}$ by Q and if you do it. It is very simple then you get this ultimately this equation and Q will come. Q is equal to $u A, T_{h2} - T_{c2} - T_{h1} - T_{c1}$. I am not unnecessarily wasting time by doing this. The simple if you do that you will get this one.

$T_{h2} - T_{c2}$ divided by $T_{h1} - T_{c1}$ automatically you will get this. If you just substitute $1/m \dot{h} T_{c_h}$ plus $1/m \dot{C} c_c$ in terms of q from here. That means here the entire thing is a mean temperature difference ΔT . Now if I define ΔT_2 as the temperature difference at the outlet here in case of parallel flow that is equal to $T_{h2} - T_{c2}$ and ΔT_1 is $T_{h1} - T_{c1}$.

Then this can be written as Q is equal to $u A \Delta T_2 - \Delta T_1$ and it is will be always positive whether ΔT_2 and ΔT_1 vary with their respective magnitudes higher and lower because plus minus will be adjusted if ΔT_2 is more than ΔT_1 both numerator and denominator will be positive and it is other way both are negative. So therefore, we can write this equation like this and this quantity is known as LMTD that is log mean temperature difference.

Because the log is coming, log mean temperature difference. The log mean temperature difference LMTD which is defined as $\Delta T_2 - \Delta T_1$ divided by that means this fashion the difference of the two ΔT at two terminal divided by the $\ln$ of their ratios, this is LMTD. So, Q is equal to therefore you see that if the exchanger is rated fully by its heating and cooling load prescribing this thing so that I know Q then I can find out the surface area if I can determine LMTD.

So, if I know the definition of LMTD which satisfies this equation that means it is that average ΔT between the two terminals which satisfies this equation and if I am provided with the value of overall heat transfer coefficient. I can determine the size of the exchanger. This is

known as typical sizing problem. Clear, to everybody. Now there is a clue if, very important it is a counter flow heat exchanger I will derive.

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The image shows a chalkboard with handwritten mathematical derivations for a counter flow heat exchanger. The main equation is:

$$Q = UA \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

Below this, the LMTD (Log Mean Temperature Difference) is defined as:

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

Then, the overall heat transfer equation is written as:

$$UA(\text{LMTD})$$

On the right side of the chalkboard, the temperature differences are defined:

$$\Delta T_2 = T_{h2} - T_{c1}$$

$$\Delta T_1 = T_{h1} - T_{c2}$$

Further down, a variable x is introduced:

$$x = \frac{\Delta T_2}{\Delta T_1}$$

And the LMTD is expressed in terms of x :

$$\text{LMTD} = \frac{\Delta T_1(x-1)}{\ln x}$$

Finally, a limit is taken as $x \rightarrow 1$:

$$\lim_{x \rightarrow 1} \frac{\Delta T}{\frac{1}{x}} = 1$$

Now first of all, counter flow heat exchanger is also the same formula but what is delta T 2 for a counter flow heat exchanger the generic formula this one. Delta T 2 counter flow heat exchanger is any side you can take counter flow heat exchanger. There is no inlet, outlet reference because it is hot fluid inlet. It is cold fluid inlet. Let us make this delta to T h 2 minus T c 1 then delta T 2 will be T h 2 minus T c 1 and what will be delta T 1?

Delta T 1 will be T h 1 minus T c 2. Clear, how to derive it. I leave it to you as an exercise very simple. How to derive this one that means I have to come to this step where this will be u A into T h 2 minus T c 1 minus T h 1 minus T c 2 divided by l n T h 2 minus T c 1 divided by T h 1 minus T c 2. So that a generic formula will come out LMTD is delta T 2 minus T 1 difference of delta is divided by L n ratios of delta T. How to do it?

Very simple, I tell you the catch because I will not waste time here in this equation if you take here one prescribed direction here positive then DTH will be minus because it is decreased in this direction take the flow direction of hot fluid then DTC will be also minus because why? Because this is increasing in the negative direction. So, therefore both the things will be minus it will be minus d T h it will be d T c.

So, therefore dT_h minus dT_c will be changed which will be $\delta Q / m \dot{c} \dot{m}$ minus $1/m \dot{c} \dot{h}$ change will be there. So, accordingly you can find out that there will be changes just only change is here. Very simple that means this equation will be minus dT_h and minus dT_c and think that you integrate this you will get a same result. This is clear because this is an energy balance all the scalar quantities counter flow if you integrate this minus DTH.

DTH will be this is preferred that is a positive direction DTH will be T_{h2} minus T_{h1} so minus DTH is T_{h1} minus T_{h2} . Similarly, DTC will be T_{c1} minus T_{c2} . So, minus DTC will be T_{c2} minus T_{c1} . Therefore integration of this with minus dT_h and minus dT_c for counter flow will give you the same result. So, finally when you will substitute these $1/m \dot{c} C_c$ minus $1/m \dot{h} C_h$ because this will come then you take the same equation.

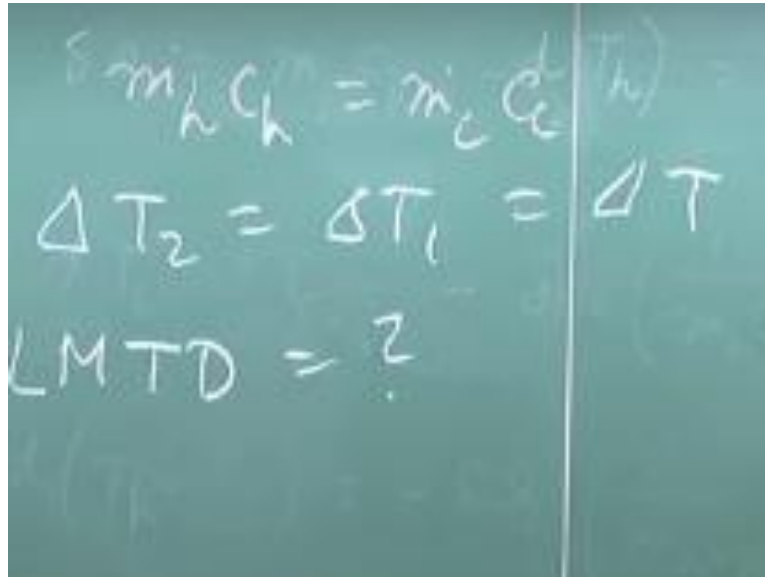
Only catches is that you have to express dT_h minus dT_c because d of T_h minus T_c , hot minus cold. So, this is an assignment everything I have told that in a counter flow also you can do the same thing. But now the most, most interesting part of this equation very, very interesting and very, very important too. I think you are intelligent you can understand. Now, the situation may come two things you must remember that a counter flow, parallel flow heat exchange.

If a heat exchanger is very infinitely long this may be equal to this. Cold fluid outlet temperature can at best become equal to the hot fluid outlet temperature I have told earlier but not the hot fluid inlet temperature. But here the cold fluid outlet temperature may be more than the hot fluid outlet temperature even in a finite length because here you see this temperature there is no restriction that this will lower than that.

Because heat is transferred this way. That means at this end of the heat exchanger the temperature differential causing the heat transfer is from the inlet temperature of the hot fluid to the outlet temperature of the cold fluid. So, as because they are flowing in the opposite direction the cold fluid outlet temperature may be more than the hot fluid outlet temperature which is not the case for the parallel flow.

Another thing is there if the product of m and c , $m \cdot c$ for hot and cold fluid becomes same that means if $m_h C_h$ is $m_c C_c$ there will be a situation for counter flow. The two curves will be parallel to each other and ultimately ΔT_2 is equal to ΔT_1 .

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$$m_h C_h = m_c C_c$$

$$\Delta T_2 = \Delta T_1 = \Delta T$$

$$LMTD = ?$$

So, what is LMTD? Anybody, who told ΔT ? How? Zero by zero form. So, you can use the L'Hopital theorem taking the limit in many ways you can prove it that it will be ΔT_1 or ΔT_2 or ΔT very simple to prove these things. That is also left to you we can take $\Delta T_2 / \Delta T_1$ as x . You can write ΔT_1 into x minus 1 divided by $\ln x$ limit as x tends to 1 you will see this becomes ΔT .

So therefore, we can do it in various ways x minus 1 / $\ln x$ L'Hopital theorem is a mental problem x minus one become one. And $\ln 1/x$ and in that case here it will be ΔT_1 this thing will be ΔT_1 into x minus one where x is $\Delta T_2 / \Delta T_1$ and we can write the denominator as $\ln x$ now limit of this as x tends to one, sorry, not limit of this as x tends to one. We have to make a L'Hopital theorem differentiation again it is zero/ zero form, until the zero/ zero form gone.

So, we have to go for number of steps with that. So, $\Delta T_1 / \ln x$, $\ln x$ is what? $1/x$ and limit x tends to one. That means there are various ways to prove it sometime you can prove that ΔT_2 minus is ΔT_1 plus Epsilon and put it and take Epsilon tends to zero. Denominator $\ln$

plus x you can expand in a series $1 + x + x^2 + \dots$ type of thing that is $\Delta T_1 / \epsilon$ you can prove that limit will be ΔT_1 only.

Therefore, it is very important these are school level task that when ΔT_2 and ΔT_1 are same. Under these circumstances mass flow rate capacity product is equal then we have to have LMTD as design one information I give you in this respect, just an information. This can be proved analytically but I am not going to prove. Just as an information for an engineer if $\Delta T_2 / \Delta T_1$ vary by a factor ratio which one is greater, depending upon that the ratio is within two point two then we can use the arithmetic average as the LMTD with 95 percent accuracy.

Usually arithmetic average is higher than the LMTD we should not use it otherwise it will be over predicted. So, therefore the LMTD and arithmetic average will be within five percent error provided ΔT_2 and ΔT_1 varies, varies means the ratio is within 2.2 and when exactly this is one ratio is one the coincide with each other that means arithmetic average is equal to the LMTD because arithmetic average is ΔT_1 or ΔT_2 clear.

Now, another interesting thing comes now into picture that in this rating even sizing problem is very simple but if it is other way. That means, I tell only $\dot{m}_h$, $\dot{m}_c$, T_{h1} and T_{c1} , neither T_{h2} nor T_{c2} is given that means this is not written. But I know A this is known as rating problem two type of problem sizing and rating. Sizing problem is the earlier one the entire heat load is given, q is known.

That means explicitly $\dot{m}_h$, $\dot{m}_c$, T_{h1} , T_{h2} , T_{c1} , T_{c2} of course this four are not required T_1 is given T_2 all known. Then we find out the area we know the LMTD, so using LMTD method we can find out but if it is other way area is given. Now this ΔT_m is now ΔA LMTD and LMTD is what? LMTD is ΔT_2 minus ΔT_1 by $\ln \Delta T_2$ by ΔT_1 . Now, if I know only this two T_{h1} T_{c1} not T_{h2} T_{c2} .

This equation cannot tell anything so how to find out this we cannot know LMTD. So, this type of problem, where the sizing is given but we have to rate the exchanger means a particular exchanger with a given surface area is suitable or not then what we have to do? We have to use

an Iterative method that means the area is given we have to rate the exchanger whether it will be suitable.

What is this (()) (31:25) we have to assume I will solve a problem today it will be more clear but still I am telling now either T_{h2} or T_{c2} any one we assume first. If you assume T_{h2} T_{c2} will be found out from this equation both are assumed helm. then you find our LMTD obviously with this assume helm then with this LMTD since A is known we find out. Now with that Q , we iterative that means we define the value of D_{h2} , clear.

So, this way we do both T_{h2} and T_{c2} until they converge with each other that means, the new value is close to the earlier value, very simple. So, today it is nothing but when it was discovered long, long back there was no MATLAB. No program not even a calculator even. So, therefor people thought that it is tremendous tough and iteration method by hand even without a calculator whereas the sizing problem is explicitly found out with the slide rule even which is not done.

So, therefore a difficulty was faced in this case that LMTD method is not that suitable because it requires tedious iterations and you gaze value is not very close you have to go for number of iterations to confront the solution. Today, it is a laughing stock but it was there, when it was discovered. Then two scientist Kays and London. They are two stalwarts in the field of heat transfer they found out another suitable method for this type of problem where seizing is given that means surface area is given.

We have to rate the heat transfer in terms his heating and cooling load that means we have to tell what are the outlet temperatures of the hot and cold fluid and this method which was found by Kays and London is known as NTU Epsilon method. So LMTD is one method another method into an employer in your placement interview we will ask this thing only. That when NTU Epsilon method, when LMT?

What is the difficulty of LMTD in rating problem and why NTU Epsilon method is difficult for sizing problem that also I will tell. Both the methods have relative merit and demerits. Now,

NTU Epsilon method. Now what is NTU? NTU I will tell you afterward when the course of deduction it will come. But now you know the meaning of NTU it is an abbreviation, Number of Transfer Unit.

And I will come to this afterward because now if I write the NTU expression like Nusselt number $h d/k$ you will know, why unnecessarily this because automatically in the way of deduction a term will come which is a none dimensional term know as number of transfer unit.

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NTU - ϵ Method.

NTU = Number of Transfer unit.

ϵ (effectiveness) = $\frac{Q_{\text{actual}}}{Q_{\text{max}}}$

$Q_{\text{max}} = (m_c c_c)_{\min} (T_{h1} - T_{c1})$ m_c is minimum

$\epsilon = \frac{m_c c_c (T_{h1} - T_{h2})}{m_c c_c (T_{h1} - T_{c1})} = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$

$m_h c_h$ is minimum

Epsilon is known as effectiveness which is also a non-dimensional term, effectiveness and this effectiveness of an heat exchanger Epsilon is defined as Q that is the heat transfer actual/ Q maximum, means ideal that is the ideal that is the maximum actual discharge by the ideal discharge is the coefficient of this type of thing, the maximum and actual. What is actual heat transfer? This is this. $M \dot{m} c_p \Delta T$, the temperature change this the actual heat transfer.

But what is the maximum heat transfer which is possible. The maximum heat transfer if you write Q_{max} try to understand this again there is a concept many people you can make people nervous by asking one question I will tell you that maximum heat transfer will be associated with the maximum change in the temperature. What is maximum change that is $T_{h1} - T_{c1}$ that means let us consider the counter flow.

I will tell the maximum heat exchange will be possible when the cold fluid will attain the inlet temperature of the hot fluid that is the maximum temperature available in the exchanger. Or other way, the hot fluid will be cooled to the inlet temperature of the cold fluid which the minimum temperature of the heat exchanger which means that $Q_{\max}$ will be associated with the maximum temperature difference $T_{h1} - T_{c1}$ of the exchanger. But there is a clue hot and cold fluid physically will not attain simultaneously the $T_{h1} - T_{c1}$.

It depends upon their capacity, that means the product of $\dot{m} C_p$. The fluid for which this $\dot{m} C_p$ is minimum that will catch it first afterward the heat exchanger operation will stop. So therefore, it is written in a generic way $\dot{m} C_p$ minimum which may be either $\dot{m}_h C_{p,h}$ or $\dot{m}_c C_{p,c}$ that I have to calculate from the value. I have to calculate from the value specific value then the mass flow rate value.

That means that ideal or the maximum heat transfer is defined as minimum capacity rate or minimum capacity that is mass flow rate times this specific heat capacity times the maximum temperature change one thing this is never achievable in a parallel in a counter flow like a reversible process. It can be achievable if we make an infinitely long exchanger in a limiting case like a quasi-static process that in a limiting place.

This T_{c2} will reach the T_{h1} or T_{h2} with T_{c1} if you keep an infinitely long. But here even if you keep an infinitely long exchanger it will never reach, $\Delta T_{\min}$ is universe it can be zero in an ideal situation of reversible process, a quasi-static process but can never become less than 0. It is impossible. So therefore, in a parallel flow heat exchanger it is never possible not feasible at all by the physics. Sad then what happen?

This happen like that this definition of effectiveness can be understood concept with respect to a counter flow heat exchanger nevertheless as a criteria of comparing the exchanger with this effectiveness we will use the same for parallel flow also. Though it does not have any physical meaning like the ratio of actual to maximum heat transfer in that case. Is it clear to everybody? Good students.

Now if I consider the cold fluid is the minimum fluid then what is Epsilon. Now if cold fluid if $\dot{m}_c C_c$ is minimum then what is this Epsilon, it is what is Epsilon you tell me if this is minimum then $\dot{m}_c C_c$ into $T_{h1} - T_{c1}$ and in that I will prefer to write this Q this equation. Why? Because this thing will cancel C_c into $T_{c2} - T_{c1}$ cancels out. Similarly, if $\dot{m}_h C_h$ is minimum then Epsilon is $\dot{m}_h C_h$ into $T_{h1} - T_{c1}$.

And I will write then here this one $\dot{m}_h C_h$ as $T_{h1} - T_{h2}$. And ultimately this will cancel $T_{h1} - T_{h2}$ divided by $T_{h1} - T_{c1}$. So, from here I can make another definition in terms of temperature change which will be more convenient and parallel flow extension will be little comfortable with this definition. It is the ratio of the temperature difference or temperature change of the minimum fluid.

Minimum fluid means minimum capacity fluid divided by the maximum temperature change of the exchanger. I think parallel flow heat exchange will be much comfortable that the definition via heat transfer is gone that means it is the change of temperature for the minimum capacity fluid divided by the maximum temperature change available in the exchanger.

Now with this I will for a simple algebraic deduction. Let us go back to our earlier deductions where we came here $\ln(T_{h2} - T_{c2})$ now we consider the parallel flow arrangement $T_{h2} - T_{c2}$ divided by $T_{h1} - T_{c1}$ is equal to I remember correctly one minus u by A just one step ahead $\dot{m}_c C_c$ plus $1/\dot{m}_h C_h$ where after this, this was replaced from there and I got expression for LMT but if I stop here in that deduction I just change the root.

Let us write this $T_{h2} - T_{c2}$ divided by $T_{h1} - T_{c1}$ you just follow you may not have to note because it is given in the text book you may or you may take but follow it. Because deduction if you logically follow afterward also taking notes you can follow then you will be able to deduce for a new condition is equal to just I write this thing in the exponential function nothing great minus uA .

And here I will make sum assumption before doing because it is case by case in a parallel flow I first assume the cold fluid is the minimum fluid that means $\dot{m}_c C_c$ is the minimum the

catch you will know afterwards first accept it. So here (43:13) this is minimum the product is less than this product then I take this as common $m \dot{c} C_c$ when I will tell the case for which $m \dot{h} c_h$ is minimum then we will take $m \dot{h} c_h$ as common.

I will tell you then it will be one plus $m \dot{c} C_c$ divided by $m \dot{h} c_h$. Now we have to play with this. Now here this equation is always used here what we will do we will express T_{h2} in terms of T_{h1} what is that write that T_{h2} is equal to from here T_{h2} will be T_{h1} minus $m \dot{c} C_c$ divided by $m \dot{h} c_h$, T_{c2} minus T_{c1} . I think I have done correctly.

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$$T_{h2} = T_{h1} - \frac{m_c c_c}{m_h c_h} (T_{c2} - T_{c1})$$

$$\frac{-T_c + T_{h1} - \frac{m_c c_c}{m_h c_h} (T_{c2} - T_{c1}) - T_{c2}}{(T_{h1} - T_{c1})} = \exp \left[-\frac{UA}{m_c c_c} \left(1 + \frac{m_c c_c}{m_h c_h} \right) \right]$$

$$\frac{(T_{h1} - T_{c1}) - (T_{c2} - T_{c1}) \left(1 + \frac{m_c c_c}{m_h c_h} \right)}{(T_{h1} - T_{c1})} = \exp \left[-\frac{UA}{m_c c_c} \left(1 + \frac{m_c c_c}{m_h c_h} \right) \right]$$

$$1 - \frac{(T_{c2} - T_{c1}) \left(1 + \frac{m_c c_c}{m_h c_h} \right)}{(T_{h1} - T_{c1})} = \exp \left[-\frac{UA}{m_c c_c} \left(1 + \frac{m_c c_c}{m_h c_h} \right) \right]$$

That means the trick is that T_{h2} is substituted in terms of T_{h1} and T_{c2} T_{c1} , T_{h2} will not work here. You will understand why. All this tricks will be understood at the end now if you substitute these at the present moment you accept that this show it has to be done only follow the algebraic continuity T_{h2} then I can write here T_{h1} minus $m \dot{c} C_c$ divided by $m \dot{h} C_h$, T_{c2} minus T_{c1} minus T_{c2} so T_{h2} minus T_{c2} minus divided by T_{h1} minus T_{c1} is equal to exponential. This space is not utilized.

$UA / m \dot{c} C_c$ one plus this one $m \dot{c} C_c$ divided by $m \dot{h} C_h$. So, what I have done. The left hand side T_{h2} minus T_{c2} T_{h2} I have substituted by this. Now what I do minus T_{c1} , here I add that is why I am writing plus because there is no place to write that. Some teacher intelligently, tactfully, give a space to minus T_{c1} and plus T_{c1} . So, what does it look T_{h1}

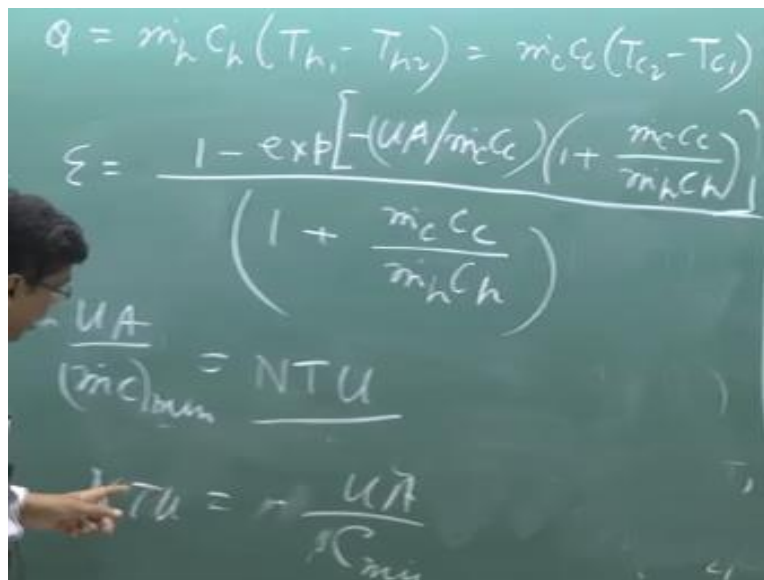
minus $T_{c1} - 1$ plus this into $T_{c2} - T_{c1}$.

So if you divided by this let me write that $T_{h1} - T_{c1} - T_{c2} - T_{c1}$ into one plus $m \dot{c} C_c / m \dot{c} h C_h$ and divided by $T_{h1} - T_{c1}$ just follow it. Right hand side I am not writing. So this becomes equal to one minus since I have considered from this step after approaching further here approaching to this step that the cold fluid is the minimum fluid. That means $T_{c2} - T_{c1}$ by $T_{h1} - T_{c1}$ is the effective place, Epsilon.

Because change in temperature of the minimum fluid divided by the maximum change in temperature, clear. So therefore, this will be one minus Epsilon into one plus $m \dot{c} C_c$ divided by $m \dot{c} h C_h$. This equals to this exponential. So, if we go at this and write this Epsilon then you will get this expression here I will write here if you now equate this one minus Epsilon, one plus this by this.

With this you get finally an expression that Epsilon is equal to one minus exponential minus $UA / m \dot{c} C_c$ into one plus $m \dot{c} C_c$ divided by $m \dot{c} h C_h$ – No, sorry okay. $m \dot{c} h$ divided by in terms of this quantity clear.

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$$Q = m_h C_h (T_{h1} - T_{h2}) = m_c C_c (T_{c2} - T_{c1})$$

$$\epsilon = \frac{1 - \exp\left[-\left(\frac{UA}{m_c C_c}\right)\left(1 + \frac{m_c C_c}{m_h C_h}\right)\right]}{\left(1 + \frac{m_c C_c}{m_h C_h}\right)}$$

$$\frac{UA}{(m \dot{c} C)_{\min}} = NTU$$

$$NTU = \frac{UA}{C_{\min}}$$

Now here $m \dot{c} C_c$ is the minimum heat capacity. Now this term minus $UA / m \dot{c} C_{\min}$ is known as number of transfer unit NTU which is a dimensionless term overall heat

transfer coefficient, surface area of the heat exchanger divided by $\dot{m} c_{\min}$. Now if you can ask me the question you should ask this question to me. Without gossiping or chatting with your friend at the back sir, if the hot fluid is the minimum fluid what could you have done?

Now if the hot fluid is the minimum fluid first I tell you this expression will be same just $\dot{m} c_c$ and $\dot{m} c_h$ will be interchange with each other so that in a generic formula I can write in terms $\dot{m} c_{\min}$ and $\dot{m} c_c$, generic formula. So this will interchange if $\dot{m} c_h$ is the minimum then it will be $\dot{m} c_h$ NTU is minus $u A / \dot{m} c_{\min}$. This will be NTU and this will be one plus $\dot{m} c_{\min} / \dot{m} c_c$.

Similarly, it will be one plus $\dot{m} c_{\min} / \dot{m} c_c$ but how to do it just interchanged. Here I was start again I started with cold fluid as a minimum fluid hot fluid as the minimum fluid I will $\dot{m} c_h$. This is not all I will first take secondly, I will not T_h to that is the second catch here T_{h2} will not be substituted from this equation T_{c2} will be substituted and in that case T_{c1} will not we added subtracted T_{h1} will be added and subtracted.

This is having a rhythm; Good boys can understand this thing better. Immediately by intuition here, we will take $\dot{m} c_h$ as common then it will be one plus $\dot{m} c_h / \dot{m} c_c$. Here T_{h2} will remain as it is T_{c2} will be replaced in terms of other temperatures from here T_{h2} T_{h1} and T_{c1} and after that I will add and subtract T_{h1} and in that case, I will use the definition of Epsilon as $T_{h1} - T_{h2}$ divided by $T_{h1} - T_{c1}$.

Because the definition of Epsilon is the change of temperature for the minimum fluid and the change of temperature maximum change of temperature available in heat exchange. Now if you write in a very generic fashion, a generic expression is like this if I make a very shortcut compact form by defining a parameter –now if I define it like this $\dot{m} c_c$ as the capital C

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$$\begin{aligned}
 m_c c_c &= C_c \\
 m_h c_h &= C_h \\
 C &= C_{\min} / C
 \end{aligned}$$

$$\epsilon = \frac{1 - \exp[-NTU(1+C)]}{(1+C)}$$

$$\epsilon = \frac{1 - \exp[-NTU(1-C)]}{1 - C \exp[-NTU(1-C)]}$$

That means the capacity rate or simply the capacity and $m_c c_c$ is the C_c and $m_h c_h$ is equal to C_h and if I define C as the ratio of $C_{\min} / c$ which may be either this by this that means this by this or this by this depending upon the case. In that case the expression will be very simple it looks like simple expression remain same $1 - \exp[-NTU]$ where NTU is $U A$. Sorry, not minus, sorry.

NTU is a scale quantity dimensionless $m_c C_{\min}$ that means $C_{\min}$ with this nomenclature where NTU , is NTU into 1 plus capital C and therefore here it is 1 plus capital C this is the expression. Where capital C is the ratio of the total capacity that means m_c specifically ratio, ratio of the total capacity. What is the ratio minimum to the other one? That means $C_{\min}$ by C so this is the expression and this expression is for parallel flow.

Counter flow expression will be again a tricky and that I leave to you to do the counter flow expression in both the cases that is left and an exercise and the expression will be ϵ is equal to $1 - \exp[-NTU]$ here it will be $1 - C$ and here it will be $1 - C$ into again $\exp[-NTU]$ into $1 - C$. Now, you see that in condensation and boiling I told one of the fluids will be parallel in case of condensation the hot fluid is a parallel line.

In case of boiling the cold fluid is parallel line and parallel line means temperature does not

change and I told qualitatively that counter flow and the parallel flow remains same but now mathematically you see that for a condensation and boiling problem this one of the fluid specific heat tends to infinity. The fluid which is boiling, and the fluid which is condensing C infinity. I showed you earlier in the figure which means that C is equal to $C_{\min}/C$ that means

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$$C = \frac{C_{\min}}{C}$$

$$C \rightarrow \infty$$

$$= 0$$

$$NTU(1+C)$$

$$(1-C)$$

$$u(1-C)$$

$$\epsilon = 1 - \exp(-NTU)$$

This higher C tends to infinity because which tends to infinity that will always be higher no question of calculation. That means an intelligent boy will tell that for boiling condensation the ratio which you denote as C will be always zero and if you put C zero in both the expression will yield the same value Epsilon is one minus exponential minus NTU this is the argument. If you put C zero because the ratio is always zero.

Because the higher heat capacity rate whatever we got the product of $m \cdot C$ is tending to infinity. Which is not a matter of hot fluid or cold fluid, which one is higher and which one is lower. But here one thing is very much open that if I do not know the temperatures explicitly then what we can do when we know the NTU because area you know overall heat transfer coefficient we know and the mass flow rate and specific heat.

We know, we can straight away find out the effectiveness and when the effectiveness is known, I can find out the temperature. So therefore this is a case where the rating will be done explicitly no iterations rating means the size is given we have to find out the heat transfer that means all four

temperatures are not known explicitly. So, we can find out the Epsilon.

And whenever we find out the Epsilon, T_{h1} and T_{c1} all right I will work then from the definition of Epsilon I can find out the temperature difference of the minimum fluid that means heat load will come in the temperature difference of the other fluid will also come. So therefore, where rating heat load and cooling load is not done but size is given NTU Epsilon method is an explicit method to find out effectiveness, very quickly temperatures are found.

But where the temperatures are given sizing is to be made LMTD is useful because from here knowing Epsilon NTU as a function of Epsilon will be little difficult you understand this may require tedious calculation or iterations.