Conduction and Convection Heat Transfer

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Lecture - 37
Boiling and Condensation - I

Good morning to all of you. Today onwards, we will discuss boiling and condensation. So far, we have discussed convective heat transfer both forced and free, and there we have seen that heat is being transferred between a fluid and the adjacent solid, when the fluid is in motion. And this motion may be caused by external agency. That is known as forced flow, where we prescribe it as forced convection.

And this motion may be generated due to buoyancy. Buoyancy driven flow where the convection is termed as free convection. But in both the cases the fluid medium is homogeneous and there is no change in phase. It is either liquid or gas, but in many practical applications in this type of convective heat transfer, the fluid medium changes its phase, which means it may change from liquid to vapour phase or from vapour to liquid phase.

Vapour is a gaseous phase as you know close to saturation state. The term vapour is a gas phase of a liquid close to the saturation state. So, either liquid to vapour or vapour to liquid, few examples I tell you so that you will be encouraged to study this boiling and condensation phenomena.

For example, the condenser of a steal power plant, you know the steam after expansion from the turbine where we get mechanical work comes or enters into the condenser and it flows over the surface of number of tubes, where the cooling water flows inside the tube. And the cooling water takes the heat from the steam and steam gets condensed.

Similar type of things happens in the condenser of a refrigerator, the working fluid known as refrigerant, which after leaving from compressor at high temperature and pressure, flows inside a, inside number of tubes and it is condensed by dejecting heat to the atmospheric air, which flows past the tubes. In this case the condensation takes place at the inside surface of the tube, unlike that at the outside surface of the tubes in case of the steam power plant condensation.

Now the other direction phase change from liquid to vapour takes place in the water tube of a boiler. As you know, the boiler receives high pressure water from the fit pump and as the water flows to the water tubes of the boiler, heat transfer takes place from the compression product known as true gas at high temperature, which flows surrounding the tube and transfers heat to the water, which in course of its flow is converted into vapour.

Now this type of processer convective heat transfer, because there is a transfer of heat to or from solid surface, that means heat transfer between the solid and the fluid where the fluid changes its phase either from liquid to vapour or from vapour to liquid. So therefore, it is a convection process associated with phase change and known as boiling and condensation depending upon the phenomena, whether it is boiling or condensation.

Now how does it differ, this thing differs from the convection without phase change. This is because of two things, number one is that as we know the condensation or boiling takes place at a constant temperature, when the fluid takes heat or dejects heat and changes its enthalpy. For example, when the steam condensed water, there is a change in enthalpy from its saturated vapour state to the saturated liquid state.

And a large difference of enthalpy is there and the difference of enthalpy becomes equal to the heat rejected by the steam. And this thing happens this conversion from steam to water, just as an example, at a constant temperature and pressure. Similar is the conversion from liquid to vapour, that means water to steam. So, this phase change was always takes place at a constant temperature and pressure.

But is associated with a large change of enthalpy, which we tell as enthalpy of vaporisation in case of boiling that is from liquid to vaporisation vapour, or enthalpy of condensation that is from vapour to liquid. And sometimes we tell it as latent heat of vaporisation or latent heat of condensation. So therefore, if a small temperature difference is created that can sustain this process.

For an example, if we have steam at atmospheric pressure we know the saturation temperature is 100 degree Celsius. Steam is ready to condense if it is cool little bit, that means if the steam is brought into contact of any surface whose temperature is little less than 100 degree Celsius, for example 90 degree Celsius, condensation will take place. And there is a huge change in the enthalpy.

That means a high heat transfer will take place, a rate of heat transfer will take place at a very small temperature difference, for which the heat transfer coefficient in phase change processes, condensation and boiling is much higher than those of free and forced convections. In sequence, the heat transfer coefficients are very low for free convection, because the heat transfer rate is low because of small flow due to buoyancy.

And forced convection is much higher and phase change process is much, much higher. This is one characteristic feature another characteristic feature is that, in a phase change process the flow is dominated by buoyancy, because of the change in density between the vapour state and the liquid state. And sometimes, surface tension plays a major role, for example in case of boiling, when liquid changes into vapour.

For example, water changes into steam in a water tube, then what happens? When the boiling is initiated that I will describe later on vapour bubbles are generated. Now the growth of the bubble, the collapse of the bubble, the movement of the bubble, that creates the flow field in such a way that heat transfer is affected depends to a large extent on the surface tension property between the liquid and vapour of that particular material, or the particular fluid.

So therefore, surface tension also makes a major contribution along with the buoyancy forces which are not common in convection without the phase change. So, these are the characteristic features which makes these two phenomena, though convective heat transfer, but little different from the pure convective heat transfer without phase change. With this, I will start first the condensation, then the boiling.

So, with this in background, let us first consider a simple case to understand the condensation heat transfer. Now before that, I have to explain again the condensation, we will start with the condensation. As I have told that a vapour, if you consider a vapour at saturated state comes into contact with a solid surface whose temperature is little less than the saturation temperature very easy to conceive that consider steam at one atmospheric pressure at 100 degree Celsius comes into contact to a plate whose temperature is lower than that.

For example, 60 degree or even 90-degree Celsius steam condenses. So now condensation takes place in two ways, how? If the liquid after condensation wets the surface, which depends upon the relative wettability characteristics between the surface and the liquid. Then what happens the drops of liquid, which are formed they spread they collies and finally a thin film, which smoothly glides over the surface and is drained out.

This is known as film condensation. But in case where the surface is such, which does not become wet by the liquid in that case what happens the liquid, the vapour after condensation forms the liquid drops, which do not spread and collies and make the film, rather the number of drops are formed and in random fashion and they are ultimately drained as the flow of drops over the surface. This is known as dropwise condensation.

So therefore, condensations are of two types, you write this, I am not writing on the board, one is film condensation where a thin film is formed when the surface is wet by the liquid, another is dropwise condensation where the film is not formed drops are formed in a random fashion and is drained. Now in film condensation it obviously apparent that this thin film does not allow the vapour to come in contact with the bare solid surface at low temperature.

That means in other way that the liquid film provides a thermal resistance or barrier to the vapour to make the heat transfer with the solid surface by coming to direct contact. There is a temperature gradient that exist in the liquid film and the vapour gets condensed at the interface between the liquid film and the vapour. So, it does not get an opportunity to be in contact with the bare surface of the solid. It is always blanketed by a liquid film.

So therefore, in this case the heat transfer rate and the condensation is relatively smaller as compared to dropwise condensation, where the vapour gets contact gets an opportunity to come into contact with the bare liquid surface because of the empty space where the drops are not there. If there are number of drops at the location of the drops the vapour cannot come into contact with the solid surface.

But when the exposed solid surfaces are there, where the vapour can come into contact. So that the rate of heat transfer and rate of condensation will be definitely more since vapour can come into contact with the bare solid surface. But, unfortunately, we have found in all practical surfaces the wettability characteristics is such if the surface is exposed or operation is made for a long, surface becomes wet by the liquid and a thin film is formed.

The effort has been made by putting coatings, adding coatings on surface by adding vapour additives to decrease the wettability characteristics but it has not been made to it a success, so that in almost all practical purposes we come across with the film condensation.

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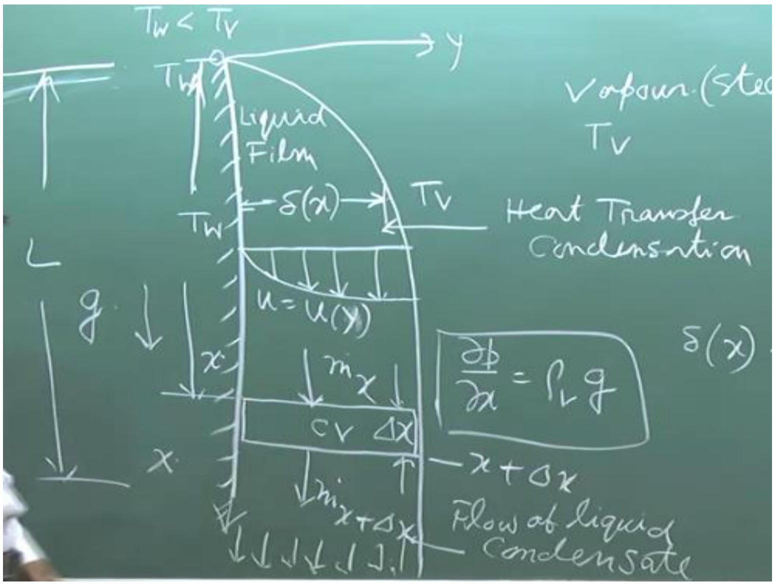

So, to understand film condensation let us consider this ways so that we can have a clear understanding. A simple case of a vertical flat plate, let us consider a vertical flat plate, which is at a temperature Tw and there is a vapour at saturated state, vapour at saturated state, whose temperature is Tv. Now this vapour you can consider as steam and if it is one ambient, one atmospheric pressure as the ambient pressure.

Then this Tv will be 100 degree Celsius. Just to understand, that means saturated vapour at a temperature Tv and the wall is at a temperature Tw, where Tw is less than Tv to enhance or to initiate the process of condensation, okay. So, what happens because of the condensation as I have told and the wettability characteristics of the plate in practice, that there is a thin film, which forms and grows in the vertical direction, downward and in a very amplified or a exaggerated to a, I am drawing this.

This is the thin film of liquid and this film grows, that means the thickness of the film increases in the direction of the flow. This is because of the condensation of vapour, more condensation. So, this is the liquid film and finally, this is the flow of liquid, this is the, this is the flow of liquid which is known as condensate, flow of liquid down the vertical plate. If I just make an axis like this, this is y and this direction for convenience of my analysis x.

Now the film has a thickness at any distance from this origin, the leading edge is zero, let us consider this as the film thickness at this location, delta which a function of x, increases with x. This is the film thickness which increases with x, why, this is because the heat transfer takes place here. Here the temperature is Tv vapour and here the temperature is Tw. So, there is a temperature gradient, Tv is greater than Tw, heat transfer takes place.

And by which it is a saturated state vapour gets condensed into liquid, sticks to the surface and makes a liquid film, which grows and flows along the surface. The thickness increases because of the more condensation, that is heat transfer, and condensation takes place, this is the physical picture. Now this liquid glides, film glides over the surface slowly, the velocity is extremely small.

But it has some velocity, and do expect that within this delta, that is within the film there is a velocity distribution. So qualitatively we can express this velocity like this, there will a no-slip condition at the surface from zero if we draw the velocity profile. So, velocity profile will be like this, u as a function of x. So, which will satisfy the no-slip condition and at the same time at the free surface, it will have a zero siestas condition.

This is a free surface this is a flow of liquid and this is vapour, because of the very low of viscosity on the gas side or vapour side, usually we impure the zero siestas, the interfacial tangential stress become zero. So, for a free surface flow the interface boundary condition is the zero siestas, so that the velocity gradient becomes zero at the free surface, which flows with some definite velocity.

So therefore, a qualitative picture of the velocity profile is like this.

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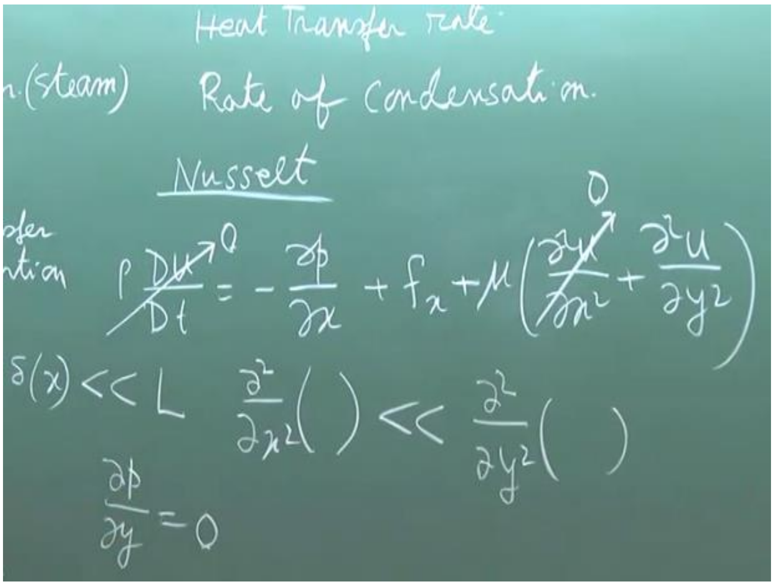

Let us first find out this velocity profile first, our main objective is to find out the heat transfer rate, our main objective is to find out the heat transfer rate, and the rate of condensation. If we know the heat transfer rate, so rate of condensation is just divided by the enthalpy of condensation. They are equal with a scale factor that enthalpy of condensation, that means if we extract heat at a certain rate from a vapour at saturated state.

So, you can find out the rate of condensation is the rate of heat transfer divided by the enthalpy of condensation, true. So therefore, the two things are almost same, we are interested in this two. Let us first, how to do that, Nusselt was the first person who derived this, and this is the classical solution, Nusselt solution of this condensation over a flat vertical surface, okay.

If you have any question you can ask me, you please, what do you want to know, do you have something to ask me, you can ask, you, do you want to ask something to me, hello, now what you are asking him, regarding this, regarding our, why do not ask me, okay, do not ask, because you will get ample opportunity to ask your friend outside, I told you at the beginning of the class. But if you have got any query please ask me, okay.

So, let us first derive the expression for velocity profile. How do you derive, if you take this co-ordinate x and y, this is the direction of gravity, I write the Navier–Stokes equations, consider a two-dimensional flow, then the x direction. This is the Navier-Stokes equation, considering the flow to be two-dimensional means what, that means flow velocity is taking place only in the x direction.

There is no y direction flow, but this in theory is a function of both x and y coordinate. Now, first of all we see that from this x direction equation of motion, what we get. The flow is extremely small, so this is zero, this is zero. We consider the flow is so small, it is a creeping flow type of assumption, probably you have been taught in your fluid mechanics class, where the inertia force can be neglected as compared to the viscous force.

Now, since the delta at any length x is always much, much less than the length of the plate, if we consider the plate has a finite length L. If we consider that the plate has a finite length L, then always one can argue with this geometrical configuration like our boundary layer approximation, where boundary layer is very thin, that is why sometime this approximation is known as boundary layer type approximation.

That the film thickness is very less than this, we can write del square by del x square of any parameter is much less than del square second derivative of y square of any parameter. Why, this is because, this is in the order of y square, and this is in the order of x square and y direction the order is delta, the scale in the y direction is delta, where the scale in the x direction is L.

So therefore, this quantity is lower by order of magnitude, by delta square by L square. This already you have done in your boundary layer approximation. So therefore, I can cancel this term compared to this term, clear. Now next part is the y direction equation, y direction, sorry this y direction, yes y direction equation zero is equal to zero. This is because there is no flow in the y direction.

And actually, the y direction gives you the boundary layer type of approximation, that del p/del y is zero. That means if you write the y direction equation rho D Phi Dt is zero, and this viscous force are totally zero, because there is no flow in the y direction. So, this gives rise to del p/del y is zero. Similar results as you got in boundary layer analysis, which is told in our language.

That pressure outside this film or in case of boundary layer, outside the boundary layer is imposed on the film. That means, if I find out the pressure gradient del p/del x, which is existing here in the outside of the film that is seen within the film since del p/del y is zero.

That is the language probably Prof. Suman Chakraborty told. We always tell the pressure in the free stream is imposed on the boundary layer, pressure in the potential flow is imposed on the boundary layer, because del p/del y is zero.

So therefore, this thing is zero, this thing is zero, I can write as dp/dx. Now our next job is to find these two term, so fx is, therefore zero is minus del p/del x plus fx. Let us find out this del p/del x, when I write this separately in the Navier-Stokes equation, where fx is the body force per unit volume. This pressure gradient pressure is the static pressure. I told you the Navier-Stokes equation is written usually without any body force, because we consider there is no any external body force filled.

But gravity we cannot neglect, but what we do tactically that gravity we taking into pressure itself and we tell that okay this is the Navier-Stokes equation. So, there is no body force, gravity is considered into the pressure, where the pressure is defined as piezometric pressure. But if you decompose it in terms of pressure gradient and the body force, then this pressure gradient is static pressure gradient.

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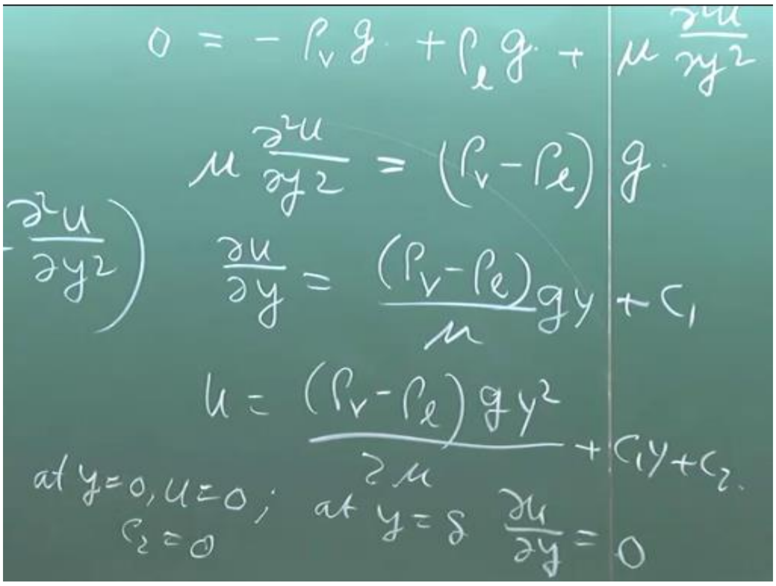

$$0 = -\rho_v g + \rho_l g + \mu \frac{\partial^2 u}{\partial y^2}$$

$$\mu \frac{\partial^2 u}{\partial y^2} = (\rho_v - \rho_l) g$$

$$\frac{\partial^2 u}{\partial y^2})$$

$$\frac{\partial u}{\partial y} = \frac{(\rho_v - \rho_l)}{\mu} g y + C_1$$

$$u = \frac{(\rho_v - \rho_l) g y^2}{2\mu} + C_1 y + C_2$$

at $y = 0, u = 0$; at $y = \delta$ $\frac{\partial u}{\partial y} = 0$

$C_2 = 0$

Now static pressure radiant in this vapour, vapour is stationary, so hydrostatic equation of pressure will hold good. So, del p/del x will be rho g, rho v, rho v is the density of the vapour, because we know del p/del x, it is because of the wet. This you know from your fluid mechanics knowledge the pressure filled in a hydrostatic. The pressure changes in the direction of the gravity.

So, it decreases vertically down in the direction of the gravity increases, decreases in the vertically upward. Since my x x is positive is downward, so minus sign is not there, del p/del x is rho v into g, okay. It is a linear variation, and it increases in the vertical direction in hydrostatic pressure distribution and since del p/del y is equal to zero, boundary layer type of approximation. So, this del p/del x will be imposed here.

So, I can write del p/del x as rho v into g with a minus sign, what is the body force per unit volume, fx, it is simply rho l is the density of the liquid into g. That is the wet per unit volume plus mu del square u del y square. So up to this the concept of fluid mechanics is over. Now we will make certain mathematical arrangements, so, therefore we can write mu del square u del y square is equal to rho v minus rho l into g.

Now I integrate del u/del y, taking mu on the other side, rho v minus rho l by mu g y plus a constant C1 and finally u is equal to rho v minus rho l, this is so simple, g y square by two mu plus C1 y plus C2. Now what are the boundary conditions tell me, this is a function of y. So, what is the boundary condition, u is a function x, I am sorry, u will be a function of y, this is a function of y, I am sorry this is function of y, u is a function of y.

So, u is a function of y and this is the expression, what are the boundary conditions, please tell me, at y is equal to zero u is equal to zero. Any other boundary condition, two conditions are required, at y is equal to delta, very good, siestas at the free surface is zero. Now the first boundary condition leads to that C2 is zero, C2 has to be zero. And second boundary condition leads that C1 equals to rho l minus rho v by mu g delta. C2 is zero.

Because y zero u zero, and del u/del y is zero means C1 is equal to rho l minus rho v by mu g delta.

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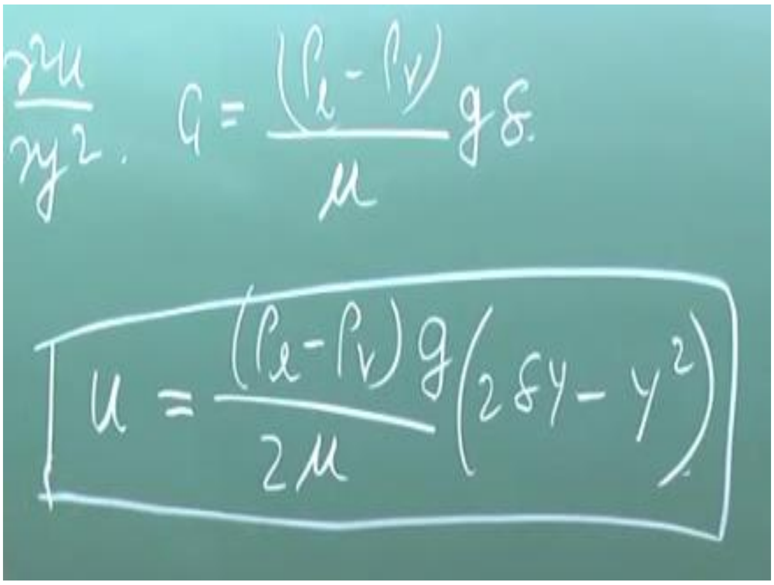

$$\frac{\partial^2 u}{\partial y^2}. \quad C = \frac{(\rho_l - \rho_v)}{\mu} g \delta.$$

$$\boxed{u = \frac{(\rho_l - \rho_v) g}{2\mu}\left(2\delta y - y^2\right)}$$

So, if you substitute this c1 here, then you get u is equal to rho l minus rho v, you take this rho l minus rho v by two mu g, then it will be two delta y minus y square. It is always positive, because rho l is greater than rho v and since y is less than delta, the quantity in the bracket is also positively proved that u is in the direction of the positive x axis, obviously, it is flow is dominated by the buoyancy, okay.

The net buoyancy force, here this is the picture where we can explain it physically that viscous force is balanced by the net buoyancy force, which we derive from the Navier-Stokes equation. Sometime we can take a control volume and can make a balance of the net buoyancy force than the viscous force, by simply explaining the inertia is zero for low velocity.

So, we neglect that and viscous force is approximated by mu del square u del y square, because of the geometry, the same thing which I derived from the basic Navier-Stokes equation, same thing, okay. So finally, this is very important thing that we have come across, u. Now what we do, okay, now what we do, we make this is the Nusselt analysis, a classical thing, I am following, make a mass balance in a fluid element of control volume, at a distance x from the leading edge, at a distance x from the leading edge, where this is a control volume.

At x, the mass flow rate is m dot x, at x plus del x let this is del x the height of this control volume or element is del x. So, at this section at x plus del x, let me write the mass flow as m dot x plus del x. And this two will not be same, because what I told for which the delta is growing, because of the m dot c, that s the mass rate of condensate. The rate of mass vapour condensed within this control volume, that due a length of delta x.

So that m x dot x plus delta x and m dot x, difference is the m dot c.

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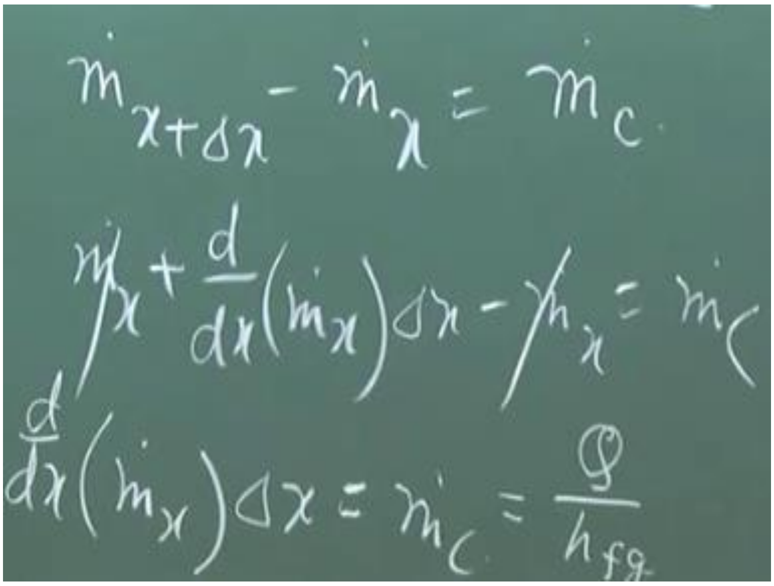

$$\dot{m}_{x+\Delta x} - \dot{m}_{x} = \dot{m}_{c}$$

$$\cancel{\dot{m}_{x}} + \frac{d}{dx}(\dot{m}_{x})\Delta x - \cancel{\dot{m}_{x}} = \dot{m}_{c}$$

$$\frac{d}{dx}(\dot{m}_{x})\Delta x = \dot{m}_{c} = \frac{Q}{h_{fg}}$$

So, a simple mass balance of this control volume tell that m dot x plus delta x minus m dot x is equal to m dot c, simple. Primary school level thing that means the gross mass balance of this control volume is that m dot x plus del x is equal to m dot x plus m dot c and I am retaining that value and if I expand this in a Taylor series by neglecting the higher order term because of delta x is very small.

This we can write m dot x plus d/dx of m dot x delta x minus m dot x is equal to m dot c, so this cancels up. So therefore, I get d/dx of m dot x del x is m dot c, okay. This m dot c also can be written as the heat transfer through the control volume divided by if h, f, g is the enthalpy of vaporization.

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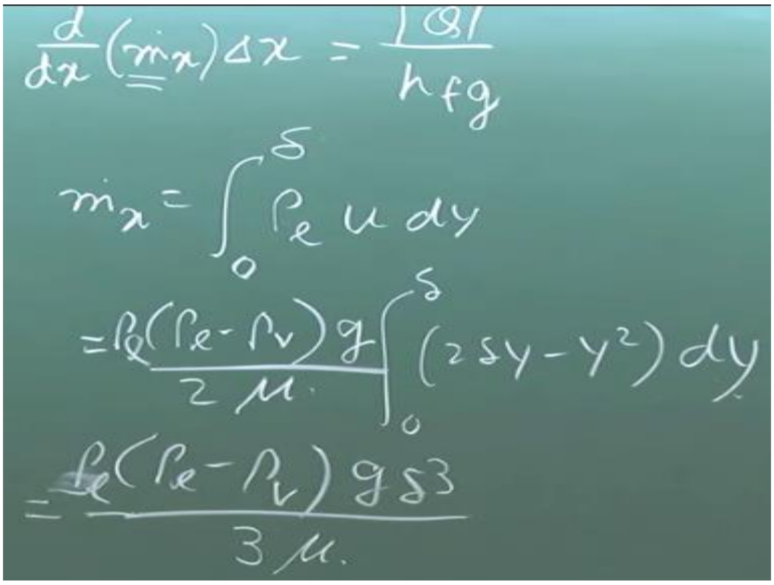

$$\frac{d}{dx}(\dot{m}_x)\Delta x = \frac{|Q|}{h_{fg}}$$

$$\dot{m}_x = \int_0^\delta \rho_l u \, dy$$

$$= \frac{\rho_l(\rho_l - \rho_v) g}{2\mu} \int_0^\delta (2\delta y - y^2)\, dy$$

$$= \frac{\rho_l(\rho_l - \rho_v) g \delta^3}{3\mu}$$

That means I can write d/dx of m dot x del x is equal to what I can write Q divided by hfg and here I deliberately put a mod value, absolute value of Q, because Q by hfg represent the rate of mass condense, which is a scalar quantity, so q may come out from some analysis with a positive and negative site, which will be so that q in the positive y direction or negative y direction of the problem.

That means of the coordinate axis taken, but I will take the absolute value to write it, because this is a scalar equation. Now, our next task is to find out the mass flow rate and heat transfer for this control volume. So now mass flow rate at any section x, m.x can be written by a simple equation as you know rho l u dy from 0 to delta at any section x. At any section x in general, at any arbitrary section the rate of mass flow is written like that we have considered a unit width of the plate in this direction, unit width of the plate.

So that, 1 into dy is the cross and u and rho l. As you know this thing by this time from your fluid mechanics knowledge rho l u dy 0 to delta is the mass flux across any section. Now, if you put this value of u, then that can be written as rho l minus rho v into g, by two mu into 0 to delta 2 delta dy minus y square dy. Now, two delta y, y square, it is y square by 2, that means 2, 2 cancels, that means this is delta Q.

You do it mentally and this is delta q by 3. 1 minus one-third is two-third, 2, 2 cancels that means this becomes rho l minus rho v g delta Q, 2 by 3 mu. This becomes d m dot x that means the mass flow rate at any section, clear. Okay rho l minus rho v g what happens? Rho l is missing, yes very good rho l, good. Rho l, rho l minus rho v g delta Q by 3 mu, okay. Now we have to find out what is Q.

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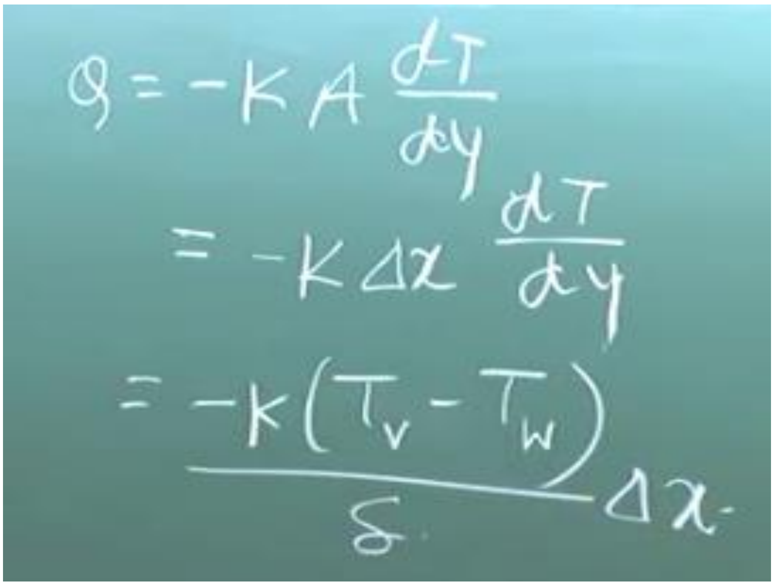

$$Q = -KA\frac{dT}{dy}$$

$$= -K\Delta x\frac{dT}{dy}$$

$$= \frac{-k(T_v - T_w)}{\delta}\Delta x.$$

We have to find out what is Q. Q is equal to minus KA dT/dx, we know, here it is dT/dy. What is A for this control volume minus A into delta x, we have taken a unit v. What is del T/del y at this interface? Because here is a temperature Tv vapour site and here Tw it is less than Tv that means there will be some temperature gradient from Tv to Ty, which is decreasing in this direction of the plane. Well, now there is an assumption.

Since the motion is extremely small, for which we have neglected the inertia force, we can consider that the effect of, as I have told, many times that convection is basically conduction affected by advection or flow, that is why it is convection. So, flow does not have any effect on the conduction. Heat transfer takes place, conduction as if this medium was station. This small velocity does not affect it.

That means I will assume these as a pure conduction problem in a stationary medium and that to in one direction. Again, because of the fact that delta is very, very less than l. So therefore, if we consider that this is a conduction in a stationary medium where the small flow has got no effect and one-dimensional conduction, then we can consider with a constant thermal conductivity the temperature gradient is linear.

That means this temperature gradient is therefore minus K Tv minus Tw, this is the temperature gradient divided by delta and Tv is greater than Tw, so therefore Q is negative, means it is in the opposite direction to the positive y axis. That means it is towards the plane, clear, okay, towards the plane. Now, delta x, clear, okay K delta x into dT/dy, so minus K. Now you put this Q and mx, then what you get, here. Then, we get that m.x is this one, finally this one.

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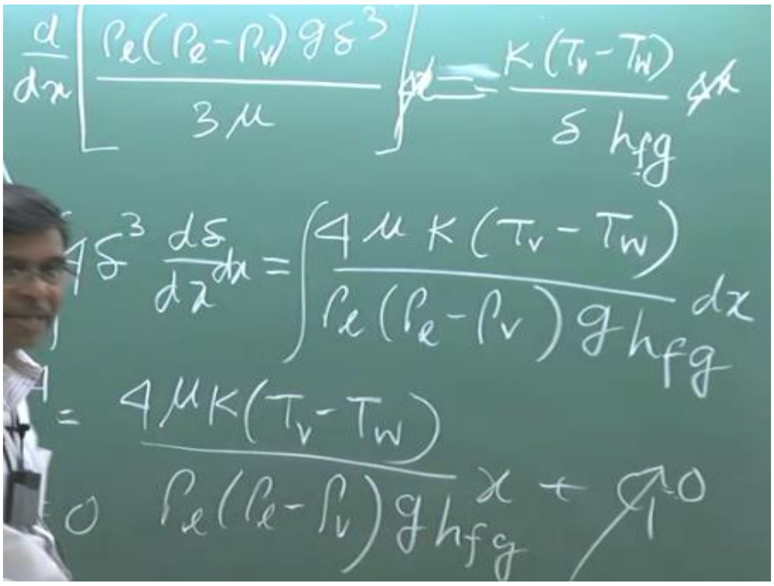

$$\frac{d}{dx}\left[\frac{\rho_\ell(\rho_\ell-\rho_v)g\delta^3}{3\mu}\right]\cancel{dx} = \frac{K(T_v-T_W)}{\delta\, h_{fg}}\cancel{dx}$$

$$4\delta^3\frac{d\delta}{dx}dx = \int\frac{4\mu K(T_v-T_W)}{\rho_\ell(\rho_\ell-\rho_v)g\,h_{fg}}dx$$

$$= \frac{4\mu K(T_v-T_W)}{\rho_\ell(\rho_\ell-\rho_v)g\,h_{fg}}x + C^{\nearrow 0}$$

So, we get d/dx of rho l minus rho v into g delta Q divided by 3 mu is equal to, now we take the absolute value that is equal to K Tv minus Tw divided by delta. Now there is a delta, I am sorry, d/dx of m.x delta x is K tv minus Tw delta x divided by hfg, because Q by hfg. That means I am substituting this mass flow rate m.x from here and I am substituting the absolute value of Q here and I get this relationship, clear?

Now delta x is cancelled. Now, everything is constant only the delta, which is varying. Now this delta Q d/dx is three delta square d delta/dx, so 3 and 3 will cancel. So therefore, we can write delta square and this delta goes there so that I can write in delta Q d delta/dx and this 3 and 3 will cancel is equal to mu K Tv minus Tw, I am just telling you, rho l into rho l minus rho v into g h f g, am I correct, d delta/dx.

So therefore, delta square, 3 delta square d delta/dx then this delta comes, so therefore delta Q d delta/dx, 3 and 3 cancels, okay. Just you do it. I have made one step jump, which can be made mentally so that mu K Tv minus Tw divided by this d delta g h f g, clear. You do it mentally, I think you can do it, okay. I give little time, clear, okay. Now if you multiply with 4 and integrate with dx, this is school level step.

Then 4 delta Q d delta/dx is d/dx of delta 4. That means delta 4 equals to 4 mu K Tv minus Tw divided by rho l into rho l minus rho v into g h f g, x plus some constant C1. Just integrate with respect to x. Because I want to find out an expression for delta as a function of x that is growth of delta. Now, the boundary condition is that at x is equal to 0 leading to delta is 0. So that this becomes equal to zero, clear. Okay, then I can write.

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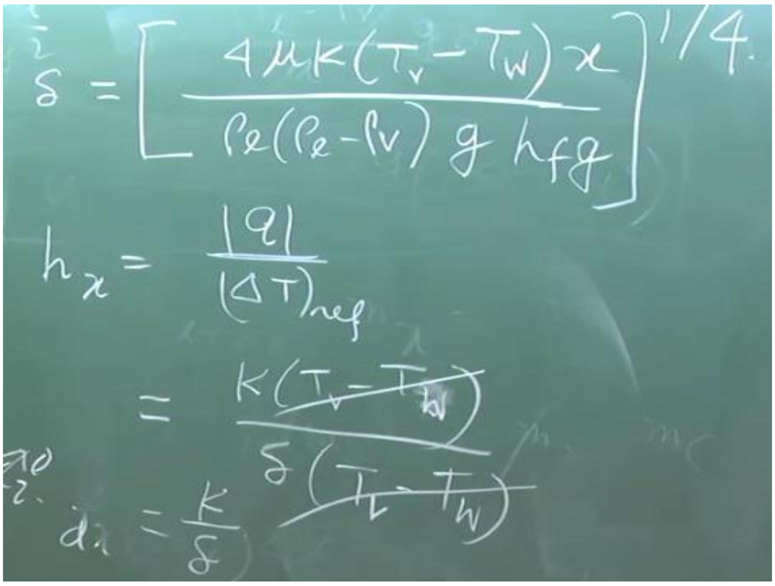

$$\delta = \left[\frac{4\mu K(T_v - T_w)x}{\rho_l(\rho_l - \rho_v)\, g\, h_{fg}}\right]^{1/4}$$

$$h_x = \frac{|q|}{(\Delta T)_{ref}}$$

$$= \frac{K\cancel{(T_v - T_w)}}{\delta\cancel{(T_v - T_w)}}$$

$$h_x = \frac{K}{\delta}$$

Therefore, I can write that delta is equal to 4 mu K Tv minus Tw x divided by rho l, rho l minus rho v g hfg, g is the acceleration due to gravity, hfg is the enthalpy of condensation whole to the power 1/4. This was Nusselt derivation for the growth of film, which is proportional to the fourth power of the x. If you compare it with the growth of boundary air over a flat plate in laminar flow.

The delta is proportional to the half power square root of it and it is proportional to the one-fourth of this. It grows very slowly. So, this is the basic definition with all those assumptions I have told. Now, engineers are interested more in finding out the heat transfer coefficient. As you know, the heat transfer coefficient h at any location. This already you know. I told you several times in my lecture or to the introduction to convection that it is heat flux at a location divided by the reference temperature difference.

Everything is a point function and therefore heat transfer coefficient is a point function, locally. That means local heat transfer coefficient. Yet, q is here also, I take the absolute value, because the heat transfer coefficient is a scalar quantity. So, I do not want this sign, which direction it is, negative direction of the axis or positive direction of the axis. So therefore, this is simply Q is K Tv minus Tg by delta.

And here incidentally the reference temperature difference also free steam temperature, sorry Tw, wall temperature, which is same, so therefore in a conduction dominated convection. Though it is a convection, but basically conduction, no effect of velocity. If you define a heat transfer coefficient, then it will be always K by delta. If the wall is at a constant temperature, this is a thumb rule, because the prescribed temperature difference is this.

So therefore, if you write this, then what you will get hx equals to what.

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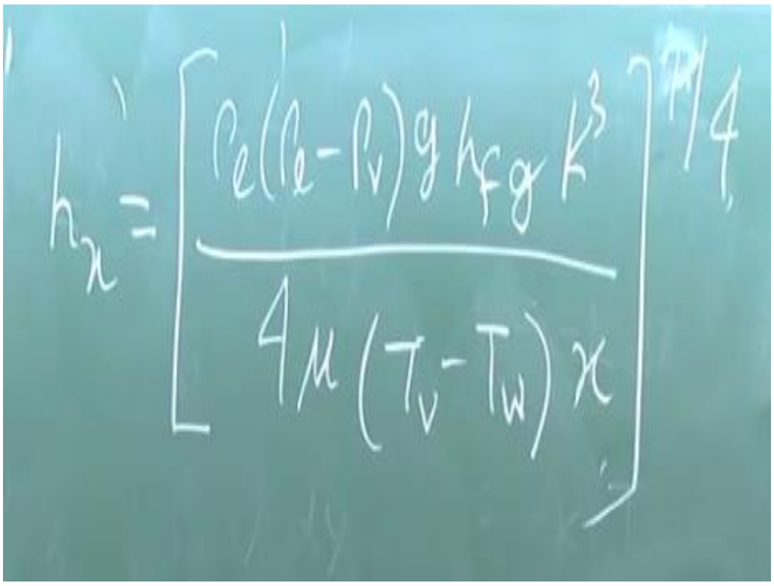

$$h_x = \left[\frac{\rho_l(\rho_l-\rho_v)\, g\, h_{fg}\, k^3}{4\mu(T_v - T_w)\, x}\right]^{1/4}$$

Then you get hx equals to K by delta. That means here I write hx is equal to this thing will go top, K by delta. That means K4 that means this will come, you just see rho l, rho l minus rho v into g hfg into KQ divided by this one will come 4 mu Tv minus Tw into x whole to the power 1/4. What happens K by delta, that means hx will be K by delta. This will be K inverse this and K4, I make and take it under one-fourth root, so it will be KQ, clear, Okay.

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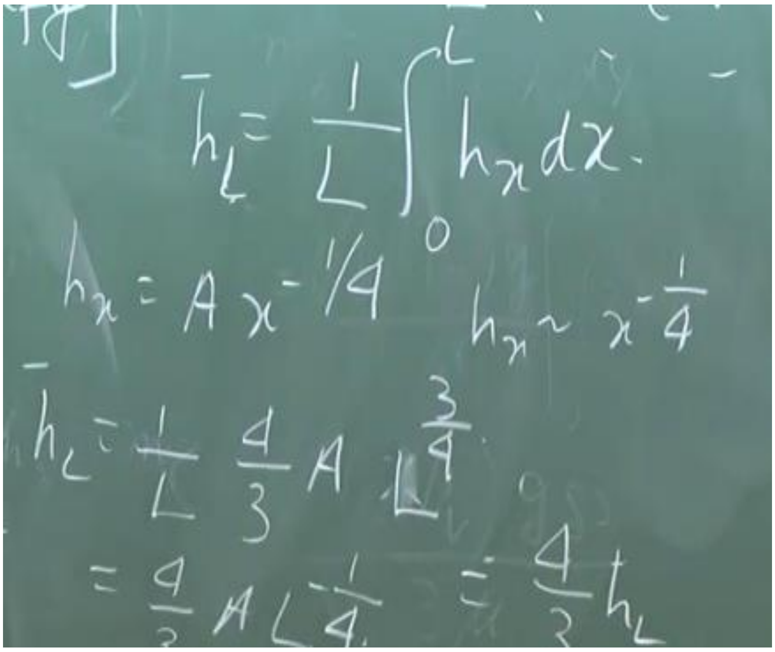

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x \, dx.$$

$$h_x = A x^{-1/4} \qquad h_x \sim x^{-\frac{1}{4}}$$

$$\bar{h}_L = \frac{1}{L} \frac{4}{3} A L^{\frac{3}{4}}$$

$$= \frac{4}{3} A L^{-\frac{1}{4}} = \frac{4}{3} h_L$$

Now, if you are told to find out an average heat transfer coefficient over a length l of the plate, which is defined as this, 0 to l. This is the definition of the average heat transfer coefficient over a flat surface, what is that value in terms of hl? Can you tell me? I told several times in the convection class. You have forgot, so I tell you again. Here, you see these things are all constant.

Obviously, the character of local heat transfers with the character of x, that means if this constant I define as some A, then I can write hx is A into x to the power minus 1/4 that means hx is proportional to x to the power minus 1/4. If you put it what we will get, hL bar is 1 by L, Ax to the power minus 1/4 dx means, 4 by 3 A x to the power minus 1/4 plus 1 3/4 and that will be L, not x. If you put this simple integration will tell this thing, clear.

That means four-third of A L to the power minus 1/4 that means it is four-third of local Nusselt number at L.

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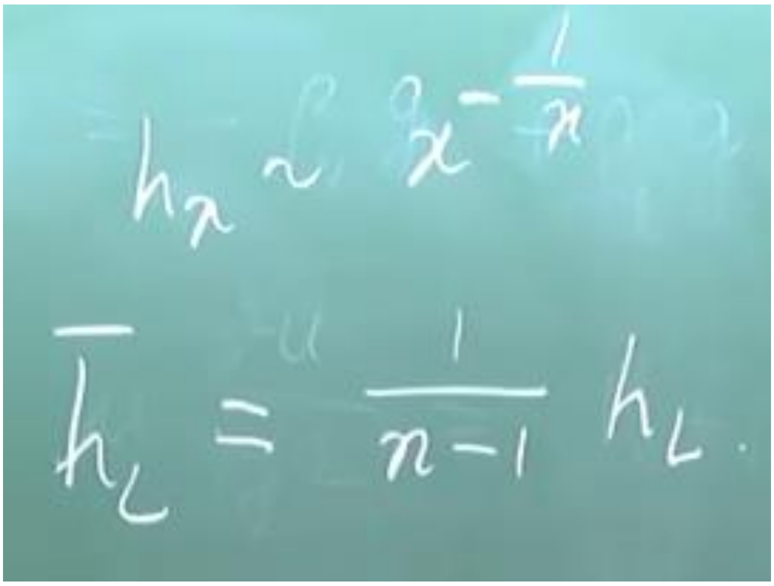

$$h_x \sim x^{-\frac{1}{n}}$$

$$\bar{h}_L = \frac{1}{n-1} h_L.$$

I told you if you if you remember in the class of convective heat transfer that if hx is proportional to x to the power minus upon n, then average heat transfer coefficient for a flat surface is always 1 by 1 minus n, minus 1 plus n, that is n minus 1 hL. Obviously instead of minus 1/4, you put as minus 1 by n, you get a relationship 1. Here, n is one fourth, one-fourth minus 1 is three-fourth, 4/3 where hx is proportional to x to the power minus half in case of flat plate heat transfer.

In case of flat plate, the heat transfer coefficient is proportional to x to the power the local heat transfer coefficient minus half and in that case, we have seen that average heat transfer coefficient over a length L of the flat plate is twice the local heat transfer. This is the thumb rule. So hx is proportional to minus one-fourth means it is four by three, the local heat transfer coefficient, 1/n minus 1, okay.

So, this thing sometimes works without, so therefore one can tell that hL average. I am not writing the expression. This four-third, this 4/3 into this with this x as L. So, one can write if it comes in the examination for example, this expression and immediately if you know this thumb rule, so immediately four by three times everything with x L, because this is four-third times the local heat transfer at x is equal to L, clear, Okay.

So, this is the entire derivation for condensation over a particle flat surface. Now, this thing can be little modified if you are told. This is the Nusselt classical deductions, which are there in all books. Now if this is little modified in case of that if the plate is inclined.

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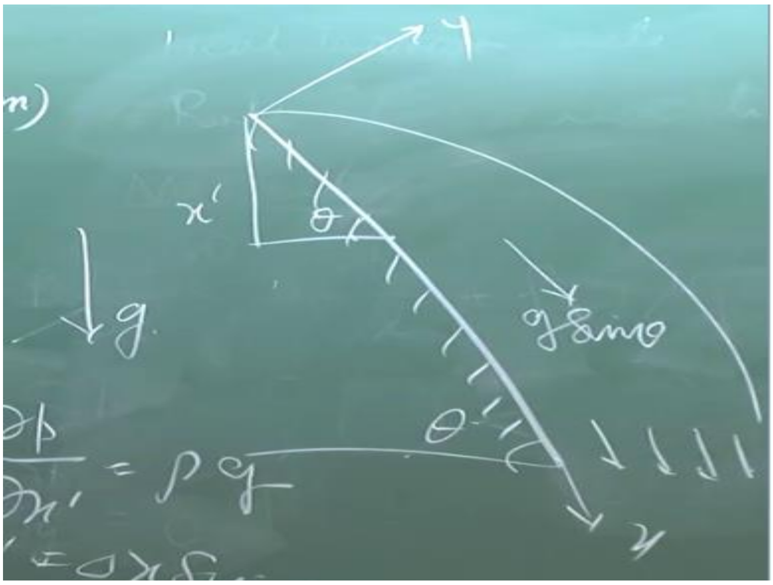

If the plate is inclined with an angle theta with the horizon, it is simply school level thing. So therefore, this is y and this is x, but g is in this direction. So entire thing will be changed by an effective g, g sine theta.

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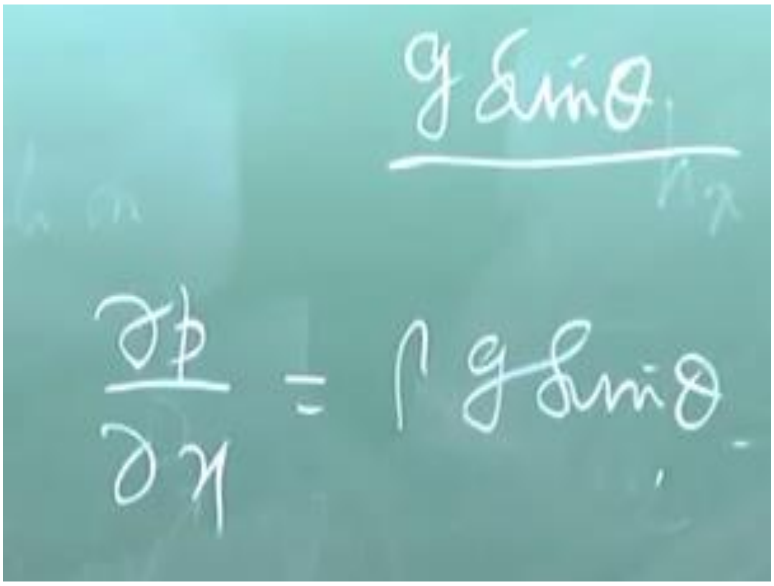

That means here the entire deduction, it will come as g sine theta. How? This is because obviously the effective g, which is working here is g sine. That means the weight, the component, the effective weight will be rho g sine theta along the direction of the flow and at the same time del p/del x. If this is x, will be rho g sine theta. So as a whole, a sine theta multiplication factor will come.

This is obvious because if this is x dash, then by hydrostatic del p/del x dash is rho g, but what x dash. If this is theta, x dash is delta x dash is delta x sine theta. This I am just telling you for your brushing up things, that is school level things. That means if there is an inclination of the plate and effective gravity g, sine theta will work, both in the hydrostatic equation pressure.

And also in the body force that means liquid, because hydrostatic pressure gradient is nothing but the weight of the vapour, so there also effective gravity comes, theta, so del p/del x dash is rho g, where x dash is this direction, vertical direction and this will be converted to rho g sine theta and the weight will be rho g sine theta. That means the effective gravity is g sine theta.

Another modification may be there. These are the things with which you should derive the Nusselt equation with these modifications, if you understand the Nusselt equation. For example, I tell the surface is made in such a way there is no slip at the surface. The fluid or the liquid slips, and I give a slip velocity, then you have to modify the distribution of velocity by taking care of the boundary conditions.

So anyway, any cosmetic change, you can do provided the basic structure of the deduction remains same. You first consider the velocity profile by neglecting the inertia term, solve the viscous terms with the buoyancy, that means pressure gradient and the body force, that is weight per unit volume and then you find out the explicit form of the velocity with the boundary conditions.

Be careful of the boundary condition, be careful of the physical situation where how the gravity is taking care of, if it is inclined plane, is it an effective gravity. So, with this, you can modify the Nusselt equation, the classical equation or classical derivation for a flat particle plate. Thank you.