Conduction and Convection Heat Transfer

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Lecture - 33
Internal Forced Convection – III

In the previous lecture we were discussing about the case of thermally fully developed flow in a parallel plate channel with constant wall heat flux. Now, we will see the consequence of constant wall temperature boundary condition for that problem.

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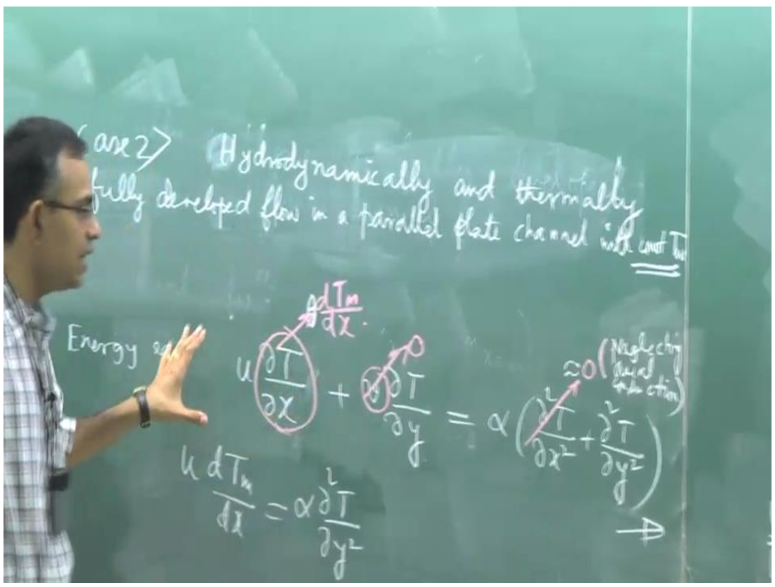

So, case 2, hydrodynamically and thermally fully developed flow in a parallel plate channel with constant T wall. So, what I have done is, I have kept in the board the derivation for the constant wall heat flux because I want to show you the contrast between that derivation and this derivation. So, wherever the things are the same as the previous case I will not erase and wherever of the things which are different from the previous case of constant wall heat flux, we will erase that.

So, we will start with the energy equation u del T del x. What is del T del x? for constant wall temperature we derive in one of our previous lectures. What is that? Theta into d T m dx for constant wall heat flux, it is d T m dx; for constant wall temperature, it is theta into d T m dx, ok.

So, this, here you have for this, this is 0. What about this one del 2 T del x 2. For constant wall heat flux, thermally fully developed flow del T del x is a constant, but for constant wall temperature, del T del x is not a constant.

Therefore, this is not identically 0 for constant wall temperature. We can at the most make an approximation that this is approximately 0. When we make this us approximately 0 what is the physics that we are considering what does this term represent, this represent axial conduction, conduction along x. So, this means we can assume this to be approximately 0 neglecting axial conduction.

So, this is an assumption for constant wall heat flux that was identically 0. For constant wall temperature, this is not identically 0, but this may be approximated to be 0 by neglecting axial conduction in comparison to axial advection. So, you can see that in the governing equation the sole difference coming out is with d T m d x, there is a multiplier of theta. ok.

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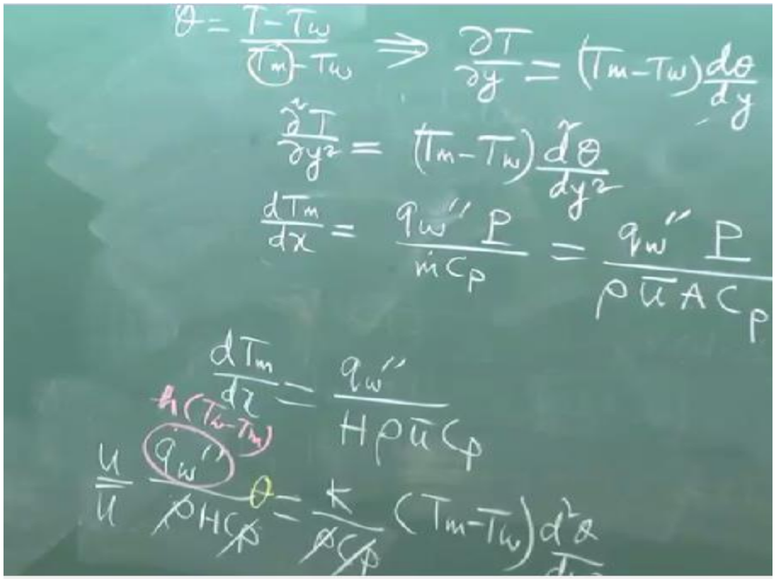

So, now let us look in to this derivation, so this does not depend on whether it is constant wall heat flux or constant wall temperature, d T m d x expression this also does not depend on whether it is constant wall heat flux or constant wall temperature. So, in the governing equation when we have substituted d T m d x. Now in this governing equation you have an extra theta multiplier with d T m d x.

So, with the previous term now we will have a multiplier of theta, right.

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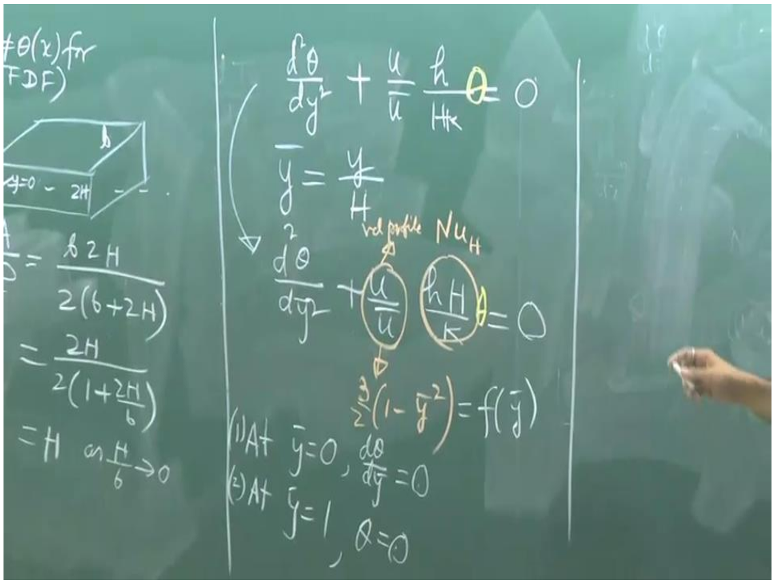

Whatever was in the previous case that was d T m d x, now it will be theta d T m d x. So, here it will be theta, so this will be theta. So, the governing equation instead of d 2 theta d y 2 plus Nusselt number into u by u average, it will be d 2 theta d y 2 plus Nusselt number into theta into u by u average. So, when this theta multiplier has come but this theta multiplier will spoil the ability of this problem to yield an analytical solution.

So, this problem can no more be addressed analytically until (()) (06:05) you have a very special case where u by u average is 1 that is plug flow. But for a general flow, you cannot solve this problem analytically in fact this kind of problem is an Eigenvalue differential equation, Eigenvalue problem in ordinary differential equation. So, you cannot solve this analytically, so I will give you an outline of how to solve this numerically.

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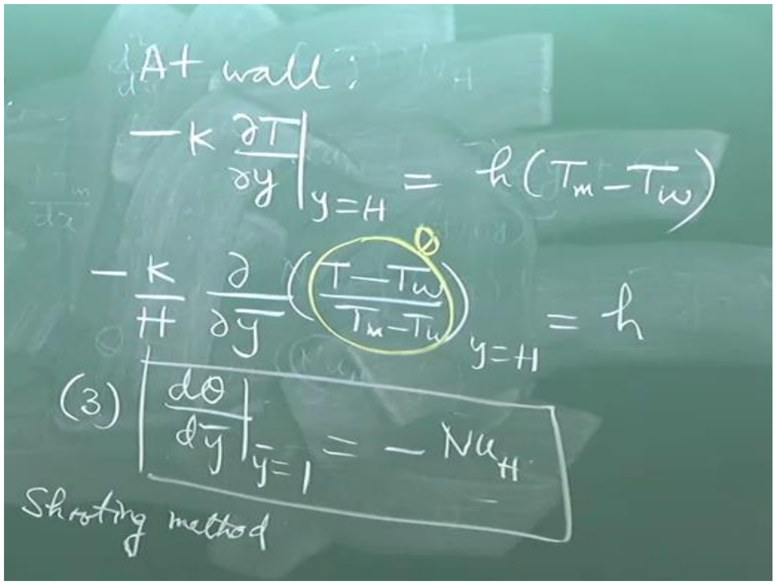

So, you have these 2 boundary conditions on the top of that at the wall, so you have this is the boundary condition at the wall, so you can write minus K del del y bar by H. So, what is this term in the bracket? This is theta, right. Can we write instead of partial derivative and ordinary derivative here? Yes, we can write because for thermally fully developed flow, theta is not a function of x. Theta is a function of y only.

So, we can write d theta d y nondimensional at y equal to 1 is equal to minus Nusselt number based on H. This is an additional constant. So, we have a constant one, we have a constant two, this a third constant. So, we can solve this problem by using a method numerical method called as shooting method. So, shooting method is a method were what you do is, so from the name it suggests that you have a target.

You shoot the target, you try to hit the target and based on your error in hitting the target, you make a revised calculation. So, what you do is that, you convert this second order ODE into a coupled system of two first orders ODE, ok. So, I am giving you the outline and your job will be to implement this in Metlab, this will be your one of the home works. See all problems cannot be analytically solved.

So, you must learn to write small programs at least to solve problems numerically. That will give you a good grasp on basic numerical method for solving the heat transfer problem. So, I will give you the outline, I will tell you what to do, but you have to implement it in the computer. So, what you do is that you write this equation as a coupled system of two first order ODEs. What are will be the variables? Theta and d theta d y, ok.

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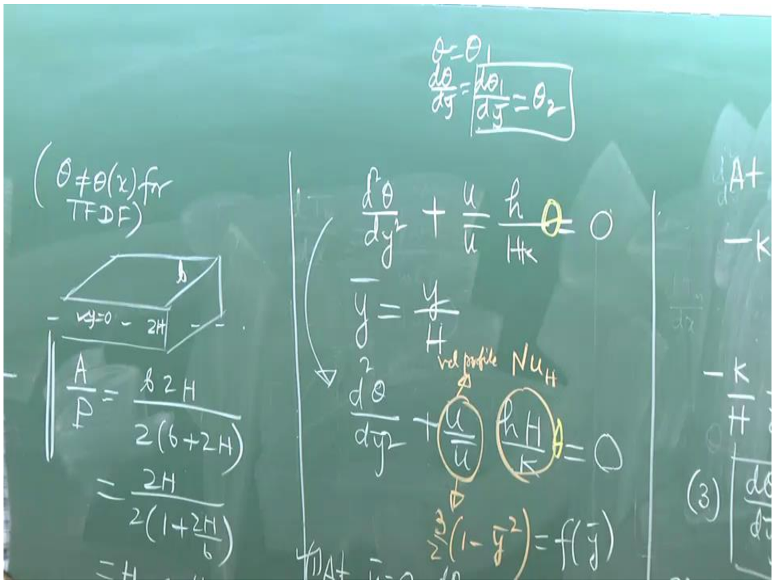

So, you can assume that theta is equal to theta 1 and d theta d y that is d theta 1 d y is equal to theta 2. So, you will get theta 2 as a function of theta 1 and another equation this one. So, these are the two coupled first order. One is d theta 2 d y plus f into Nusselt number into theta 1 equal to 0, right and the other equation is d theta 1 d y is equal to theta 2.

So, you will get a coupled equation 2 equations with theta 1 and theta 2, but two first order equations. So, this is a trick of reducing (()) (10:36) order equation into coupled n number of first order equation. Here, we are reducing the second order equation to coupled system of two first order equation. Then, what are the boundary conditions at y equal to 0. See, if we have both the conditions for theta 1 and theta 2 at y equal to 0, then we call it an initial value problem.

That is, we know at the start of the domain, what is theta 1 and what is theta 2. So, at the start of the domain, we can use the start of the domain at y equal to 1 also. So, at the start of the domain at y equal to 1. Let us say y equal to 1 that is the wall. We considered the start of the domain, so theta 1 is 0 and at y equal to 1, this is theta 2 at y equal to 1 is minus Nusselt number, but you do not know Nusselt number.

So, you can give the boundary condition, if you guess the Nusselt number, right. So, the first step is so I am writing the broad steps.

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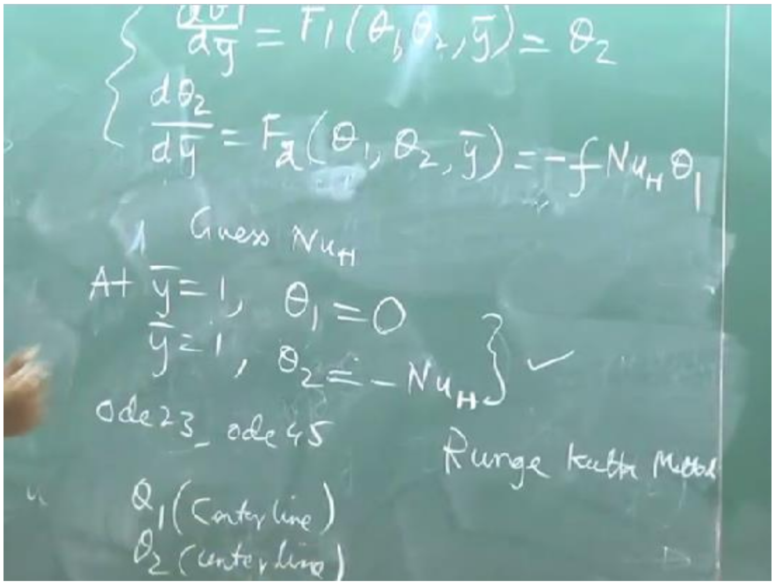

So, you have d theta 2 d y bar as the function of theta 1 theta 2, y bar and d theta. What is this f 1? This is theta 2 and what is this? Minus f into Nusselt number into theta 1, right. So, these two equations. Now you use the boundary condition, these are the two coupled equations. Now guess Nusselt number. Once you guess the Nusselt number then what you will get, at y equal to 1.

You have boundary condition theta 1 equal to 0 and at y equal to 1 theta 2 is d theta d y is minus Nusselt number. So, these 2 now will become known if you guess Nusselt number. So, with this you can use the initial value problem solver. In Matlab, you have some built in functions like ODE23, ODE45, this kind of functions, built in functions in Matlab, you can use to solve for theta 1 and theta 2.

Then, using these so these are basically using a method known as Runge–Kutta method. This is the method of numerical solution of initial value problems. ok. So, once you calculate this, then you can calculate what is theta 1 at center line and theta 2 at center line this you can calculate from wall, you come to the center line and you calculate theta 1 at center line and theta 2 at center line.

So, these initial value problems are marching problems that is in a particular direction you march. The direction may be in time, the direction may also be physical direction I mean, x or y direction. So, here in the direction from the wall to the center line we are moving, so at the center line you can calculate theta 1 and theta 2 starting from the wall you march. Now, theta 2 at the center line you calculate.

And what do you expect, see these boundary conditions is that at the center line you expect theta 2 to be 0. So, you ask now the question is it 0. If it is 0, then you have guess the Nusselt number correctly, but if it is not 0 you have to do a define guess based on what deviation from 0 you have got from this. So, that is what is shooting method that is your target is to hit the bull's eye. The bull's eye is 0.

So, if you do not hit the bull's eye you look into the deviation from the bull's eye and then retake your shooting so then if not correct the Nusselt number guess and (()) (0:16:49). So, in this way you go on correcting the Nusselt number guess till you (()) (16:58) to a convert solution that whatever guess of the Nusselt number gives theta 2 equal to 0 at the center line that is the convert value of the Nusselt number. So, that is the procedure for solving this problem.

There is no short cut way of doing this you have to write the program by yourself you have to solve and I give you the answer. You have to check the answer. So, let be give you the answer for this. So, the Nusselt number is 7.54. That is the answer for this problem. So, you write a problem and then get then value and check whether you get this. Of course, now you know the answer, so you can play a trick with me by giving these are initial guess.

And then in one step without iteration, you can get the answer. But I mean all of us understand that life is not as simple as this.

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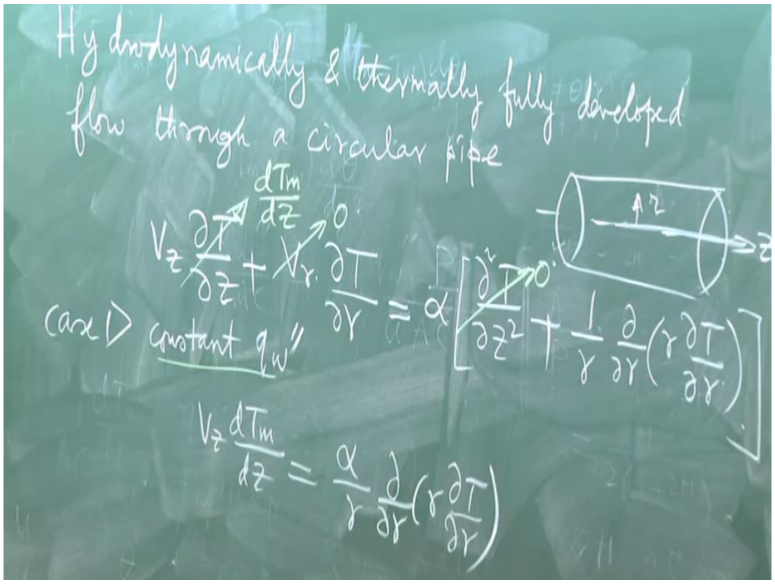

So, anyway now move onto the other geometric which is the circular pipe. Hydrodynamically and thermally fully developed flow through a circular pipe. So, we have now seen the parallel plate channel situation just now we have seen, now will see the circular pipe. So, the concept wise, there is absolutely no difference only the mathematics wise, it will be little bit different which we will see that what different it is

And what is the consequences in the solution but physical concept wise that is why first I started with the parallel plate channel that it maybe a very simple physical situation, but it gives you the physical understanding of the problem completely. Now for flow through a circular pipe, let us start writing the governing equation. So, u del T del x in place of u, what will be this? So, the circular pipe, let us say this is the pipe with this as the z axis and this as the r axis, ok.

So, in place u, it will be V z del T del x will be del T del z plus in place V, it will be V r. So, the only difference is that in the cylindrical coordinate system, Laplacian will take these form that is when you consider the derivative with respect to r, so this is 1 by r del del r of r del T del r that del square term will involve this. If it was a spherical system, it would have been r square. So where from this come r, r square all these?

Because in the cylindrical system the elemental area involves 2 pi r d r and in the spherical system, it is proportional to r square 4 pi r square. So, the elemental volume is 4 pi r square dr for the spherical system and here 2 pi r d r into L. So, when you divide the all the terms per unit volume that 2 pi r becomes 1 pi r and 4 pi r square becomes 1 pi r square that is all this terms come.

So, it just a matter of changing from one coordinate system to the other, it has I mean no special significant. So, case 1, constant wall heat flux which will work out and constant wall temperature I give you the as the homework. Just like the previous case you have to use the shooting method to solve this problem, but I will work out the constant wall heat flux case. So, constant wall heat flux del T del z will become d T m d z, right.

What will be V r, V r will be 0 for fully developed flow. What will be this just like del T del x is equal constant for thermally fully developed flow with constant wall heat flux in parallel plate channels. Similarly, del T del z equal to constant. So, this will be 0. So, you have left with z d T m d z what is d T m d z. We have derived it for a general cross section, so we can use it for a circular pipe. So, what was that q double dash p by m dot C p.

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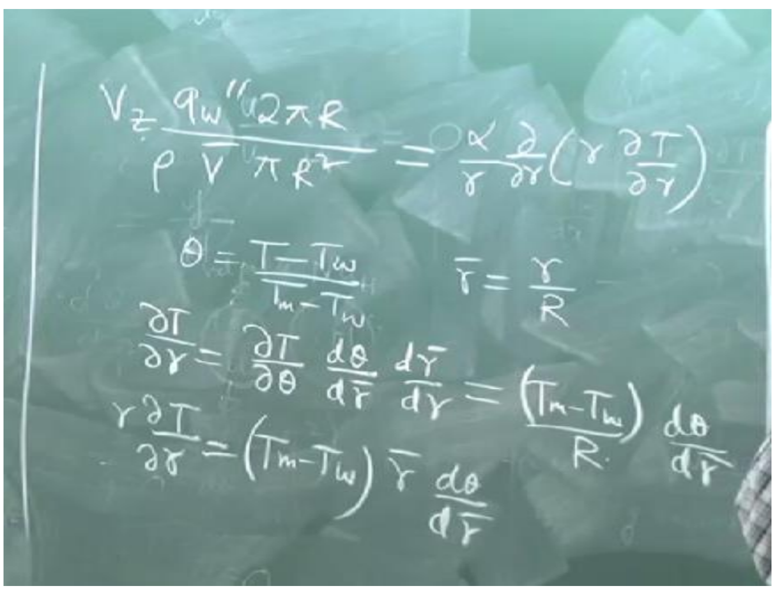

So, you can write this as V z into q double dash. P is 2 pi r perimeter of a circle for is 2 pi r divided by m not is rho into V average into A pi r square is equal to alpha into 1 by r del del r of r del T del r. Now, we will define theta is equal to T minus T wall by T m minus T wall and nondimensional r is equal to r by R. Where R is the radius of the pipe. So, we can write del T del r is equal to del T del theta d theta d r bar into d r bar d r, right, just like chain rule.

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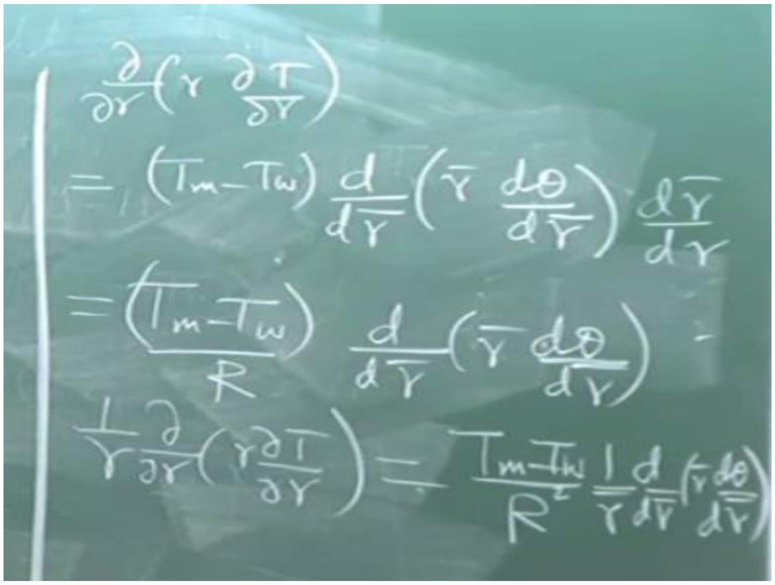

Changing the basis from T2 theta and r2 r bar. So, del T del theta is T m minus T wall and d r bar d r is 1 by R. So, what is r del T del r then what is del del r of r del T del r. So finally, 1 by r del del r of r del T del r. right. T m minus T wall by r square into because this r you can write r bar into R So that R into this R makes it R square.

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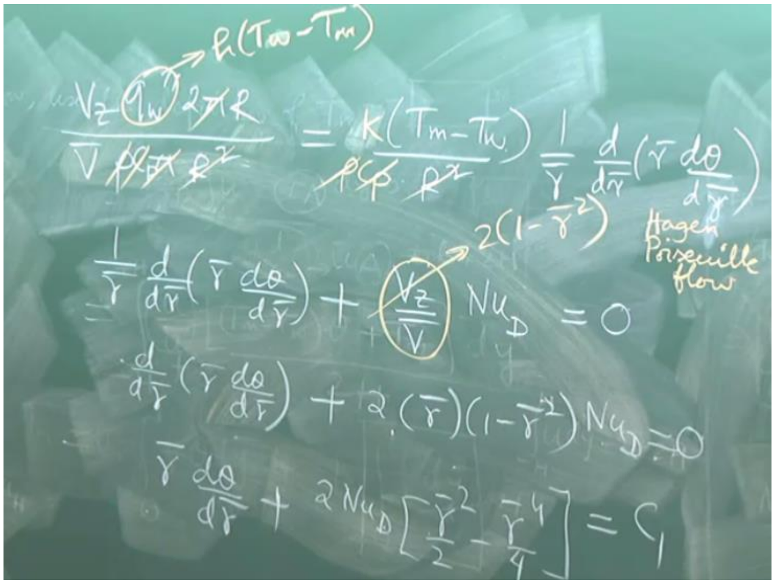

So, we can write Vz into q double prime into 2 pi R by V average C p rho pi R square is equal to alpha into T m minus. Now what is the next step in place of q all we will write h into T wall minus T m and alpha is K by rho C p. So, rho C p gets cancelled, pi gets cancelled, R square also gets cancelled so 1 by r d d r r d theta d r plus Vz by V average. So, you see here this h into 2 r by K, what is 2 r? 2 r is the diameter of the pipe.

So, it is h d by K that is Nusselt number based on the diameter d. Normally, the convention is for a pipe, the reference length scale is consider to be the diameter of the pipe, whereas for a the flat plate it is the length of the plate. So, these are physical reference links governing the physics of the problem. So, for a pipe it is never the length of the pipe that will seriously govern the physics of the problem, but it is the diameter of the pipe or radius whatever.

So, normally in engineering it is the diameter that is considered as a reference scale. If somebody takes radius also it is ok there is no problem it is just a matter of convention that take as the diameter in engineering. So, what are the boundary conditions, so let us integrate this d d r of now where does fluid mechanics come into picture here what is Vz by V, this you tell. This is the solution from Hagen-Poiseuille flow, what is that? What is the velocity profile?

This is 2 into 1 minus r square by R square this is Hagen-Poiseuille flow. Hydrodynamic fully developed flow through a circular pipe. Now, can you tell the value of c1 what is d theta d r at r equal to 0, it is 0. So, if you substitute d theta d r equal to 0 at r equal to 0 you will get C 1 is equal to 0.

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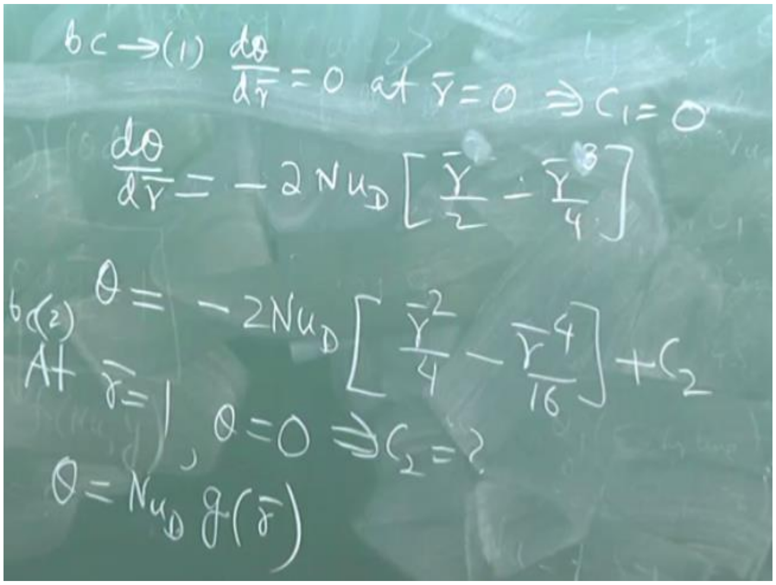

So, boundary condition 1, d theta d r equal to 0 at r equal to 0 that will tell you C 1 equal 0. So, d theta d r is equal to minus 2 Nusselt number into r square by 2 minus r 4 by 4 divided by r that is r by 2 minus r cube by 4, right. Then, we can right boundary condition 2 at r equal to 1, what is theta? Theta is T minus T wall by T m minus T wall, r equal to 1 means wal, so theta is 0. So, these will give you what is C2.

So that means you can write theta is equal to Nusselt number into some function g of r, right.

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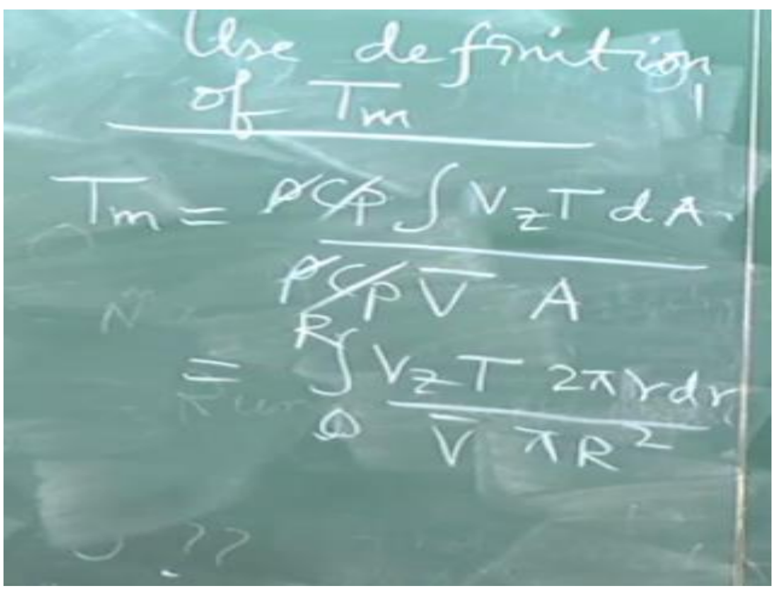

Now how do you calculate the Nusselt number just like the parallel plate channel case, we will use the definition of bulk mean temperature, so use the definition of bulk mean temperature. So, what is the definition? rho C p is constant we are assuming. So, what is d A for a circular pipe 2 pi r d r. So, integral of V z into T into 2 pi r d r by V average into pi r square 0 to r. So, you can write T m is equal to V z by V average into T into 2 r bar d r bar 0 to 1.

So, what we have done is, we absorbed one R with r and 1 R with d r r. So, it has become r bar d r bar. Now in place of T, we will use the definition of theta and write.

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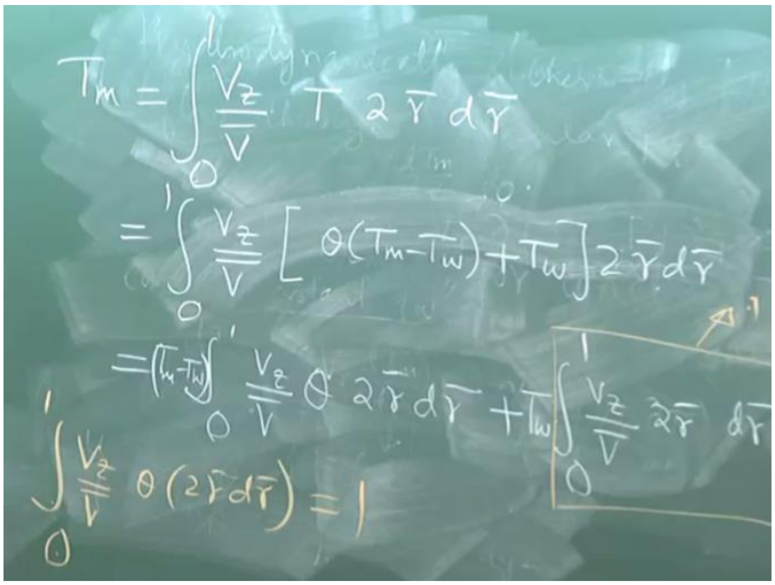

What will this last term? Remember integral of V z into 2 pi r d r is equal to V average into pi r square. So that is the nondimensional form of that ratio which becomes 1. So, that integral of Vz into 2 pi r d r by V average into pi r square that ratio is 1. So, you see I am trying to generalize it, it does not depend on what is the velocity profile, this becomes always 1. So, you can write integral of V z by V average into theta into 2 r bar d r bar 0 to 1 that is equal to 1.

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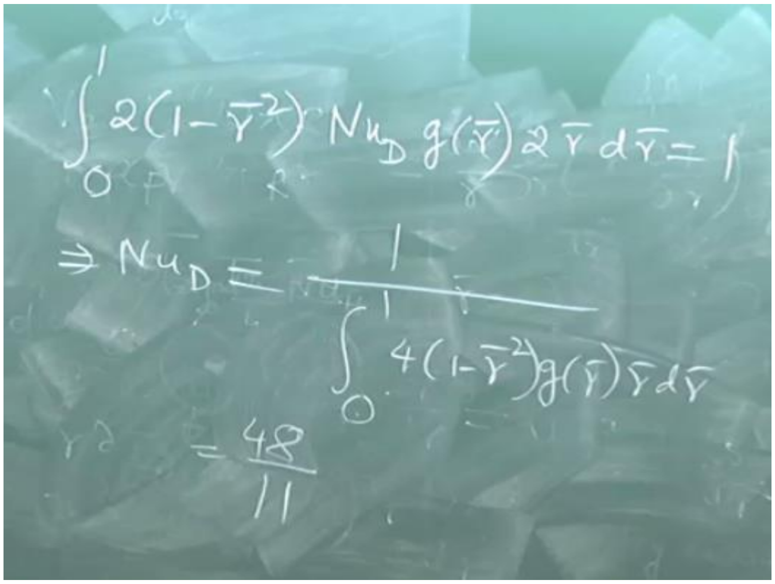

So, V z by V average is 2 into 1 minus r bar square then theta is Nusselt number into g of r bar into 2 r bar d r bar equal to 1. So, that means Nusselt number is equal 1 by integral of 0 to 1 4 into 1 minus r bar square into g r bar into r bar d r bar, right. So, this is the number once this is the simple polynomial integration, there is no trick in this integration because this is a simple polynomial.

So, once you do this integration you will get the value of the Nusselt number and the value of this Nusselt number is 48 by 11. So, I am just giving you all the answers because I think a good exercise that you complete all this things by yourself by doing the small missing algebraic calculations. Because I have given you entire framework, the entire method, but some little bit of polynomial integration and those things are not numerically evaluated.

So, you can numerically evaluate those and check this answer that will give you a good confidence of how to approach these problems. The other case is the constant wall temperature. I am giving you the answer you have to do by the shooting method, so again the important difference between the pervious case.

And this case is that when you are considering the equation what will be the change in the equation if it is constant wall temperature this will be multiplied by theta that will be the only difference just like the parallel plate channel and then you have to work it out by the shooting method and I am giving you the answer, so Nusselt number, so this case is constant wall temperature.

Nusselt number is equal to 3.66 based on the diameter. So, we have worked out 4 cases or we have discussed about 4 cases, constant wall temperature and constant wall heat flux, each for parallel plate channel and circular pipe. So, these 4 cases we have covered and velocity profile we have considered to be fully developed pressure driven velocity profile which you can derived from the Navier-Stokes equation assuming fully developed flow.

Now in practical industrial applications often the laminar flow constraint is not satisfied. So, then if the flow is turbulent flow, in turbulent flow through a circular pipe is very common in industry. So, if the pipe is either heated or cooled then what type of relationship you will have between the Nusselt number and the other parameters. So, that it is given by many correlations. For turbulent flow, you cannot work it out analytically.

So, based on experimental correlations, many popular correlations are there. I do not expect that you remember many correlations, but at least one correlation which is industrially used very extensively much be learn at this level. Because at the end we are interested to solve engineering problems and that is one of the important correlation which we used for turbulent flow through circular pipes for solving engineering problems and that is given by a correlation called as Dittus-Boelter correlation. So, let me write down this correlation.

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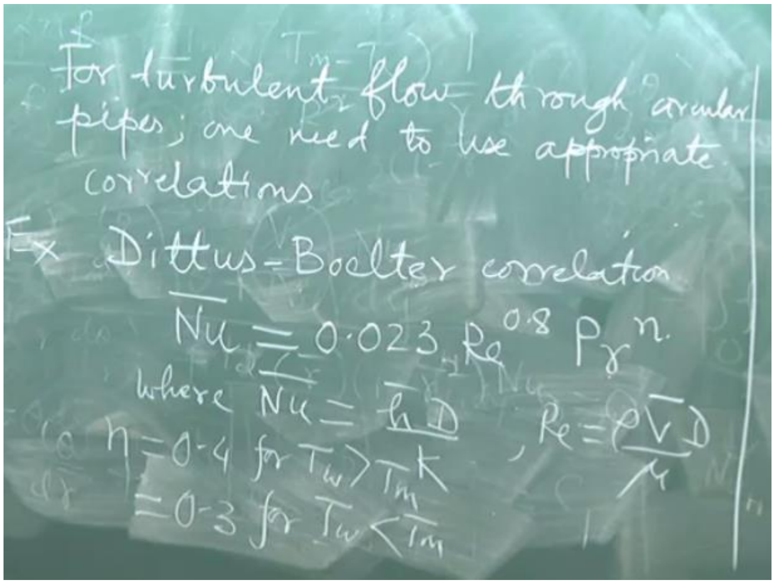

So, for turbulent flow through circular pipes; one need to use appropriate correlations. So, in this correlation the value of this, so again you see the Nusselt number is of the order of Reynolds number to the power n into Prandtl number to the power n. So, that fundamental understanding still remains. Now this Prandtl number to the power n, value of n depends on whether it is heating or cooling.

If the wall temperature is greater than the bulk mean temperature, n is point 4 and if it is less than the bulk mean temperature that is cooling that n is point 3. Remember that this is average Nusselt number because in a turbulent flow you do not achieve a situation where you have constant heat transfer co-efficient. So, you have this h varying continuously with x, so you have h average into d y K that is Nusselt number average and Reynolds number.

Of course, rho into V average V by mu. Now, a very important thing see as engineers when make calculation you have to use the properties mu, rho, K. These properties are functions of temperature. So, at what temperature you will calculate these properties. So, not at the bulk mean because close to the wall the temperature will be mostly driven by the wall temperature, mostly govern by the wall temperature.

Far away from the wall, it will mostly govern by the bulk mean temperature. So, the engineering practice is to evaluate properties at T wall plus T m by 2, ok. If it is for flow over the flat plate then instead of T m, we can use T infinity. So, T m see now you have learnt some aspects of forced convection, you tried to unify the concept. In the forced convection for flow over the plate, whatever was the role of T infinity, the same role is played by T m at the bulk mean temperature for an internal flow which is confine between boundaries, ok.

So, I do not want to bother you with many of these correlations because as I promised you that we will not discuss in the undergraduate level some formula which we cannot derive in the class, but this is a sort of an exception as an example because this is so commonly used by engineers, I believe as engineers you must know this correlation. Now, so far, we have discussed about what let us summarize.

We have discussed about forced convection with flow over the flat plate and flow in a channel or a pipe. We have derived the momentum equations, the energy equations, we have solved the fluid mechanics problem and the heat transfer problem, but in all these problems we have neglected viscous dissipation which was one of the terms that appears in the derivation of the energy equation.

Now, what happens what are practical engineering situations when the viscous dissipation term is important and how can you analyze those situations mathematically. So, you learnt that in the next lecture when we discuss about cases where viscous dissipation terms maybe important. Thank you very much.