Conduction and Convection Heat Transfer

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Lecture 34
Internal Forced Convection – IV

So far, we have discussed the cases in which the viscous dissipation is not important. We have discussed forced convection over surfaces and we have discussed forced convection within a duct or a pipe but in all those cases we have not considered the viscous dissipation to be important. Now in Engineering, there are many practical situations when the viscous dissipation is important.

So, we will now try to analyze one or two example scenarios where the viscous dissipation becomes important.

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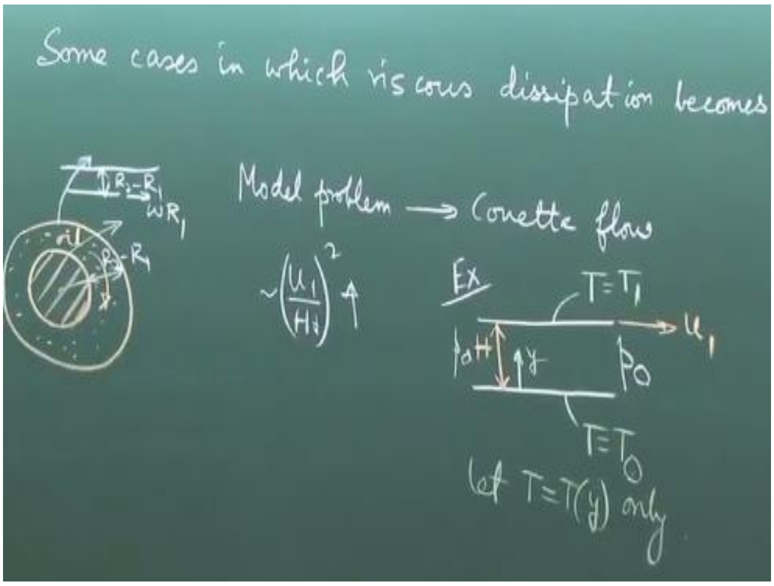

So, our agenda now is, some cases in which viscous dissipation becomes important. To analyze this scenario, we will consider a model problem. For every physical scenario, we consider certain representative problems like for internal flow we consider flow through a parallel plate channel. So here we will consider a model problem which is called as Couette flow. So, this kind of fluid flow is often discussed in fluid mechanics in this particular fashion.

Let us say you have two parallel plates. This is a very simplistic representation of the scenario. You have two parallel plates separated by a narrow gap of H. This gap is narrow and there is a relative translational motion between the two plates. That means, let us say the top plate is moving towards the right with a velocity u1 and the bottom plate is having zero velocity. Okay?

So that means there is a relative velocity, so it does not matter whether this is moving towards the right, left or whether the bottom plate is moving towards right or left. Important message is that there is a relative motion between the two plates. So, two important things. One is a narrow gap between the two plates. The other is there is a relative motion between the two plates.

Now, many times when things are introduced through lectures often we do not ask ourselves a question that why this situation? Why we are studying this problem? When Couette flow is first introduced in say fluid mechanics, have you ever thought that have you ever seen somebody cooling up late in a fluid. I have never seen. I do not know whether you have. I mean, there are situations which are similar to this.

But exactly this situation where you have two parallel plates separated by a narrow gap, someone is cooling one plate relative to the other. I mean, this kind of scenario is not very commonly seen, but then whey are we studying this. So, it is not that this kind of scenario is absolutely not there in practice. I mean this kind of scenario is there but I want to mean is that it is not a very common industrial scenario like flow through a pipe is a very common industrial scenario.

Like if you go to any process industry, power industry you will see that there are lots of pipelines and fluid is flowing through a pipe. But in those industries, you will not see that somebody is pulling a plate relative to the other within a fluid. So, what is the motivation. So, I would like to give you a motivation behind this. So, I will give you one fundamental scientific motivation and one practical engineering motivation.

And we will merge those motivations together to see that why such a flow is very important in the context of fluid mechanics and heat transfer. Now as mechanical engineers you must be familiar with bearings. Or at least you have heard of the term bearing. So, if you have a shaft, at least you know what is a shaft, right? So, if you have a shaft which is rotating and transmitting power, now to support that shaft you require a bearing.

So, let us say this is a shaft which is rotating, say clockwise, anticlockwise whatever, with a particular angular velocity. Now, this may be the rotor but this may be a stator just an outer casing to support the shaft. Now we have actually magnified this figure. This gap is very small. But at the same time this gap is finite. Why this gap is finite because during the rotation of the shaft.

I mean, it will acquire certain eccentricity at certain times and there is always a tendency that the shaft may be in contact with the bearing, then there will be metal to metal contact. So that should be awarded. So in between to avoid that high friction through metal to metal contact a lubricating oil is kept. See, this is practical engineering. You must understand this. Always, the issue would be that, at the end in heat transfer.

We will boil down the situation to mathematical equations, solution of the mathematical equations and all, but we should case a model practical problem towards that direction not that we are just as mathematicians interested to solve those problems arbitrarily. So, you have oil here, separated by this gap. Now this gap is very small. When this gap is very small, then what happens? Then effect of curvature of these two is not important.

Effect of this curvature will be important when this gap is relatively large, but when the gap is relatively small then the effect of curvature is not important. When the effect of curvature is not important we can model this by almost like a flat surface, this by another flat surface separated by, so if this distance is r2 minus r1 where r2 is the radius of the outer and r1 is the radius of the inner.

Then this is r2 minus r1 and the inner one is having a velocity which is what omega multiplied by r1, where omega is the angular velocity. So, this is having a translational velocity omega multiplied by r1. The upper one is having no velocity and there is a narrow gap. So, you can see that this problem which appears to be hypothetical is actually a very practical engineering problem.

One has to have the mindset towards understanding the problem in this way. Now this s of course the practical way of looking in to this problem, but this problem also has a very fundamental scientific insight because of which this problem needs to be studies carefully. So far you have seen that, can you tell whether this an internal flow or an external flow? This is an internal flow, because eventually these two plates form a channel. How is the flow driven?

When you have two plates in a channel or two plates forming a channel and there is a fluid which is flowing between the two plates, then how is the flow actuated? In the previous examples we have seen that the flow is actuated by a driving pressure gradient, that is called as a pressure gradient driven flow or pressure driven flow. Here there is not pressure gradient, of course you can create a pressure gradient.

But we will consider a simple Couette flow when this pressure is P0 and this pressure is also P0. That means there is no pressure gradient. But still there is a fluid flow in between the two plates, how? This is driven by shear. So, this is a classical example of a shear driven flow. So just like we have a classical example of a pressure driven flow, this is a classical model problem of a shear driven flow.

And in many physical scenarios shear is a very important dominating mechanism. So, if shear creates a flow, what is the signature of that flow that can be studied by this very simple problem. So, one fundamental scientific motivation is studying shear driven flow. So, in any practical scenarios starting from engineering to biological applications, wherever there is a shear driven flow you can consider this model problem and start with that.

On the other hand, as a mechanical engineer, the problem of lubrication in bearings is very important and for such problems you can first start with the fluid mechanics and then of course heat transfer because, see, in one way this narrow gap, how this narrow gap manifest in analysis of the problem? Of course, here we have discussed that if the gap is narrow then this curvature effect may be neglected.

But that is not all because even if the curvature effect is not negligible you can analyze this problem taking the curvature into account, that is not a very big thing. But the narrow gap means, see, what is the velocity gradient? What is the order of magnitude of the velocity gradient? Order of magnitude of the velocity gradient is u1 by H and if you recall the viscous dissipation terms the viscous dissipation terms scale will square of the velocity gradients.

So, your viscous dissipation term will scale with this. So, if this is small then this will be large and then viscous dissipation will have a big role to play. So, what will viscous dissipation do? It will convert the viscous effects or the shear between the work done due to shear between various fluid layers irreversibly into intermolecular form or energy or internal energy that will rise the temperature.

We have shown during our derivations that it will trivially rise the temperature. It is trial heating and not cooling. So just like if you rub your palms you do not expect that your palms will get cooler, right? You expect that these will become hotter. So, the same mechanism works here. And when it becomes hotter than that means that will act like a heat source in this oil. Okay.

And when there is a heat source in the oil, the oil temperature may rise and that might have several practical consequences in the performance of the bearing. So, it is important to understand that what is the temperature distribution within the system. So that is our motivation. So, as we are seeing that in convection when we are interested about the temperature distribution.

We first start with the fluid mechanics analysis because we need to get the velocity field and we will use the velocity field to get the temperature field. So, we will consider this model problem and we will set up the coordinate axis, y axis like this. We will assume a fully developed flow so that x dependence is not there. So, the momentum equation, so it is just like flow in a parallel plate channel. Only thing is that the flow actuation mechanism is different.

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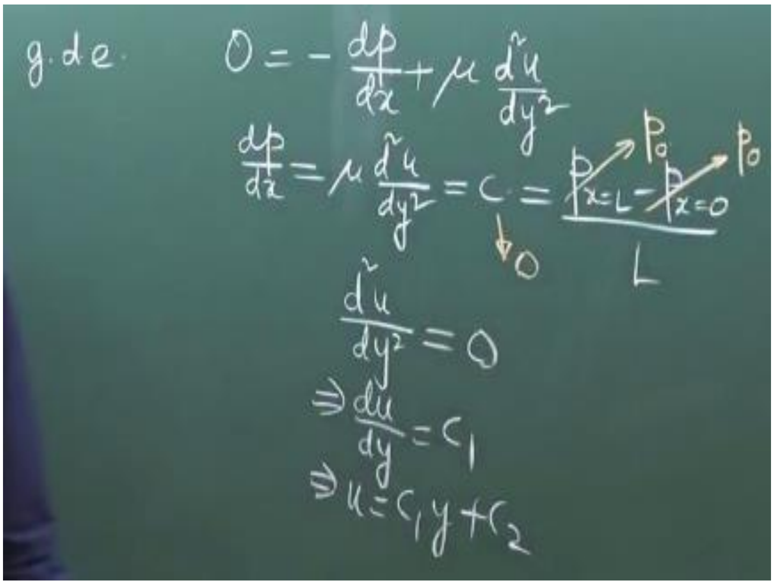

So, what is the governing differential equation? So, if you recall, for fully developed flow, del u del x is equal to zero and if there is no penetration at the wall that is there are no holes at the wall, you have v equal to zero also. So, left hand side, del u del x plus v del u del y that term will be zero and because of steady flow the unsteady term will also be zero. So, total acceleration is zero.

So fully developed flow is a flow where all the forces are balanced, so that there is no net acceleration in the flow. So therefore, you will have zero is equal to…So I will write dp dx instead of del p del x, okay? This is the equation that we had when we were having pressure driven flow through parallel plate channels, that equation is applicable here. So why I have written, I have actually jumped one step.

I mean here in the first step you write partial derivate and you write partial derivate because this is a function of y only and because this is a function of x only, that means each equal to a constant. So, I have jumped all those steps because we have already done that in the previous examples. So, you have dp dx is equal to mu d two u d y two, is equal to a constant, right? Now, what is dp dx? Because dp dx is a constant that means pressure versus x is linear.

So, what is this? This is p at x is equal to L minus p at x equal to zero divided by L, that is dp dx because dp dx is a constant, p versus x is linear, so dp dx is delta p by delta x, right? So, what is this? This is P0 and this is also P0. This is called as simple Couette flow. But it is not necessary that you have a Couette flow where the pressures at the inlet and the outlet are always the same. If not, it is a combined Poiseuille and Couette flow.

So, then you can solve this problem very easily by noting that this is a linear differential equation, right? So, you can consider it as a super position of two problems. In problem one you have only pressure gradient but no shear driven flow. In problem two, no pressure gradient and only shear driven flow and the combination of these two is the solution of the problem.

But just to get the implication of pure shear driven flow we are considering that there is no pressure gradient. So that means what is the value of c, c is equal to zero. So, u is equal to c1 y plus c2.

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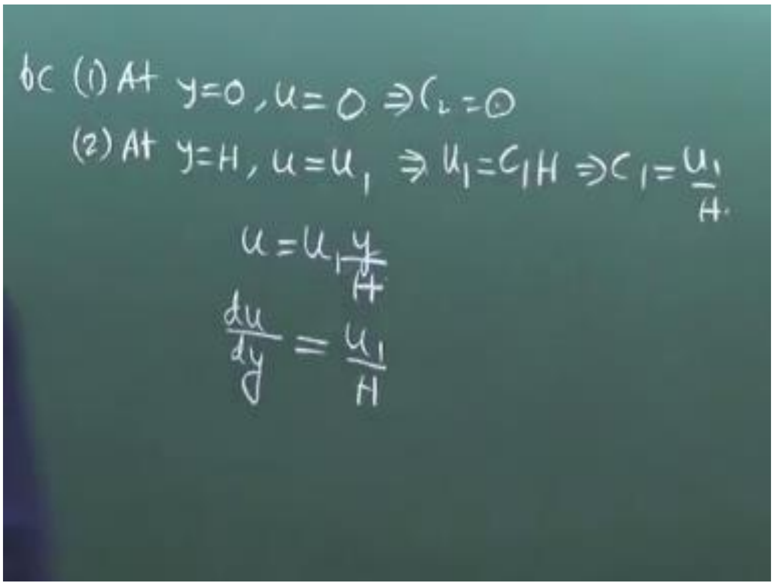

So, you apply the boundary condition at y is equal to zero, u equal to zero. That means c2 is equal to zero and boundary condition two at y equal to H and u equal to u1. What boundary condition is this?

This is no slip boundary condition, right? So, no slip does not mean zero velocity of the fluid. No slip means zero relative tangential component of velocity between the fluid and the solid boundary and that originates from viscous effects. So, at y equal to H u equal to u1, that means u1 is equal to c1H, that means c1 is equal to u1 by H. So, the velocity profile is u equal to u1y by H. So, what is du dy?

What is the rate of deformation of fluid in a plane like this, that is del v del x plus del u del y. So here there is no v, so it is basically du dy. So, this is the rate of deformation. So, this is the rate of shear. So, rate of deformation is u1 by H which is a constant, that means if somebody prescribes a rate of deformation, then you can use that rate of deformation to model this problem because if the rate of deformation is given.

And the gap is given you can find out what is u1, so the rate of deformation the value is given. If the value is given, given a particular gap you can find out what is u1. So, with that boundary condition you can stimulate a practical situation where the flow is purely driven by shear of a given magnitude, okay? So, this is the magnitude of that shear. So that is why this is a pure shear driven flow. Now next what you will do.

This is the fluid mechanics part of the problem, next we will analyze the heat transfer part of the problem. So, let us say that this is isothermal T is equal to T0, is the bottom plate and T equal to T1 is the top plate. And let us assume that T is a function of y only. Eventually if you have long parallel plates, infinitely long and two plate, top and bottom plates are isothermal.

Then it will come to a situation where temperature ceases to be a function of x, if you have such infinitely long plate. So that is the situation that we are studying. See, all these idealizations are there to simplify the mathematics and bring out the essential physics. So always in mathematical modeling, good physicist what they do is that, see, the actual practical problem is very complex.

But what good physicist try to do is they try to simplify the problem to an extent that the mathematics becomes simple enough to bring out the essential physics and that physics is utilized for design and other analysis. So that is what we are attempting here.

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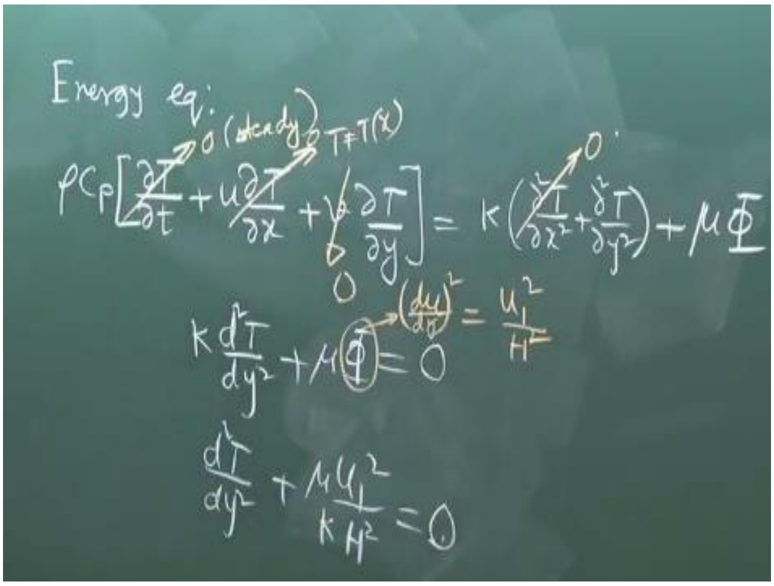

Now, for the heat transfer part of the problem we need to solve the energy equation. See, we have considered the viscous dissipation term but we will analyze later on whether this term is at all important or not. This term we have kept in the analysis because at least it has the potential of being important because H being small the square of this maybe large. So that gives the motivation of keeping this term in the analysis.

So, this phi is the viscous dissipation function which let me write the expression of this in an index notation. This we have derived earlier but let me just write it.

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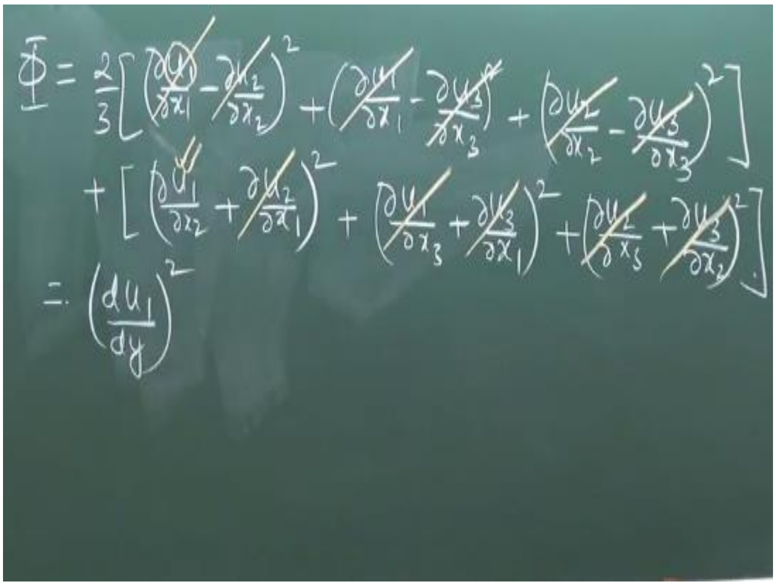

So, phi - so this we have derived earlier. So, can you recall that what are the assumptions under which this is valid? This is not a general, for any general situation this is not the expression for viscous dissipation. For what kind of fluids? Yes. Homogenous, Isotropic, Newtonian and Stokesian fluid because we had used lamda is equal to minus two third mu in this derivation.

So, Stokes hypothesis has to be valid. So, in the usual index location u1 means u, u2 means v, u3 means w, x1 means x, x2 means y, x3 means z. So now let us see, first of all u is a function of y only and there is no other velocity component. So u1, terms with u1, with derivative with respect to x2 will only be there, right? So only del u1, del x2, that term will be there. All other terms will be zero.

So, all these terms will be zero. So, this will be there. This will be zero, this will be zero and this will be zero. So, this becomes this for this particular problem. Now let us come to the energy equation. This is zero because it is steady flow T is not a function of x, so this is zero and this is zero because v is zero. So, if there is a hole in the plate, then you will have the dT dy term.

So not that the left-hand side always become zero. You have to be careful about the situation and then put that understanding into the mathematical terms because T is not a function of x, this is zero. So, you are left with k d2T dy2 plus mu phi is equal to zero. What is this phi? Phi is du…du dy square, so this is u1 square by H square. So d2T dy2 plus mu u1 square by k H square is equal to zero.

This is our governing differential equation. Now for the boundary conditions we have to assume one of the plates to be hotter than the other. Let us take the example that T1 is greater than T0. If the two plates are at the same temperature, then it is not an interesting problem for heat transfer. But even then, we will see that if the two plates are at the same temperature there is some interest in the heat transfer scenario because of the existence of this term.

So, there is a heat source. Now, let us first complete the solution of this problem.

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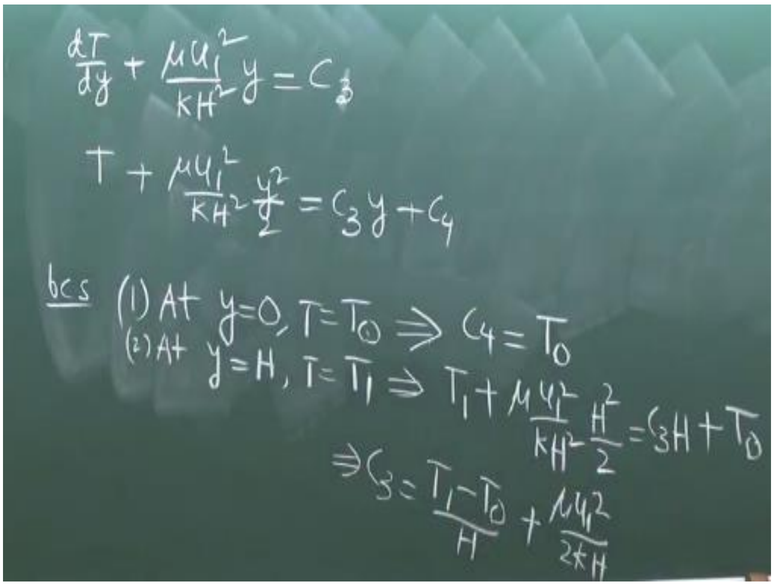

So, let us integrate it once, dT dy plus mu u1 square by kH square y is equal to c1, c1 c2 we have used, so c3, T plus mu u1 square by K H square, y square by 2 is equal to c3, y plus c4. What are the boundary conditions? At y is equal to zero, T equal to T0 and at y equal to H, T equal to T1. So, the first boundary condition tells, that c4 is equal to T0. Right, at y equal to zero, T is equal to T0, so c4 is equal to T0. At y equal to H, T equal to T1.

So, at y equal to H, you have T1 plus mu, mu one square by k H square into H square by 2 is equal to c3H, plus c4 is T0. So, you have c3 is equal to T1-T0 by H plus mu u1 square by 2 k H. So, the solution is T plus.

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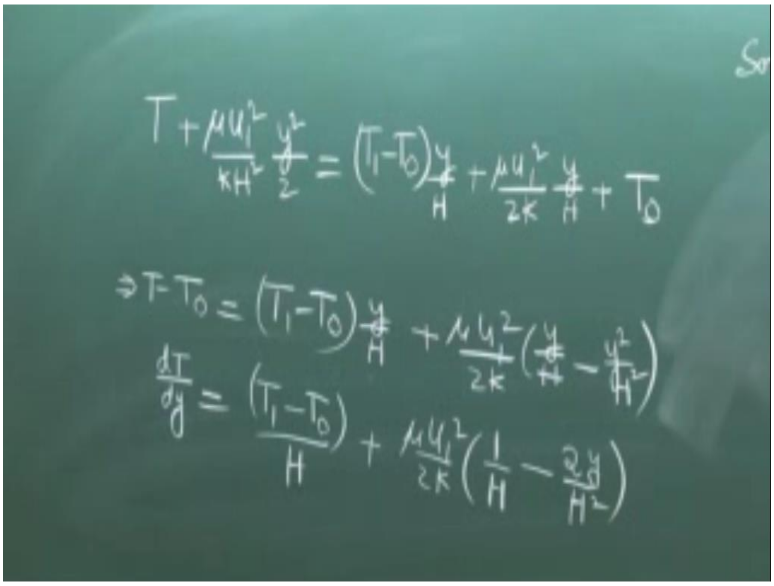

So, the solution is T plus mu u1 square by k H square, y square by 2 is equal to c3y plus c4. So, we can write, T minus T0 is equal to T1 minus T0 multiplied by y by H plus mu u1 square by 2k multiplied by y by H minus y square by H square. So, you can see that the temperature distribution is because of two components. This component is because of what? This is because of heat conduction. You can see this is a linear temperature profile.

This is because of heat conduction and this is because of viscous dissipation. So, the net change in temperature is due to a combination and here a linear combination because the governing equation is linear, is because of a combination of heat transfer due to conduction plus heat transfer due to viscous dissipation. Now what is the matter of our interest is to calculate the heat flux.

So, what is our intuition. Intuition is that this top plate is at a higher temperature than the bottom plate, so heat has a natural tendency to flow from the top plate to the bottom plate. That is the common intuition. Now let us see, let us find out what is the heat flux, so dT dy. dT dy is T1 minus T0 by H plus mu u1 square by 2k into one by H minus 2y by H square.

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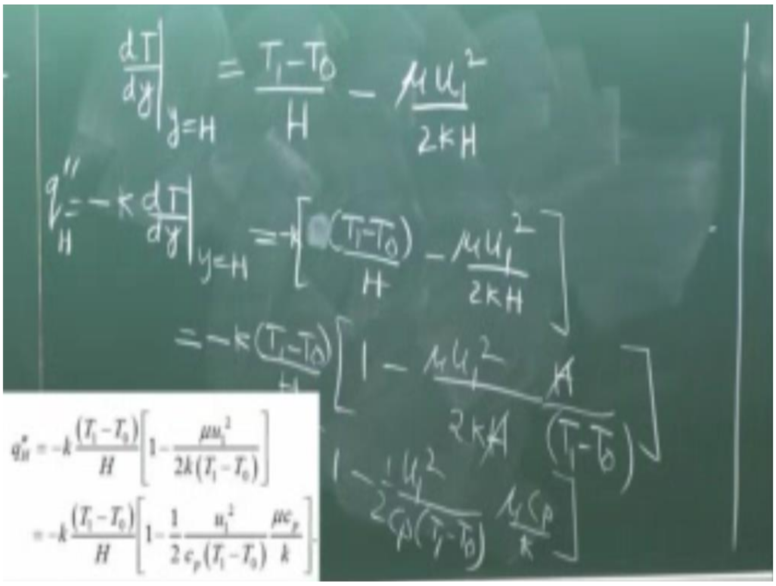

So, let us find out what is dT dy at the top wall. So, what is dT dy at y equal to H? First term is due to conduction and the second term arises due to viscous dissipation present within the fluid. One very important thing to note from this equation is that when the two plates are at same temperature or T1 is equal to T0, then also one can obtain a non-zero temperature gradient at the top wall.

So, the viscous dissipation which acts as a source of thermal energy causes this temperature gradient. Now, the heat flux at the top wall can be obtained by using the above temperature gradient in the following way. qH is equal to minus k dT by dy at y is equal to H is equal to minus k multiplied by T1 minus T0 by H minus mu ul square divided by 2kH. Rearranging different terms.

We obtain the heat flux at the top wall as qH is equal to minus k multiplied by T1 minus T0 divided by H multiplied by one minus mu multiplied by M1 square divided by 2k multiplied b T1 minus T0 is equal to minus k multiplied by T1 minus T0 by H multiplied by one minus cp multiplied by T1 minus T0 multiplied by mu cp by k.

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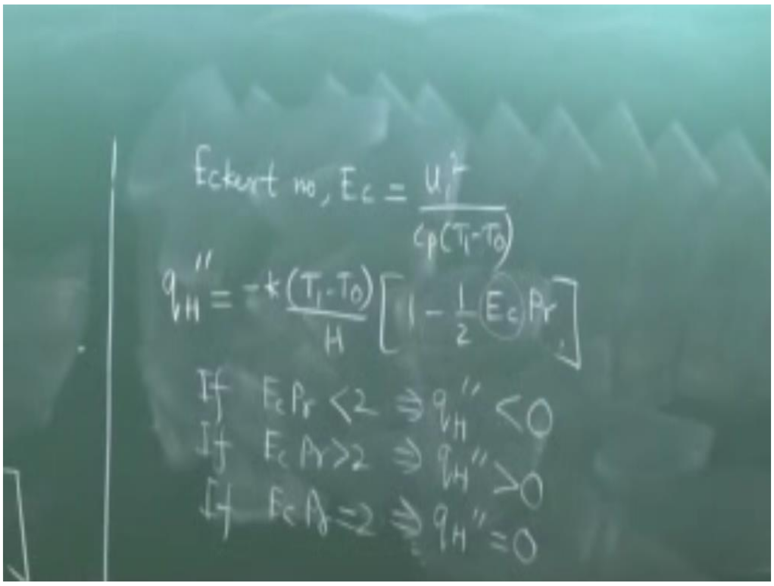

The above representation is of prime importance from the view point of understanding the physical process in terms of non-dimensional numbers which are the combination of different physical parameters, combined effect of which govern the heat transfer process. The right-hand side of this equation can be represented in terms Prandtl number defined as Pr is equal t mu cp by k and Eckert number defined as Ec is equal to u1 square by cp multiplied by T1 minus T0.

Prandtl number, Pr represents the ratio of momentum diffusion and thermal diffusion, while the Eckert number (Ec) is basically the ratio of kinetic energy associated with the fluid velocity and the thermal energy associated with the temperature difference. So, the heat flux at the top wall can be written as qH is equal to minus k multiplied by T1 minus T0 by H multiplied by one minus Ec multiplied by Pr divided by 2.

Now we have expressed the heat flux in terms of temperatures at the two walls and the two important non-dimensional numbers which are Ec and Pr. A closer look into this equation reveals that the direction of heat flux is solely governed by the following two terms T1 minus T0 and one minus Ec Pr divided by 2. Relative sign of these two terms decides the direction of heat flux.

So, for the case T1 greater than T0, the direction in which the heat through the top plate will depend on the magnitude of the dimensionless group Ec Pr that is a, if EcPr less than two implies qH less than zero and the heat will flow from the upper to the lower plate as per intuition. If on the other hand Ec Pr greater than two implies qH greater than zero then interestingly the heat flow from the lower plate to the upper plate even though the upper plate is at a higher temperature as compared to the lower plate.

So, in this particular case there is no heat transfer between the top plate and the bottom plate because that will violate the second law of thermodynamics. So, in this case due to viscous dissipation, locally, the temperature near the upper plate becomes more than T1 and hence heat transfer takes place from the lower plate to the upper plate.

Here the viscous dissipation basically acts as a heat source. So as if there is a heater sitting in lower plate which is making the local temperature more than T1. This is something which is non-intuitive. Finally, if EcPr equal to two implies qH is equal to zero and the upper plates acts as an insulator. Here despite a temperature gradient being present between the two plates, there is no heat transfer.

This is because the temperature difference created due to the viscous dissipation is exactly nullified by the conduction between the two plates. So, there is zero net heat flux. So, the product (and not the Eckert number alone) determines the strength of viscous dissipation in heat transfer. Based on the value of this product, the direction of heat transfer between two plates will change.

This kind of analysis is very important in determining the heat transfer in bearings. In bearings, the temperature of the shaft and its outer casing is very important as the viscosity of the lube oil is a function of temperature (although we have considered the viscosity to be a constant for the present case). Top wall towards the bottom wall, at the top wall, that is something very little because of the natural temperature difference between the top plate and the bottom plate.

However, when Eckert and Prandtl number becomes greater than two, then despite the top plane greater than, I think the temperature greater than that of the bottom plate there is actually the heat transfer of the bottom to the top. This is something which is non-infinitive. So actually, there is no direct heat transfer between the bottom plate and the top plate because that will violate the same wall boundary linings.

So, what is happening is that because of viscous dissipation locally here the temperature is becoming greater than T1. Viscous dissipation essentially is in line with heat source. So, I think there is a heater setting here that heater is making the local temperature greater than T1. So, there is a heat transfer from T1 to T0. But this is something which is not included. The third case is interesting that despite our temperature gradient being created by a natural boundary condition there is no heat transfer.

So why it is happening like that? It is happening like that because this heat transfer because of the temperature difference created by viscous dissipation is exactly nullified by the heat transfer because of the heat conduction between these two. So, there is zero break heat flux. So that you can also see mathematically. This is the heat transfer; this term is equationally what the heat transfer through conduction.

This term is the representation of the heat transfer in viscous dissipation and you can see that when they are exactly nullified each other the heat will pass this. That is what is the physically founded. So, the very important message is that, the Eckert number into Prandtl number is done by the Eckert number, but the Eckert number into Prandtl number that decides the strength of viscous dissipation in heat transfer.

And based on the value one can have interesting upper edge inversions of the direction of the flux. So (()) 48:36) what is happening is, there is local rise in temperature because of viscosity and that creates the (()) (48:46). So, this kind of analysis is very important for analyzing heat transfer in bearings. So, one can understand the temperature distribution, the heat flux and so on in bearings.

And why temperature bearings temperature in the gap between the shaft and the outer casing is important. It is important because the entire functionality of the oil, which is say a lube oil, it depends strongly on the viscosity of the oil. And the viscosity of the oil is a strong function of temperature. So here, for simplicity we have assumed that mu is a constant. But in practice mu is not a constant.

Viscosity of the oil, if it varies the temperature of the oil, it can vary thus dramatically and that will dramatically alter the performance of the bearing. So, in the practical engineering scenario also this appears to be very important. So, to summarize the discussion so far, in convection, we started with derivation of the energy equations. Of course, before that we had a preliminary discussion on fluid mechanics and then we had derivation of energy equation.

And we analyzed cases of flow over flat plate for different Prandtl numbers and internal flow that is flow through parallel plate channel and circular pipe. In boundary conditions like constant volume and constant volume plus and we also have considered the examples where viscous dissipation is important as has been discussed today. So, we will be having some (()) (50:34) problems for this portion of the scenes of nature that we had, these projects will be uploaded.

And this will be discussed extensively in the tutorial sessions. As I told you the type of problems that you will face for this part of the discussion is not dramatically different from a derivation that we have done. These derivations all are actually problems. So, in some cases boundary conditions may be changing, some cases one additional term may come in, some case velocity profile will change.

Please try to (()) (51:15) but the entire (()) (51:18) these problems is what exactly what we have discussed in the class for this portion. So, we will stop this lecture now.