**Conduction and Convection Heat Transfer**
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**Lecture-35**
**Free Convection – I (Natural Convection)**

So far, we had discussed about force convection, where the objective is to have a convection by driving fluid flow by a force mechanism like, for example by a pressure gradient or by some external forcing mechanism. On the other hand, there may be situations, when you do not have an external forcing mechanism, but you can still drive a fluid flow and one such example is natural convection or free convection.

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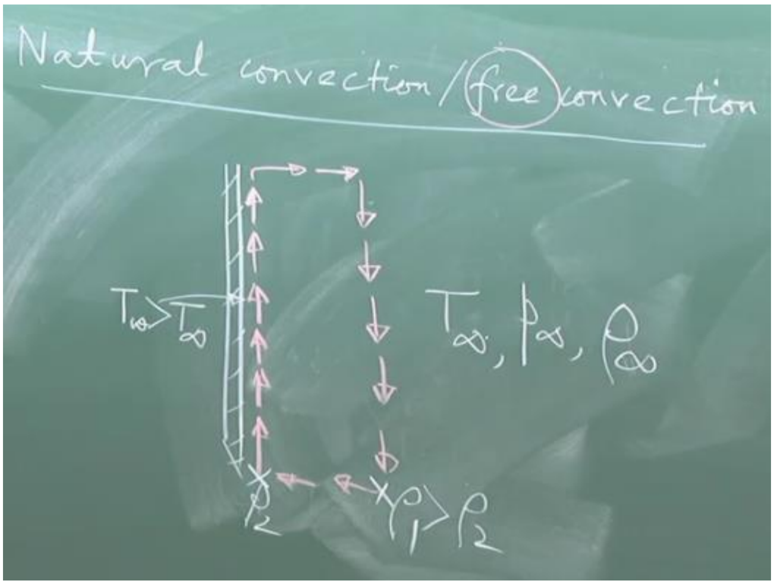


So, let me give you some physical insight on natural convection, before we entered into the mathematical analysis. So, let us say that you have a vertical plate like this, although we give an example of vertical plate or a vertical surface mostly to introduce natural convection, it does not mean than it has to be a vertical surface always to have the effect of natural convection.

But this is a classical demonstrative example by virtue of which we can show that how convection current can be generated without any external driving flow? So how that is possible? let us say that the ambient temperature is T infinity, ambient pressure is p infinity and the density is rho infinity. Let us say that the temperature of the wall is T wall, which is greater than T infinity.

The medium can be any fluid, let us say there is air, okay, which is the very common medium and why we are considering this? Because, see one of the terminology is associated with this, this is free convection. So, free is an English word, which resembles the situation when we are getting something free of cost, whether we actually getting free of cost or not, is something which is the matter of further consideration.

But at least air we can get free of cost, one vertical plate you can put free of cost, you are not able to heat a vertical plate free of cost. But I mean there can be see scenarios when there can be a boundary which is heated free of cost like, for example, in sunlight, if you expose a surface to solar energy or something like that, actually you do not pay any cost.

Let us say there is any heating within an electronic device. So, there is an electrical or electronic device, because of Joule effect, there is a heating. So, then you can get a heated surface, say heated printed circuit board, you can get; that you can get, that heating is free of cost. Because you did not intend to get a heating, that heating is an artifact of Joule effect. So, let us say that, it has the situation is like this.

Now if the air which is in contact with the solid boundary, air again, I am giving an example, it can be some other fluid, it gets hot, it gets lighter, right. If it gets hot, its density decreases and it tends to go up. Now when it reaches here, it still has its inertia, right. Because it was going up, now one possibility is that by maintaining its inertia, it can go to the left or it can go towards the right.

It cannot be penetrated the solid boundary to the left, so it has the only option of going toward the right, it cannot go further up, because heating is stopped here. But then, once this effect is gone, then gravity will play its role and it will come down, so once it comes down, it has to complete the loop to satisfy continuity. Because when fluid has gone from here, some other fluid should replenish back to maintain continuity.

So, in this way, a circulation is created. This is called as natural convection circulation. Okay. So, these kind of flow is, although we are saying natural, but it is actually again a pressure driven flow fundamental. Why it is a pressure driven flow? Here you have, let us say here you have density rho1, here you have density rho 2. Which density is more? Rho1, let us assume for the time being that these air is treated as an ideal gas, so p/rho is RT, right.

So that means, this density rho 1 greater than rho 2 means, p1 greater than p2, so that means, there is actually a driving pressure gradient that is driving the flow. But the catch of the story is that this driving pressure gradient is not an externally impose pressure gradient, but and intrinsically created pressure gradient, because of density gradient which in turn is created, because of the temperature gradient within the domain.

So, it is possible that because of temperature gradient, there is a density gradient and that drives the flow, but density gradient can be created because of other gradients also, for example, there can be solutal concentration gradient in a multi component system. So, let us say there is a salt water system and the salt water system is freezing. So, when the salt water system is freezing, then there is a concentration gradient being created all the time as the salt water system is freezing.

As the concentration gradient is created, there is a density gradient created. Because, depending on the salt concentration, the density of the mixture would change. So, it is possible that there is a density gradient created not just because of temperature gradient, but may be because of a solutal concentration gradient, but whatever may be the reason.

If there is a natural or intrinsic density gradient that is created and that can drive this kind of flow may be due to temperature gradient, may be due to concentration gradient that we call as natural convection. Okay. Now what are the important intricacy of these natural convection, we will understand, natural convection for mathematical analysis is much more involved topic than force convection.

So, in the under-graduate level, we will not get into all the details of mathematical analysis of natural convection, but at least we will try to identify or try to derive the basic governing equations and do some order of magnitude analysis. We figure out that what are the various parameters which affect the heat transfer? So, one of the important thing to consider is that when you have a force convection, we have seen that the Nusselt number is the function of Reynolds number and Prandtl number, that much you have learn by this time.

Now here the Reynolds number is no more important, because there is no forced mechanism of driving the flow. So, the Nusselt number, of course it is expected to be a function of Prandtl number, it may or may not be, we will see, but in addition to that, the Nusselt number is likely to be a function of some other dimensionless number instead of Reynolds number. What are those dimensionless numbers and how the Nusselt number is related to that?

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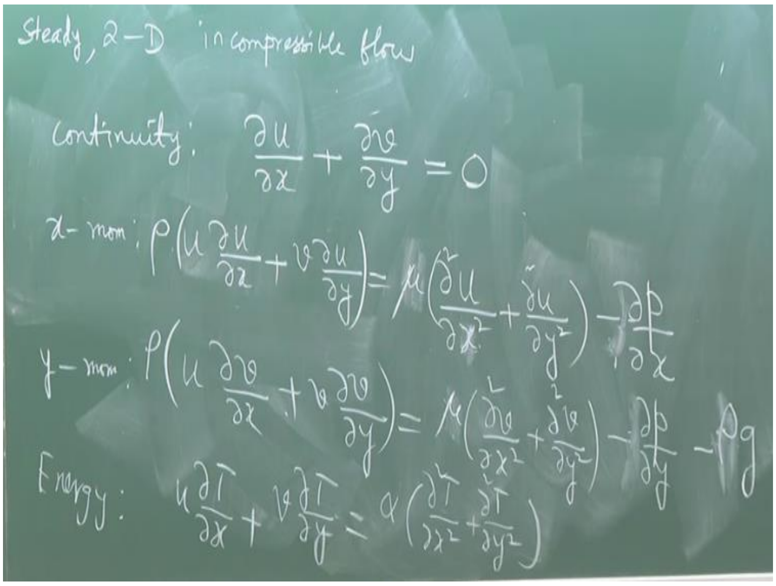


That will be one of the objectives of our further investigation. So, we will try to figure out, so let us figure out the governing equations; let us say that it is a 2-dimensional incompressible flow, so when I am telling that it is incompressible flow, you can pounce on me immediately and say that on one side, you are saying density is changing, on another side, how can you say that it is a incompressible flow? Right?

So, if you recall that during one of our preliminary discussions on recapitulation of fluid mechanics; I have talked about one example, which is called variable density incompressible flow. So, this is an example, where we are talking about that, so the incompressible flow does not necessarily mean that the density has to be constant, incompressible flow means that the volumetric strain rate is 0.

So, constant density flow is a special case of incompressible flow, so we are assuming steady 2-dimensional incompressible flow. Of course, you can have natural convection with unsteady 3 dimensional compressible; all these additional complexities taken into consideration, but to begin with; I don’t want to complicate the situation with those and again just captured the essential physics of interest of natural convection.

So, the continuity equation; now let us say our x axis and y axis, say this is a x axis, this is y axis and we will be considering a boundary layer type of flow just like we have consider the flow over horizontal flat plate. Instead of horizontal flat plate, it is the vertical flat plate. So, why we are made it vertical? Is to make sure that the body force is important assuming that gravity in these direction.

X momentum; let us write del p/ del x for pressure gradient along x. The X momentum will not be important. Let us also write the energy equation; let us write all the equations. There is a rho; because rho is an important parameter here, let us prefer to keep the rho in the left-hand side and multiply all the terms by rho, so that you know, because rho is an important variable here.

I want to preserve the sanctity of rho. So, this will become mu, and this is rho, this is mu, okay. Now first let us try to qualitatively understand the inter linkage of these equations. First of all, these rho is a function of what? These rho is a function of temperature. How will you get temperature? You will get the temperature from the energy equation. Therefore, in the momentum equation, you have a term which contains temperature.

This rho, may be say polynomial function of temperature. So, rho be the function of temperature, you cannot now decouple the momentum equation and energy equation, so far in force convection, what we have done? We have solved the fluid flow velocity profile, we are substituted that velocity profile in the energy equation to get the temperature distribution, that is what we have done.

Now here we cannot do that, because while solving the fluid flow through the continuity and the momentum equations, you have a term which depends on temperature. So, the energy equation and the momentum equation becomes explicitly coupled not that you can solve the momentum equation separately and substitute that in the energy equation to get the temperature; that you cannot do for it in this case.

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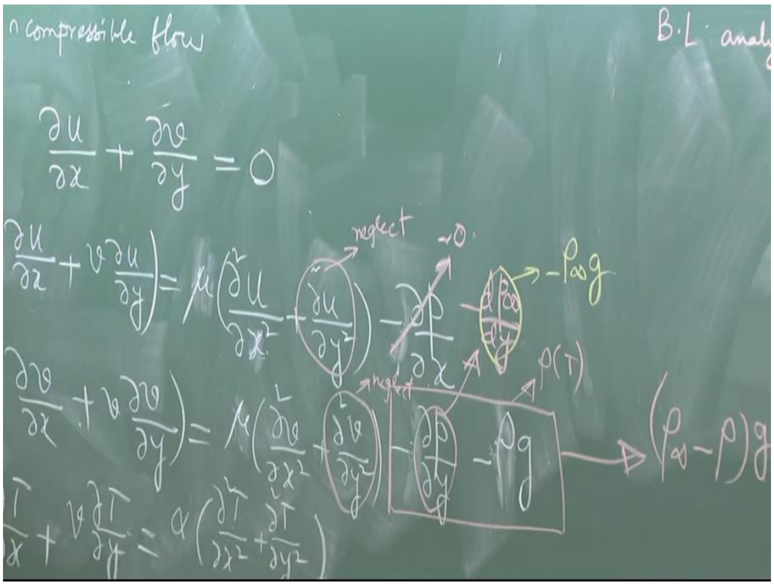

Now, question is that can we make certain simplifications, of course we cannot decouple these, but just for further analysis, can we make certain simplifications? So, one simplification is the boundary layer analysis. So, in the boundary layer analysis, first of all, out of these 2 terms which term will be important? What is the x reference length scale, that is the boundary layer thickness and what is the y reference length scale? That is the height of the plate.

Let us say, height of the plate is h. so assuming that delta R, delta T, that is either the hydrodynamic or thermal boundary layer thickness is much less than the height of the plate, which is very similar to the kind of assumption that we made for force convection boundary layer. We can neglect this. Similarly, for y momentum equation, also we can neglect this. What is the other important substitution?

Whatever is the pressure gradient; see out of these 2 momentum equations just from pure intuition, can you tell which momentum equation is going to be more important for you? Y momentum equation, right. Because that is where the body force is coming into the picture. So, in the y momentum equation, so in the momentum equation for the dominant flow direction, if you have a pressure gradient, that pressure gradient is, a pressure gradient that is imposed from outside the boundary layer.

Just like so in the for the force convection over a flat plate, where the direction of plate was x direction, then you had the – dp/dx=-dp infinity/dx. So here similarly, for the boundary layer analysis –del p/del y is –dp infinity/dy and this del p/ del x will be 0. Just draw the analogy with force convection over horizontal flat plate and natural convection over a vertical plate. I am only a little bit confusing to you**.**

Because of the fact that in a force convection case, we consider the x axis to be along the plate, now y axis is along the plate, that is the only difference. But if you are more comfortable with putting x axis along the plate, you can do that. I mean it should not matter, because which term is important or not, that should be decided from the physics and not from whether it is x or y. So that should be kept in mind.

So, you can see, now what is dp infinity/dy? What is infinity here? So, there is a heated flat plate, there is a convection current close to the plate but far away from plate what is there? Far away from plate, there is no motion of the fluid, right. So, there is a big difference between natural convection and force convection. Force convection at the age of the boundary layer, u is u infinity, right.

Force convection over a flat plate, but in natural convection at the age of the boundary layer, what is u? u is 0, u means here v; here instead of u, you have to consider v. So, the velocity outside the boundary layer is 0, because there is no mechanism that is forcefully driving the flow. On the other hand, for force convection, you have u=u infinity at y=at at delta, basically y or x depending on the coding.

So then if there is no fluid motion in the fast stream, then what is d p infinity/dy? This is governed by what? This is governed by fluid statics, right, because their fluid is under rest, so if fluid is under rest, what is dp infinity/dy? –rho infinity*g, right. This is the fluid statics, possibly the first chapter in fluid mechanics that you have studied. So, if you consider these 2 terms together, so what is the net term?

These – with these -, will become +, so rho infinity-rho g. Question is that when we are having a rho which is a function of temperature, then in the left-hand side also there is a rho which should be a function of temperature, but out of these 2 effects, which effect is primary? Which effect is secondary? right. The question is that if rho is a function of temperature for governing the physics of the problem.

Rho is the function of temperature in these term or rho is the function of temperature in these term, which is the primary effect and which is the secondary effect? Which is actually driving the flow? See, this body force that is driving the flow, so this rho being the function of temperature, is the primary effect, these rho being the function of temperature is also a physical effect but that is the secondary effect to the problem.

The primary effect in these rhos being the function of temperature. So, to simplify the problem, one important assumption was done, made by Boussinesq, what he made is an, he made an assumption that he considers the density variation with temperature to be important only for the body force term and not for the other term. So here, he considers rho= rho infinity, this is an assumption.

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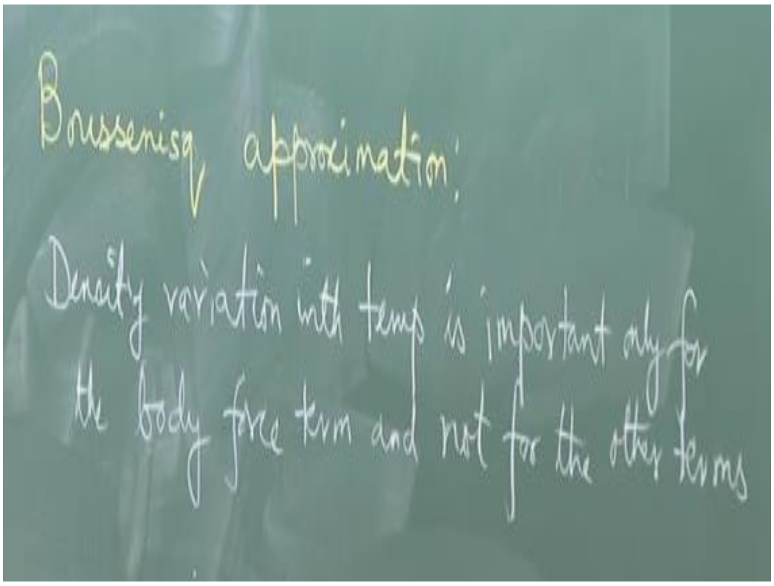


This is not the reality, here also rho will change, but you know these are certain levels of simplifications to make your analysis more and more mathematically attractable. So, let me write that, Boussinesq approximation; density variation with temperature is important only for the body force term and not for the other terms. So that means, for the other term you can write, rho as rho infinity only in these term, you do not write rho as rho infinity.

So, this is a very special term. So now how do you bring? So, you can see from the equation, you can make out that it is these term that is driving the flow. It is the difference between rho infinity and rho, right. So how do you bring the temperature dependence into account; everything is not an ideal gas so you can, so that you cannot write it simply by writing the equation of state of an ideal gas.

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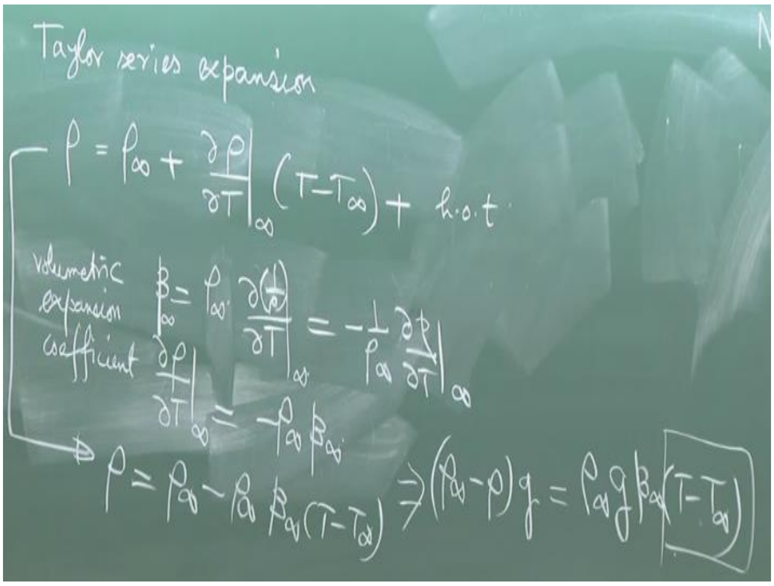


So, you have to make some mathematical analysis to tract these rho as a function of temperature. So that is often done by expanding rho, about rho infinity using a Taylor series expansion as a function of temperature. So, let us do that. So, if the temperature difference is larger and larger, the higher rod at tone terms will be more and more important. But if the temperature difference is not that large, only up to the linear term may be good enough.

Many practical problems are like that, where up to the linear term is what is good enough. Now we can write these in terms of the volumetric expansion coefficient beta. So, beta at the infinity or the fast stream condition is 1/v change in volume, per unit volume for each degree change in temperature. So typically, when you consider the partial derivatives, some other variables you keep as constant.

But, I mean depending on that you could have beta p, beta S, depending on various thermodynamic processes but we are generally calling it as the volumetric expansion coefficient without getting into that details. Now instead of specific volume, we will write 1/ density, okay. So, d of 1/rho will become -1/ rho square, so that, -1/rho will come here. So instead of del rho/del T at infinity, we can write –rho infinity beta infinity.

So, in these equation, you have rho=rho infinity- rho infinity beta infinity* T- T infinity. So, rho infinity – rho *g becomes rho infinity g beta infinity* T- T infinity. This is the body force term you can see there, so rho infinity-rho *g rho infinity- rho, you can take, these in the other side, - will become +, so rho infinity g beta infinity *T – T infinity. So, this is a sort of linearization of the problem.

So, you have a driving force, this driving force will decide that what is the strength of; related strength of various convection mechanisms. Now, if these driving force is very large, then even in a problem, where force convection is there, natural convection may be the dominant mechanism, so I will show you later on that how do you decide that given a physical problem, what is the driving mechanism?

Because a practical problem or an industrial problem. Nobody will give you a tag that this is force convection this is natural convection, like that. May be there are some effects of force convection, there are effects of natural convection, so together force convection and natural convection, if they are there, that is called as mixed convection. So, in a mixed convection, now which effect is important and which is not that you have to figure out.

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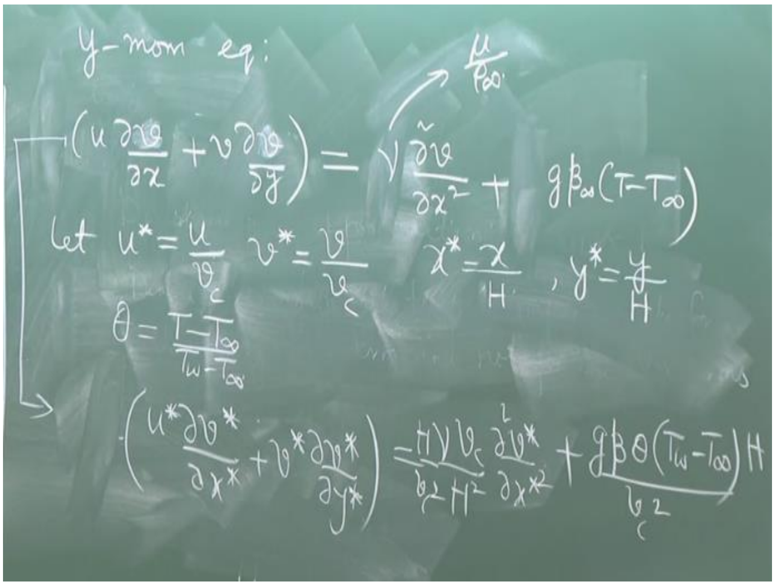


So, to do that we will first not considered the analysis of these equations but we will just nondimensionalised the equations. So, let us write the y momentum equation again. This is the equation. So, our first objective before making any analysis is to ask ourselves a question that, which mechanism of convection is important? Is it force convection that is important, is it natural convection that is important, given a practical scenario.

How will you understand that? So, we will non-dimensionalised these equations, let u star =u/ H, these v star, sorry u/ the characteristic velocity, v star = v/ the characteristic velocity, this is vc is the characteristic velocity along y. The velocity at which, a reference velocity at which the fluid is flowing, some characteristic velocity. Yes, u del v/del x. So, before that, let us divide both sides by rho infinity, so that it becomes nu at infinity.

But fundamentally it is not nu at infinity, but mu/rho at infinity, this mu can be function of temperature. But let us not complicate the problem in that way by putting everything as a function of temperature, I mean of course mu is a function of temperature but it may not be a very strong function of temperature over the range of temperature considered for the problem, but if the range of temperature considered for the problem is very large, then nu as a function of temperature must be taken into account.

Then you cannot take mu as a constant. So, all these, these are practical things like depending on the practical situation as an analyst you have to take the decision. Nobody will tell you, its not an examination problem, that somebody will tell you, assume mu= constant. Practical engineering does not talk in that way. So, given the practical situation, as an analyst you have to decide that it is judicious to take mu as a constant.
If it not judicious, then you have to take mu as a variable, there is no way out. So, let us call x star = x/H, y star=y/H and theta is T- T infinity by T wall – T infinity, non-dimensional temperature. So, what will be these term? u will be vc, vc square/H, right? This term by this term u *-. Sometimes in notation, beta infinity is written as beta, this infinity is subscript is- If you look into books, you will find that mostly, people do not use the subscript infinity, so I am omitting it here.

So now you can multiply both sides by H/vc square, right. So, H/vc square. One important thing, it is a very important conceptual thing. I have not here committed that what is the source of this vc? It may be force convection also. See is this equation valid for force convection. Yes, why not? Have you omitted any term which could be important? potentially important for force convection.

We have not omitted any term; we have added a body force term. But other than that, we have not omitted any term. So, this equation will be valid for force convection as well as natural convection. So, in general, a combination of these 2 effects, which is called as mixed convection, where both force convection and natural convection may be important. So, this vc, may be due to force convection.
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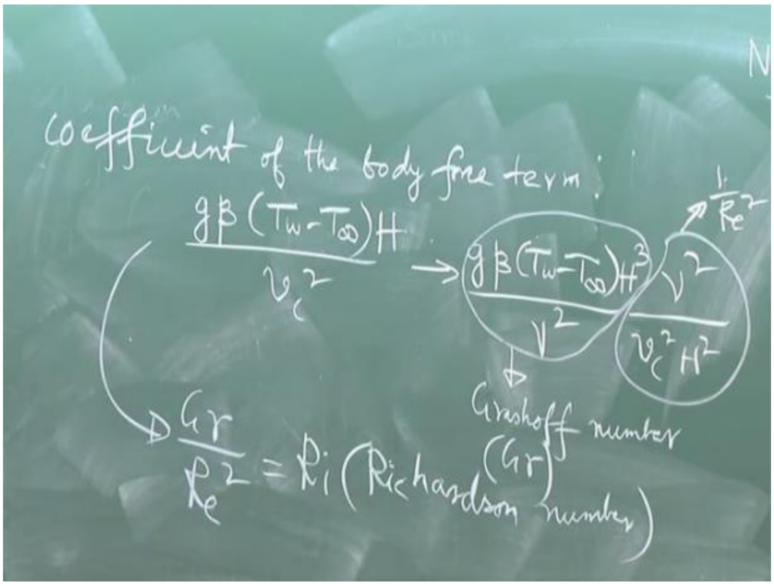


So, if, force convection effects are present, so coefficient of the body force term, let us write; coefficient of the body force term, so theta is the dimensional, sorry; dimensionless parameters, so it varies between 0 to 1. So, its coefficient is, its coefficient is the factor that decides that how strong is that term? So, g beta T wall –T infinity/; there is a H; there was a H, so we multiplied by what? There is a H, right. So, g beta T wall – T infinity/.

So, this you can write; right. So, what we have done is? Why we have made this manipulation? Because velocity characteristic velocity into the lens scale, divide by nu is a Reynolds number. So, if force convection effects are there in a flow, in a problem, then Reynolds number may be an important non-dimensional parameter. So, this is 1/ Re square. Now see by non-dimensionalising all the terms, we are made all the terms non-dimensional.

So, this being non-dimensional its coefficient, these also must be non-dimensional, right. So, this is a non-dimensional term, you can check by putting the dimensions, you will see that, it will come out to be non-dimensional, but it is very clear, so this is a non-dimensional term, so these also must be a non-dimensional term. So, this is called as Grashof number. So, this becomes Grashof number by Reynolds number square.

Again, in subjects like fluid mechanics and heat transfer so many scientist, mathematicians, they have contributed that whenever there is a non-dimensional number appearing, it is a custom to honour some of those by giving it a name as one of the; as the names of one of those scientists. So that is how, like all these scientists are honoured by giving a name to some of the non-dimensional numbers.

Again, this is called as Richardson number. So, this also to honour the contribution. Now what? What is the name; this is important but it is more important that now from here can you tell that given a physical problem how do you analyse, how do you access whether force convection effect is important? Whether natural convection is important? Or whether you have to modulate as mixed convection, where both forced and natural convection are important.

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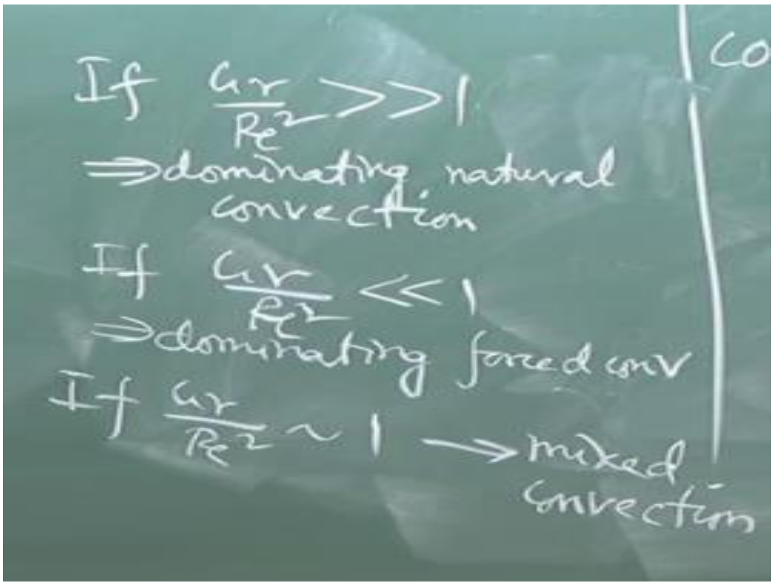


So that clue is given by these term. So, if Grashof number/Reynolds number square is much greater than 1, then what does it signify? It signifies that natural convection is much more important than force convection, dominating natural convection. If Grashof number/ Reynolds number square is much less than 1. Then that means what? Dominating force convection.

A very interesting situation if Grashof number/ Reynolds number square is of the order of 1. Then you cannot discard either of these modes and you have to consider both natural as well as force convection, that is called as mixed convection. So, from here we have seen that Grashof number is a very important non-dimensional number in natural convection, which gives as a clear indication of whether natural convection is important or not.

So, before analysing any problem, we must be convinced that yes natural convection is important, otherwise we should not take the border of, taking an additional body force modelling that and so on. We can simply neglect that. If you we find that the natural convection effect is not important, but we find the natural convection is important, then we have to model it properly.

In the next lecture, we will see that how to make an order of magnitude analysis for modelling natural convection problems. Thank you.