**Conduction and Convection Heat Transfer**

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**Lecture 36**
**Free Convection – II (Natural Convection)**

In our previous lecture, we were discussing about some fundamental, physical and mathematical considerations for natural convection. Now today what we will do is we will perform some order of magnitude or scaling analysis to unveil some important physics associated with natural convection.

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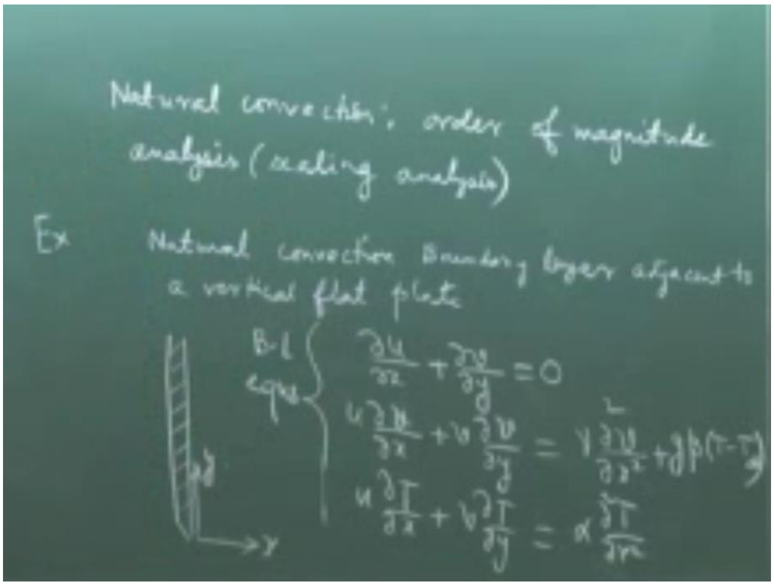


So, we will as a model problem consider the natural convection, order of magnitude analysis or scaling analysis with an example of natural convection boundary layer adjacent to a vertical flat plate. So, this is exactly the situation that we discussed in the previous lecture, but we will now try to get some deeper insight. In the previous lecture we had a broad understanding and now we will try to get a deeper insight.

So, this is the vertical flat plate and we set up the coordinate axis, so that this is x axis and this is y axis. So, what are the boundary layer equations for the continuity equation? Sorry, it is the y momentum equation that is important, so u del v del x. These are the boundary layer equations. And these equations are coupled because you have to know the temperature field to solve the velocity field. This sense of equations we discussed in our previous lecture.

Now we will take it up from there and try to make an assessment of the various parameters.

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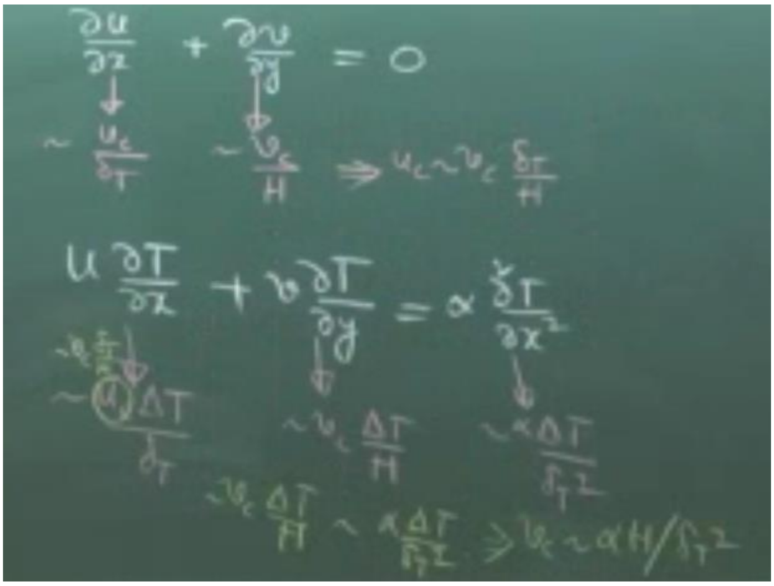

Now let us say that we make a scaling analysis of the continuity equation first. So, what is the order of magnitude of del u del x. Let us say you see is a characteristic velocity along x. Let us say that the height of the plate is H, so del u del x what is this? See you are analyzing the natural convection, that means where you are analyzing it? Within what boundary layer? When we say boundary layer equations, there is a little bit of sense of vagueness about it.

Because it does not tell whether it is hydrodynamic or thermal, I mean one obvious reason is they are coupled, but when you are writing these equations or analyzing these equations, what is your domain of interest, is it hydrodynamic boundary layer or thermal boundary layer. It is the thermal boundary layer; the reason is the temperature gradients are existing only within the thermal boundary layer and that temperature gradient is actually driving the flow.

It is T minus T infinity. So, if you look at the source down, this is T minus T infinity. So, T is different from T infinity only within the thermal boundary layer. So, the important body force in the momentum equation comes into picture only within the thermal boundary layer. That does not mean that the hydrodynamic boundary layer and thermal boundary layer thing is the same because the fluid also has its inertia.

So, when it attains its peak velocity not that just outside the thermal boundary layer it suddenly comes down to zero. So, it takes a bit of a distance to come down to zero, so that is because the fluid has its inertia. So even when the heating or cooling effect is stopped still there is some motion. Therefore, hydrodynamic and thermal boundary layers are not of the same thickness but what is important is that the hydrodynamics is not the continuity equation in the hydrodynamic boundary layer that is of importance for this boundary layer analysis

But it is the continuity equation analyzed within the thermal boundary layer. So uc divided by delta T, right? Not delta. This is, what is the order of magnitude of this? Let us say vc is the characteristic velocity along y, vc divided by H. So, as we have discussed earlier if these two terms are of the same order of magnitude then what it means is that uc is of the order of vc multiplied by delta T by H. So, if delta T is much less than H.

Then the velocity scale along x is much less than velocity scale along y, but it is still not zero. Then we will analyze the energy equation before entering into the momentum equation. What is the order of magnitude of this? Uc, this is some characteristic temperature difference by some Characteristic length. What is the characteristic temperature difference? T wall minus T infinity. That is the triggering temperature difference for heat transfer, right?

So, in short hand notation we write it as delta T. So, delta T is nothing but T wall minus T infinity. So uc multiplied by delta T by delta T, this is of the order of vc multiplied by delta T by H and this is of the order of alpha delta T by delta T square. And uc in place of uc we can write vc delta T by H. So, you can see that this two terms are of the same order of magnitude. You can see that this is philosophically such an important inference that from the momentum equation.

From the energy equation, from any physical scenario, we have seen that when we come to the boundary layer analysis, in the advection terms no term is dominating with respect to the other. Until and unless one is identically zero. Like for internal flow, the situation is different because for fully developed flow v is identically equal to zero. But if v is not identically equal to zero, then no matter how small the, u or v whatever, how small is it?

You will see that these two terms are of equal order of magnitude. This is something which is very intuitive. So, these two terms together are of the order of vc delta T by H, that is of the order of alpha delta T by delta T square. So, delta T square or you can write vc is of the order of alpha H by delta T square. See, why we have come up with an expression for vc is because this is what is our matter of interest. That is see, try to understand it practically.

You are investing by hitting a plate, hitting a vertical plate. So always just like any business you want a return of investment. So, in heat transfer business, in natural convection, your return of investment is what flow you are getting out of this temperature gradient that you are creating. So, this is a scale of velocity that you are getting by creating a temperature difference delta T.

And interestingly that temperature difference delta T does not appear anywhere in this expression. That is again something which is not very intuitive.

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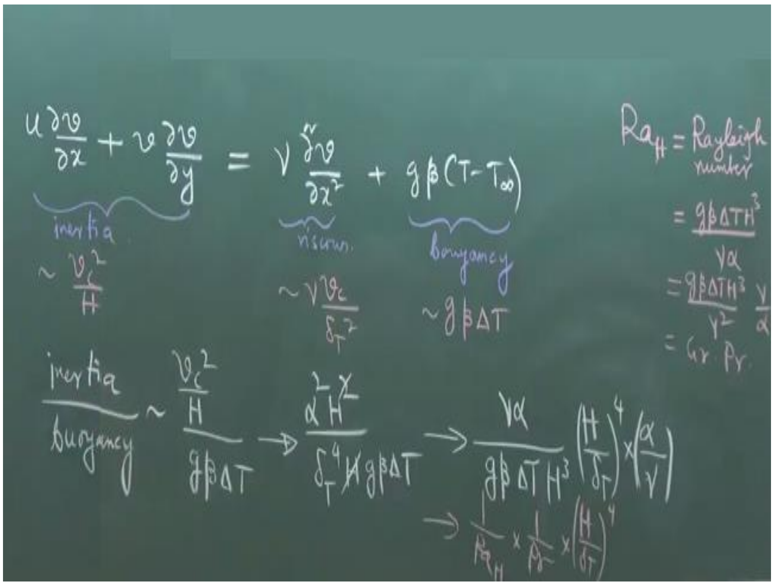


So, we will keep this expression in consideration, now we will come into the momentum equation. So just to give you a perspective, this term physically is what? This is inertia. This is viscous. And this is buoyancy. So, let us try to get order of magnitudes of these three terms. So, what is the order of magnitude of the inertia term? So, you can calculate any one of these because we have already seen that they will be eventually the same order of magnitude.

So, it is more straight forward because this does not involve delta T. So vc square by H. This, what is the order of magnitude of this? Nu. This is a very important thing. See, we are making our analysis within the thermal boundary layer. So, it is delta T and not delta, although this is momentum equation. When we will be analyzing this momentum equation, for the hydrodynamic boundary layer then this will be replaced by delta.

But we are using the momentum equation for analyzing the physical situation within the thermal boundary layer and that is why this is delta T and not delta. And this is of the order of g beta…Now can you tell out of these 3 forces there is at least one force which you expect to be important in natural convection, what is that? Buoyancy, because this force is the driving force for natural convection.

Now given that this force is an important parameter for natural convection, the question is out of the other two forces which one is dominating because out of the other two forces whatever is dominating that will compete with this, right? So, we will find out inertia by buoyancy and viscous by buoyancy and see which is more and which is less. So, inertia by buoyancy is of the order of, now in place of vc you can write alpha H by delta T square.

So, this is alpha square H square by delta T to the power 4 by H g beta delta T. So, one H gets cancelled. Because here you have delta T to the power 4, so to make it compatible and non dimensionalized, the numerator should have H to the power 4, then you will get H by delta T whole to the power 4. So, for that you have to multiply both numerator and denominator by H cube.

So, this becomes g beta delta T H cube by mu alpha multiplied by H by delta T to the power 4 multiplied by mu by alpha. Why we have done this is because again this is our by the time so familiar, non-dimensional number, the Prandtl number. This is non-dimensional. So, this is another non-dimensional number. A very similar non-dimensional number we discussed in the previous lecture.

That was g beta delta T H cube by mu square, Grashof number and this is g b delta T H cube by mu alpha. This is called as Rayleigh number. So, this you can write as g beta delta T H cube by nu square multiplied by nu by alpha. So, Rayleigh number is nothing but Grashof number multiplied by Prandtl number.

“Professor - student conversation starts” No. That is alright. I am just writing this one. g beta delta T H cube by nu alpha. Oh, these term you were talking about. So alpha by nu, sorry, not nu by alpha. But I am just talking about this Rayleigh number. This term. “Professor - student conversation ends" So Rayleigh number is, again you can't see, this is Grashof number multiplied by Prandtl number.

So anyway, this we can write g beta delta T H cube by nu alpha, so one by Rayleigh number multiplied by one by Prandtl number, this is one by Prandtl number multiplied by H by delta T to the power 4, right? So, this is inertia by buoyancy. Let us calculate viscous by buoyancy.
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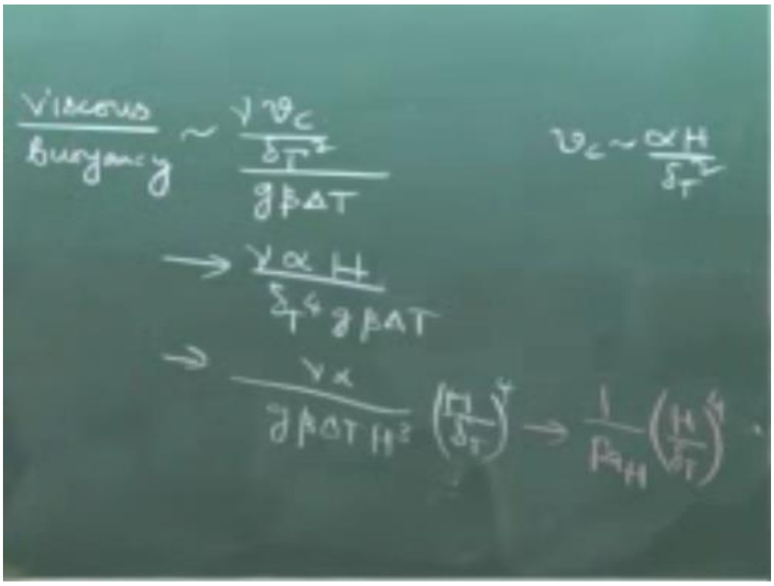

Viscous by buoyancy, viscous force nu vc by delta T square divided by g beta delta T. What was vc? Alpha H by delta T square, right? So, this is nu multiplied by alpha multiplied by H by delta T to the power 4 g beta delta T. So, this you can write it as g beta delta T, H cube by nu alpha multiplied by H by delta T to the power 4, right? So, this is of the order of one by Rayleigh number multiplied by H by delta T to the power.

Now can you tell that by this analysis how can we figure out that which force will compete with buoyancy in the thermal boundary layer, is it inertia force or the viscous force, depends on Prandtl number, right? Because you see the difference between inertia by buoyancy and viscous by buoyancy is the factor one by Prandtl number in the inertia by buoyancy. The remaining factors are the same.

That is one by Rayleigh number into H by delta T to the power 4, they are the same. Only difference is one by Prandtl number. So again, the situation boils down to whether Prandtl umber is much greater than one or Prandtl number much less than one.
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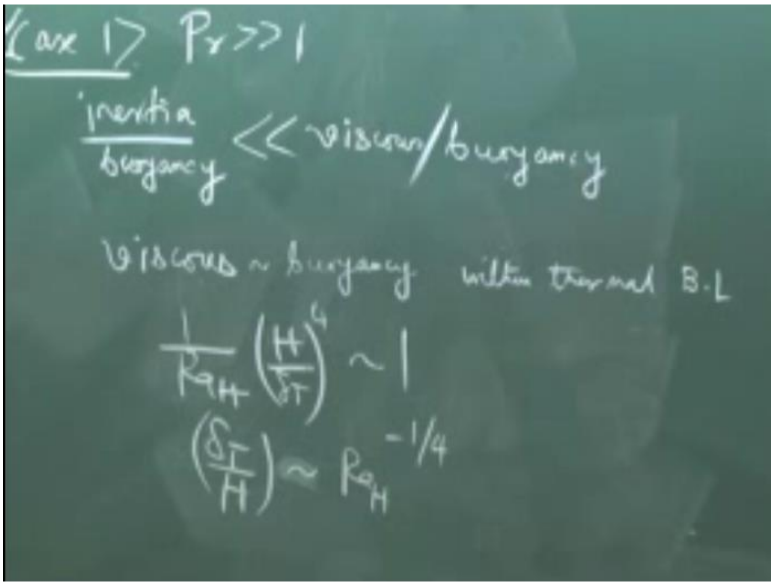


So, we will consider the first case when Prandtl number much greater than one. If Prandtl number is much greater than one then inertia by buoyancy is much less than viscous by buoyancy, right? Because other factors are the same. That means which forces are competing? Viscous and buoyancy within which boundary layer, thermal boundary layer.

So viscous is of the order of buoyancy means one by Rayleigh number multiplied by H by delta T to the power 4 is of the order of one, right? These two are of the same order. That means, delta T by H is of the order of Rayleigh number to the power minus one fourth, right? From this, delta T by H to the power 4 is of the order of Rayleigh number to the power minus one. So, delta T by H is of the order of Rayleigh number to the power minus one fourth.

So just like the assumption the boundary layer assumption that in the boundary layer theory for forced convection what was the assumption? For hydrodynamic boundary layer, delta much less than L whatever we consider the length of the plate. Delta much less than that. For thermal boundary layer, delta T much less than that. For thermal boundary layer delta T, much less than L.

So here also for natural convection, delta T much less than 8 will be our assumption, why? Because if you recall, the energy equation we have neglected del square T, del y square as compared to del x square. That assumption is based on the fact that delta T is much less than 8. So, delta T is much less than 8 only when the Rayleigh number is large. Just like for the hydrodynamic layer analysis for flow over a flat plate is valid for large Reynolds number, similarly here the situation is coming for large Rayleigh number.

For thermal boundary layer, in forced convection, it is Reynolds number multiplied by Prandtl number, that. So here- but it is basically related to Reynolds number to the power n multiplied by Prandtl number to the power n, where n and n maybe two different coefficients. So here also it may depend on Rayleigh number to the power n multiplied by Prandtl number to the power n. Here Prandtl number is not coming into the picture.

But for the other case where Prandtl number much less than one, Prandtl number will come into the picture because you can see the existence of Prandtl number in the denominator. We will come into that subsequently but before that we will try to find out what is the Nusselt number. Again, in any problem of convection, it is Nusselt number that is our matter of interest.

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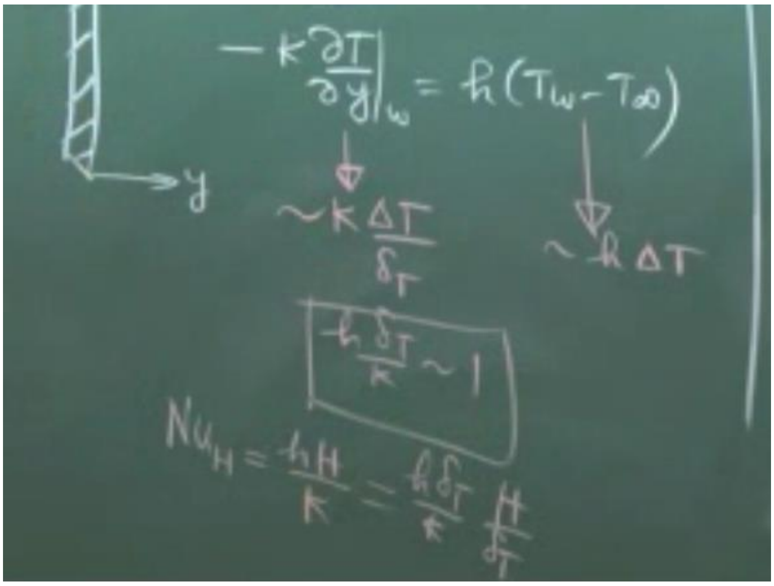

So, if this is the plate and this is the y axis, at the wall we can write minus k del T del y at wall is equal to H multiplied by T1 minus T infinity. Nusselt number is this H expressed in a non-dimensional form. So, what is the order of magnitude of this? Yes, what is the order of magnitude of this? K. The characteristic temperature gradient takes place within the thermal boundary layer.

And that is T wall minus T infinity by the thickness of the thermal boundary layer and this is h delta T. This is exactly equal to h delta T, you need not put as order because this is h and this is delta T. So, this two terms must balance each other, so that you can write h delta T by k is of the order of one. This inference does not depend on the value of Prandtl number. So now the Nusselt number which is hH by k based on the length h is equal to h delta T by k multiplied by H by delta T.

So, this is h delta T is of the order of one and H by delta T is of the order of Rayleigh number to the power one fourth. That means, Nusselt number is of the order of Rayleigh number to the power one fourth. If you do very rigorous mathematical analysis and what new thing you will get is, you will still get this with the constant c which you are not able to capture through this order of magnitude analysis.

So, you will get Nusselt number is equal to some constant multiplied by Rayleigh number to the power one fourth, that constant's value you can get by more detailed analysis. But entire physics you can capture through this except for the value of that constant.

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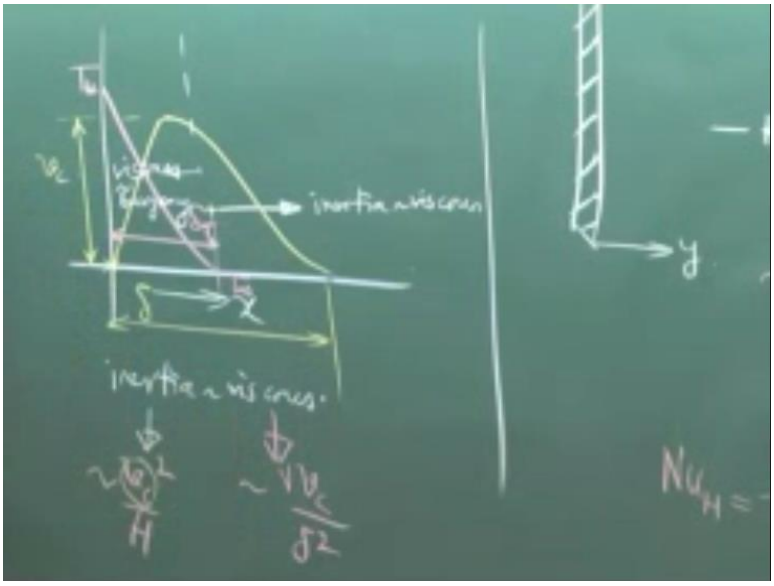


Now we have discussed about thermal boundary layer. But what happens for the hydrodynamic boundary layer. So, let us try to make a sketch of the phenomenon, the velocity and the temperature profile. So, this is x, so how will the velocity profile look? So, the velocity at the wall will be zero and then because of buoyancy effect the velocity will go up an again in the fast stream the velocity will come down to zero, seem critical.

So, this is our characteristic vc and these is your so-called delta, not delta T. So, from T wall, it will come to T infinity and this is what is your delta T. Okay? Now the question is, how do you estimate what is the thickness of, what is the order of magnitude of delta. So, for that you have to make your analysis within the hydrodynamic boundary layer but outside the thermal boundary layer.

So now if you make an analysis within this thermal boundary layer, then within this thermal boundary layer what forces are important? What forces are competing? So, on this side, viscous and buoyancy force is competing, right? What happens in this side? That is the question. Which forces are competing? So, there are 3 forces which could in general interplay with each other. Inertia viscous and buoyancy.

Outside this buoyancy has no role because the temperature has become already T infinity. So what forces have role to play? Viscous and inertia. So, in this… And this is the physical scenario. Because of inertia only, outside the thermal boundary layer still you get some velocity because the fluid has inertia, it does not suddenly come to a zero velocity when the temperature gradient is mid zero. So, it still tries to maintain its motion.

The viscous force tries to oppose its motion. So there comes a situation where the viscous force is completely successful in opposing the inertia and then the velocity comes to zero. Then you come to the age of the hydrodynamic boundary layer. So, inertia is of the order of viscous if you write. What is the order of magnitude of the inertia at all? vc square by H. What is the order of magnitude of the viscous term? Yes. Nu.

Not delta T square, delta square. Because this analysis is valid over a length scale over a region for which the length scale is delta and non-delta T. So, these are fundamental conceptual things. Concept is not algebra where some fourth power will come in the numerator or denominator all those things. It is important that, I mean, this is where if are not careful you will make mistake.

So, when you are doing the analysis where the length scale is, appropriate length scale is delta, it is that delta you should substitute. Now you tell me in place of vc should we substitute alpha H by delta T square or alpha H by delta square. Alpha H by delta T square because it is the temperature gradient within the thermal boundary layer that is responsible for this vc.

So, it is not the expression for delta, it is the expression for vc that we are substituting there.

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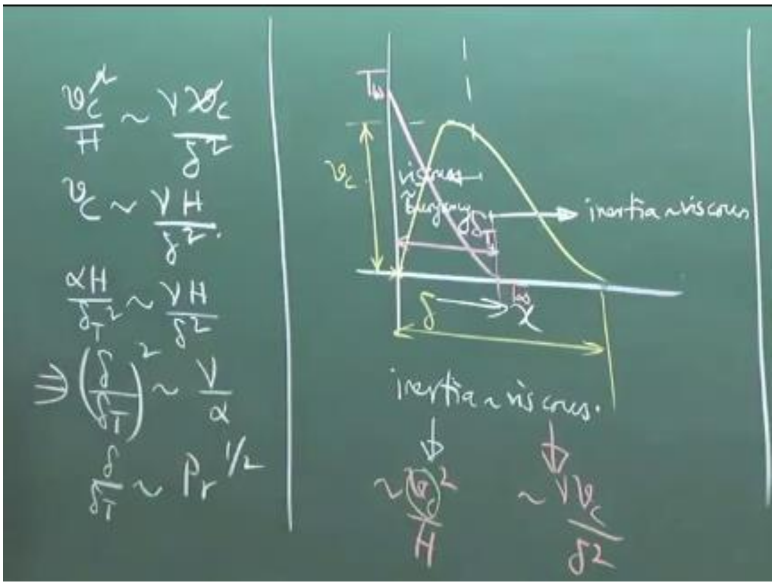


So, these two forces, now if they come of the same order of magnitude, vc square by H is of the order of new vc by delta square, so vc is of the order of new H by delta square and vc is alpha H by delta T square. So, delta by delta T square is of the order of nu by alpha. That means delta by delta T is of the order of Prandtl number to the power half, right? So, this picture diagrammatically is justified.

We have considered Prandtl number greater than one, so delta is greater than delta T and the meaning of the Prandtl number comes out to be in this example, for the case of Prandtl number much greater than one, very similar to forced convection, right? But we will see that it is dramatically different if we consider the other case Prandtl number much less than one. So, we will consider that case now.

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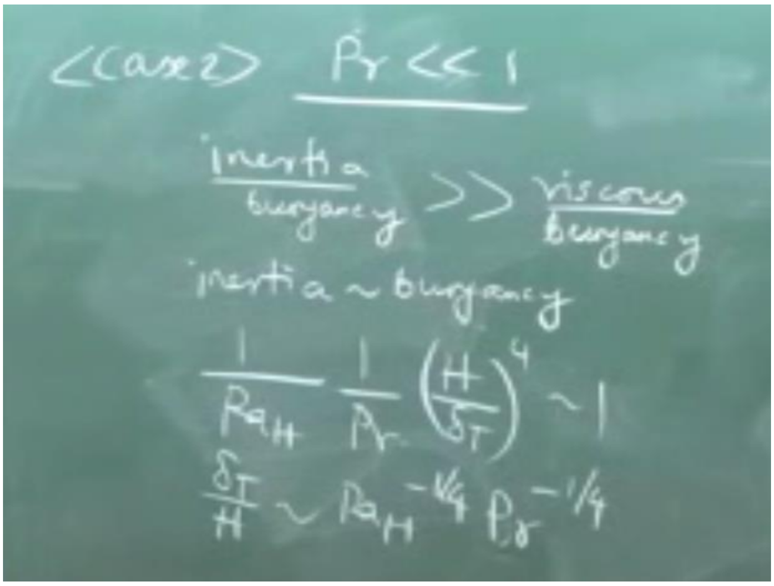


So, for Prandtl number much less than one, you have inertia by buoyancy is much greater than viscous by buoyancy, okay? So, inertia by buoyancy is much greater than viscous by buoyancy because the Prandtl number being less than one, one by Prandtl number will be much greater than one.

So that means inertia is of the order of buoyancy. So, inertia is of the order of buoyancy means you have one by Rayleigh number into one by Prandtl number into H by delta T to the power four is of the order of one. That we get from this expression. So, delta T by H is of the order of Rayleigh number to the power minus one fourth multiplied by Prandtl number to the power minus one fourth.

See the Prandtl number dependence comes into the picture, for Prandtl number much less than one but not for Prandtl number much greater than one and how do you calculate the Nusselt number?

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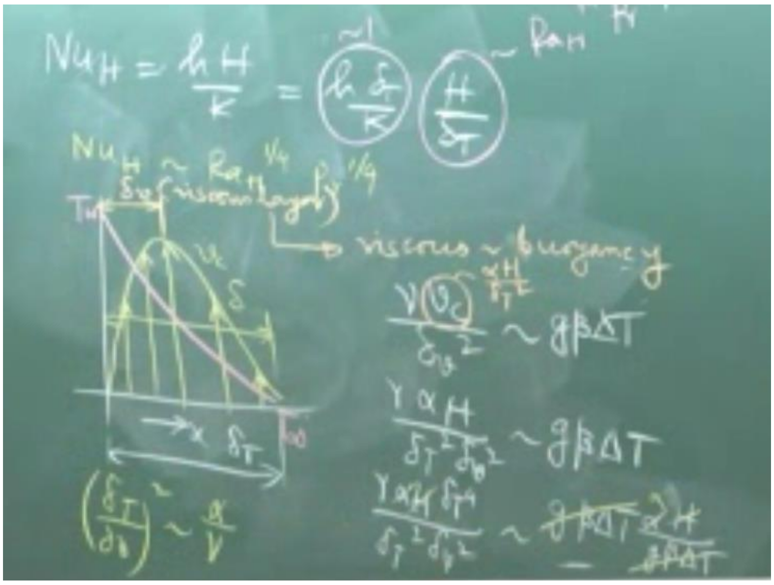

For Nusselt number is equal to hH by k which is equal to h delta T by k multiplied by H by delta T. This is of the order of one. We have shown that this of the order of one does not depend on whether its Prandtl number much greater than one or less than one and this is of the order of Rayleigh number to the power of one fourth to the Prandtl number to the power one fourth.

So Nusselt number is of the order of Rayleigh number to the power of one fourth multiplied by Prandtl number to the power one fourth. Now let us try to again qualitatively draw the velocity and the temperature profiles. So, velocity profile is just like the other case we will draw it. What I forgot to do in the last case is to put vectors for the velocity, so I am just putting the vectors for the velocity. So, this is the y component of velocity.

So, similar thing, we will do for this case. Now, I have drawn a sketch with an intention to confuse you a little bit. So, when you look into the sketch? there is a first thing that you apprehend to be incorrect, what is that? From your past experience of interpreting physically what is Prandtl number? So, your past experience of interpreting what is Prandtl number is the ratio of delta by, it is an indicator not exactly the ratio of delta by delta T.

But it is an indicator of the ratio of delta y delta T. So, if Prandtl number much greater than one, delta must be much greater than delta T. So, this sketch is fine. But if Prandtl number is much less than one, then delta should have been much less than delta T, but that I have not drawn, but this inference is based on the mental block that Prandtl number is always indicated of delta y delta T.

And we will now show that it is not necessarily true for all cases. So we will wait for a moment and our further analysis will resolve this paradox. So now can you tell that, if you now analyze the thermal boundary layer, can you recall what forces are important in the thermal boundary layer? Inertia is of the order of buoyancy, but there is always a layer within which viscous effect must be important.

Otherwise it will be like inviscid flow. It is not an inviscid flow. It is a viscous flow. So, although within the thermal boundary layer, the inertial and the buoyancy forces are competing with each other. Within that layer there should be at least a thin sub layer. It may be very thin, but it is not vanishingly zero. It will be some of some thickness where the viscous force will be important and if the viscous force is important, which layer is it?

It must be the wall adjacent layer. So, you can imagine a layer like this which we call as delta v which we all as viscous layer. This is not the delta. It is just a name that we are giving. This is defined as a layer where you have viscous force of importance, viscous forces of importance with which it will compete, inertia or buoyancy? Buoyancy because near the wall you get the maximum temperature reference.

So, near the wall it is the buoyancy that is driving the flow. So viscous will try to oppose the flow and buoyancy will try to drive the flow. So viscous is of the order of buoyancy. So, what is the viscous force in terms of order of magnitude, nu vc by what? What is the order of magnitude of this? Nu vc by, you see what?

Everything is written in the board here. It is your interpretation. Nu vc by delta v square, right? Because your layer under consideration is the delta v layer, is of the order of buoyancy. So, g beta delta T. Again, vc is of the order of, what is the vc, alpha H by delta T square. So, you can write nu alpha H by delta T square multiplied by delta v square is of the order of g beta delta T.  To get a ratio of delta T by delta v you have to basically multiply it with delta T to the power four.

Then you will get delta T square by delta v square. So where from you will get delta T to the power four you will get from this. Inertia and buoyancy are of the same order that means these and these are of the same order. So, from here you will get a scale of delta T to the power four. So, what is the scale of delta T to the power four? Is of the order of alpha square H by g beta delta T, right? Because this is of the order of one.

So, delta T to the power four is of the order of alpha square H by g beta delta T. So, you multiply both sides by delta T to the power four. So, nu alpha H by delta T to the power four by delta T square delta v square is of the order of g beta delta T multiplied by alpha square H by g beta delta T, right? So, g beta delta T gets cancelled. H gets cancelled. So, what we infer from here?

Delta T square by delta v square, or delta T by delta v whole square is of the order of one, alpha also get cancelled from both sides, another alpha remains. So, one alpha remains. So, this is of the order of alpha by nu. Instead of alpha square it becomes alpha which cancel off one alpha from this side. Okay? So, delta v by delta T is of the order of Prandtl number to the power half. Okay? So Prandtl number is not a measure of delta y delta T but delta v by delta T. Okay?

So, some examples, experience with some examples prompt us to think that Prandtl number is always an indicator of delta y delta T, but that need not always be correct. This is an example where we see that Prandtl number is not an indicator of delta y delta T but it is an indicator or some other delta v by delta T. So, the figure that we have drawn with some arbitrary delta and delta T does not conflict with this, right?

So, to summarize we have done some order of magnitude analysis for natural convection across a vertical flat plate with two limiting conditions, Prandtl number much less than one and Prandtl number much greater than one. Other approaches of analyzing natural convection are the two approaches that we have discussed in the forced convection that is the similarity solution technique and the integral approach.

These two are not there in your syllabus for this particular course because, not that it is conceptually very much different from what we have discussed but mathematically it is little bit more involved because now the momentum and the energy equations are coupled. But the basic philosophy is the same for example in the integral method, you have to integrate the respective governing equations.

So, I will give you a home work that for delta equal to delta T that is a special case, very special case, you find out the Rayleigh number, Nusselt number as a function of Rayleigh number and Prandtl number using integral method. So again, I am repeating the homework that for delta equal to delta T, find the relationship between Nusselt number, Rayleigh number and Prandtl number using the integral method. Okay?

So basically, when delta is equal to delta T, I mean you can integrate either from o to delta or 0 to delta T all the time. Now the question is what velocity profile will you assume? Now can you tell that what is the difference between this velocity profile and the forced convection velocity profile? So, at the wall the velocity satisfies no slip, but at the free stream the velocity is zero. There it was u infinity, here, it is zero.

Not only that, how do you calculate the second derivative of velocity at the wall. So, if you recall, let us say that you are interested to have a velocity profile v equal to A0 plus A1 y by delta plus A2 y by delta square plus A3 y by delta cube, like that. So, to evaluate these constants you require some boundary conditions. One is velocity here equal to zero, no slip. Another is velocity here equal to zero. Third is the velocity gradient here is zero.

The fourth one, what is the fourth one? Here there are four constants. So again, the principle is the same. How did we explain the fourth boundary condition for the first convection? We applied the momentum equation at the wall. So, at the wall, both u and v were zero, so from that we got an expression for the second derivative. So here also, if you apply the momentum equation at the wall, you will get the fourth condition.

So, using this four conditions, these four constants can be evaluated. These constants you can substitute in the integral equation and then you can find out the temperature gradient at the wall and hence the Nusselt number and you will see that again the Nusselt number will scale in the same way as with Rayleigh number and Prandtl number in the way in which, in a manner in which we have seen through the order of magnitude analysis.

But you will get some fitting coefficient with that. So, order of magnitude analysis does not give you the coefficient, the constant along with the variation with Rayleigh number. That constant we will get by the integral approach and even if you do similarity solution you will get the same thing. So, let us summarize, we are almost towards the end of our discussion on convective heat transfer. So, let us summarize what we have done so far.

So, we have discussed, we started our discussion with convection with some physical explanation or physical interpretation of what is convection and how does it relate with fluid mechanics and then some basic derivatives of fluid flow equations. So, in particular we have derived the Navier-Stokes equation, then we have discussed about some exact solution of the Navier-Stokes equation.

Then we have discussed the hydrodynamic boundary layer for flow over a flat plate. Then we derived the energy equation and applied that for analyzing the thermal boundary layer over the flat plate. Then we discussed forced discussion internal flow that is forced convection through plate, pipes, channels, etc and then we discussed natural convection. So, the remaining topics in convection.

So, we have discussed so far, this where no phase change is involved. But there are many practical engineering situations when phase change is important and two such examples are condensation and boiling. So, condensation and boiling will be discussed next and then the discussion on convective heat transfer will end with one practical engineering application where all this experiences of handling the various mathematical analysis of convection will be useful, is design and analysis of heat exchanges.

So that will more or less wind up this particular book course on conduction and convection heat transfer. So, for the remaining lectures on this particular course Prof. Som will take over. So, from the next lecture he will be starting with the new topic. Thank you.