Conduction and Convection Heat Transfer
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Lecture - 32
Internal Force Convection - II

In the previous lecture, we were discussing about the concept of hydrodynamically and thermally fully developed flow. We discussed about the consequences of hydrodynamic and thermally fully developed flow in terms of expressing a non-dimensional temperature and a non-dimensional velocity. Now, we have seen the term delta T delta x can be expressed in terms of theta and d T m d x, where theta is T minus T wall by T m minus T wall.

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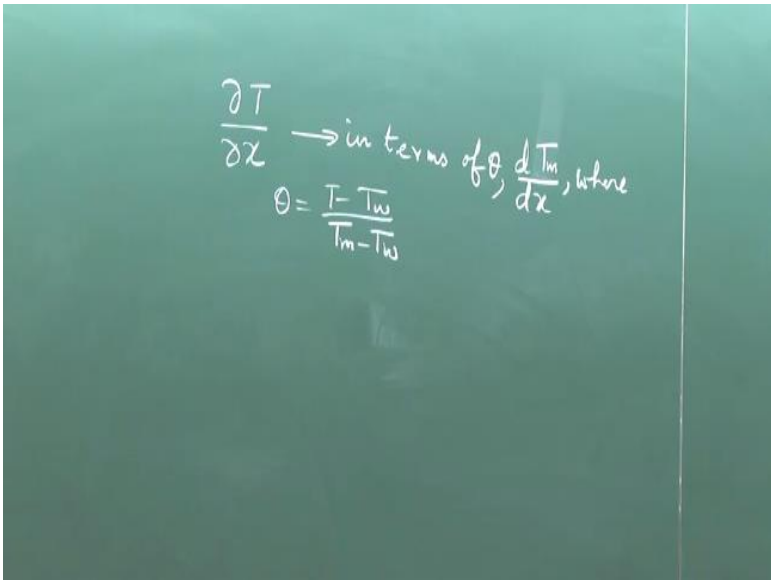

So, for constant wall temperature it is theta into d T m d x and for constant wall heat flux, it is just d T m d x without involving theta that much we discussed in the previous class. Now, irrespective of the boundary condition, this parameter is always involved. So, one thing we have been successful in reducing the dimensionality of the problem that when we are expressing delta T delta x in terms of d T m d x.

We are converting like a 2-dimensional or a 3-dimensional problem to (()) (02:37) 1-dimensional problem. But the question is how to calculate this d T m d x, so for that we will make an overall energy balance.

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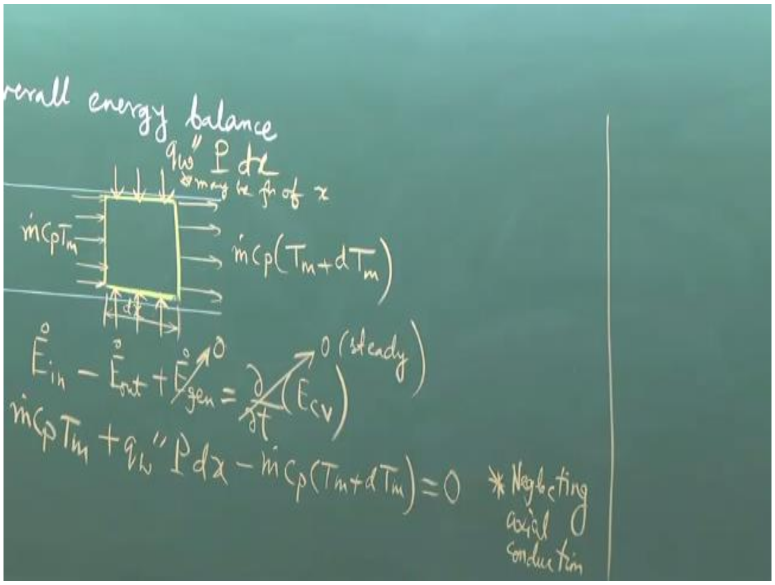

So, let us say that this is the channel and we take a control volume like this. Across this control volume, we are going to write an energy balance, so when we are going to write an energy balance, there is some energy that is entering. Let us say that this length is d x. There is some energy that is leaving, so what is the energy that is entering in terms of the bulk mean temperature.

The bulk mean temperature is an equivalent temperature which would have existed uniformly across the cross section to make the flow of the same energy as that of the actual case. So, if T m is the bulk mean temperature then what is the rate at which energy is transferred across this section, it is m dot into H, m dot into H is m dot into C p into T m. We are considering incompressible fluid and let us say this one is m dot C p T m plus d T m.

Then, we have heat transfer at the wall, what is the heat transfer at the wall? If the wall heat flux is q double prime this may be a constant or it may be a variable. Let us say that the wall heat flux is q double prime. So, what is the heat transfer rate across the wall. Let us say this is a circular pipe of radius r. So, if the wall heat flux is q double prime, what is the rate of heat transfer? q double prime into 2 pi r d x, right. 2 pi r d x is the surface area of the pipe.

So, 2 pi r d x is nothing but perimeter into d x. So, to generalize it for any section, we will write it as q double prime into perimeter into d x. So, we can write for this control volume, rate of energy in minus rate of energy out plus rate of energy generated. So, this is zero because it is steady flow and steady state. Rate of energy generation is zero that we are not considering in this problem. If there is some rate of energy generation, we can accommodate this in this formulation very easily.

Rate of energy in is m dot C p T m plus q double prime Pd x. What is the rate of energy out m dot C p T m plus d T m. Now, this equation is not perfect, but it has an approximation. My question is what is that approximation that is there in this equation. This is not exactly correct. "Professor - student conversation starts" Yes, no, no, no, it is not taken as constant. "Professor - student conversation ends".

Even if it is a function of x it is true because we have taken a small element over which it will be constant. Q double dash, this may be function of x. So, in the axial direction we have considered that there is heat transfer due to fluid flow, but we have not considered that there is heat transfer due to axial conduction. So, we have neglected axial conduction, so ideally, we should have taken a heat flux minus K d T m d x here and minus K d T m d x at x plus d x here.

So, we had not taken any axial conduction. So, we have neglected axial conduction and that is valid in many practical problems where advection is much more dominating than axial conduction. So, this very importantly is valid neglecting axial conduction.

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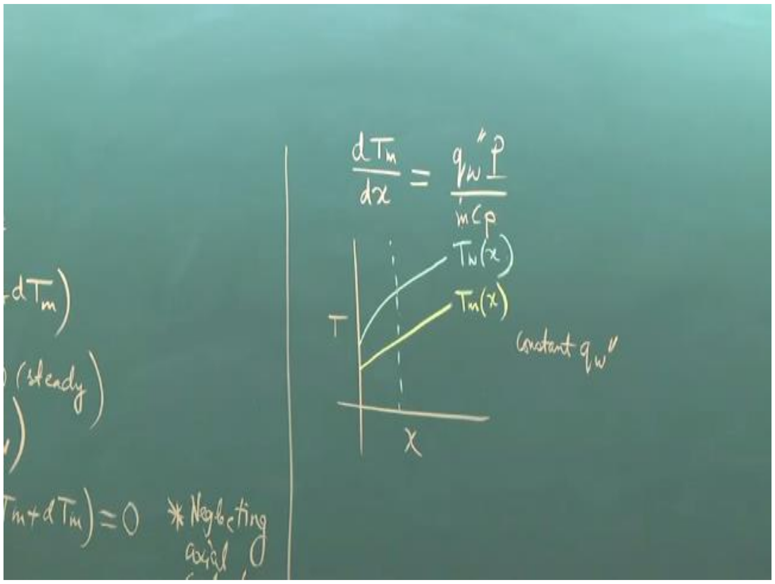

So now from this equation, you can write d T m d x is equal to q double dash P by m dot C p. Now, can you tell from here that d T m d x is constant if wall heat flux is constant, yes or no, yes. Because for steady flow rate is m dot is constant, we assume that the properties are constant then the perimeter of the cross section, it is the constant for the geometry that we are considering. So, if the wall heat flux is constant then the d T m d x is constant.

So, we had earlier shown that for the constant wall heat flux, delta T delta x is equal to d T m d x is equal to d T w d x is equal to constant, ok. But not for any general constant wall heat flux. For constant wall heat flux and thermally fully developed flow, ok. So now, we come to an inference that this is the constant for constant wall heat flux not just in the thermally fully developed flow region, but about the entire region.

Because this overall energy balance theory is very general it does not take into account whether it is thermally fully developed or not. It is just the simple energy balance using the definition of bulk mean temperature that is converting the multidimensional problem to 1 dimensional problem, but it does not take into account whether it is thermally fully developed or not. So, the conclusion is that even if the flow is not thermally fully developed for constant wall heat flux d T m d x is the constant, but this is still an approximation.

Because it has neglected axial conduction. If you do not neglect axial conduction that is not true. Whereas this is exactly true, does not matter whether axial conduction is there or not, ok. So, we can make a graph of say T versus x for constant wall heat flux. So, when we write plot T versus x, we essentially want to plot basically T m and T wall because we are now converting it to 1 dimensional problems where the functions of x are T m and T wall.

So, let us say that T wall is greater than T m now what is the graph of T m versus x, it will be a straight line because d T m d x is a constant assuming that axial conduction is negligible it is a single straight line throughout otherwise in the thermally developing region this may not be a constant, so it may be a curve and then in the thermally fully developed region, it will be a constant that formula.

So, let us make as sketch of this is T m versus x assuming that the wall is heated, now what will be the graph of T wall versus x assuming T wall is greater than T m. So first that graph should be above this if it is heated then how will the graph look like, see d T wall d x is a constant only if it is a thermally fully developed flow. So, let us say that the flow becomes thermally fully developed from here.

So, up to this it will be a curve beyond this it will be a straight line. What straight line, straight line parallel to this because d T m d x is equal to d T wall d x. So, the slopes of these 2 straight lines are equal in the thermally fully developed region. So, this is T wall as a function of x, ok.

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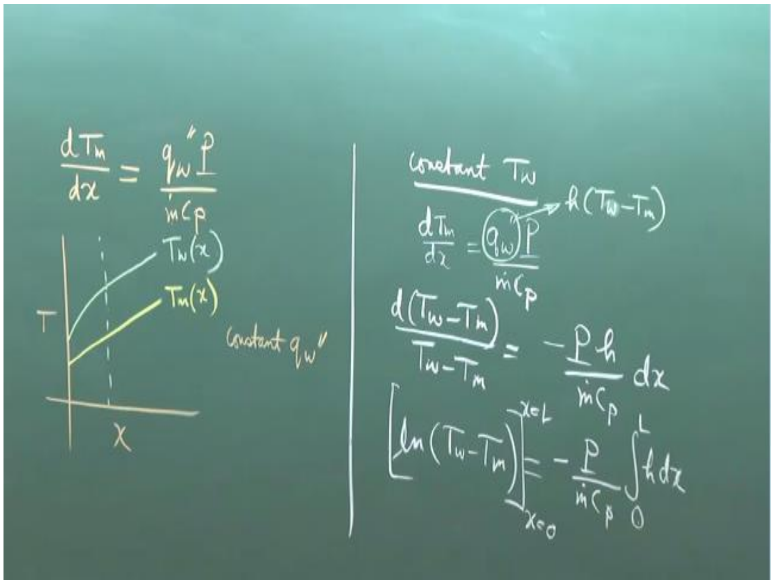

Now, this is the case of constant wall heat flux. Let us stud y the qualitative behavior of constant wall temperature may be first we studied quantitatively and then we show it in a plot. So, the case of constant wall temperature. So, d T m d x is equal to q double prime p by M dot C p. How can you write q double prime in terms of the heat transfer coefficient h, q double prime equal to what h, h into T m minus T w or T w minus T m.

So, when you are considering q double prime, look at this figure, here the sign convention is the heat transfer is from the wall to the fluid that means T wall minus T m, right. If it is the opposite the sign itself will take care. So, in place of q double prime, we will take h into T wall minus T m, so we can write d of T wall minus T m divided by T wall minus T m is equal to minus p into h by m dot C p d x.

So, what we have done is, we have written d T m d x as T of d x of T m minus T wall because T wall is a constant it does not matter whether we take it inside the derivative or not. It will make no difference because d T wall d x is 0 for constant T wall, ok and then this minus sign is observed because we have converted T m minus T wall to T wall minus T m.

So, it is of the form (()) (18:21) it will be log of this, so Ln of T wall minus T m is equal to minus p by m dot C p integral of h d x from x equal to 0 to x equal to L, where L is the total length of the channel or the pipe. This of course we have to put a definite limit from x equal to 0 to x equal to L. X equal to 0 is the starting and x equal to L is the ending of the channel.

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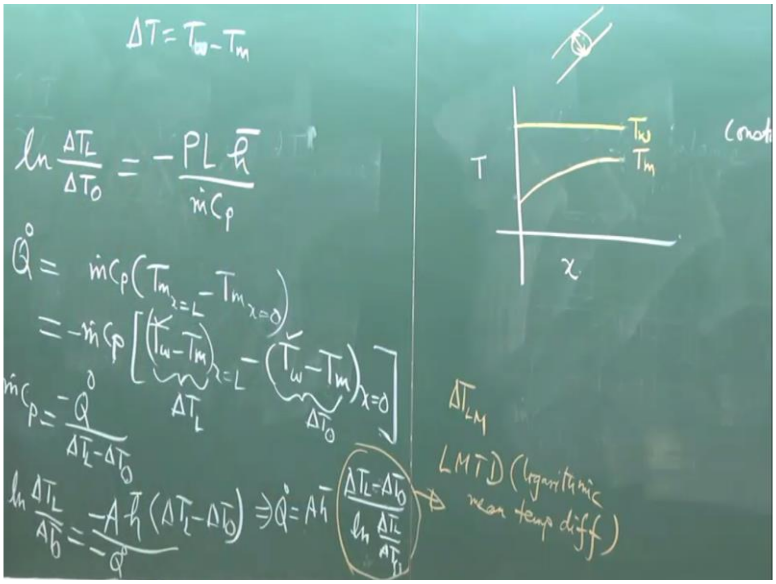

So, we can write Ln of delta T L by delta T 0 where delta T is T wall minus T m is equal to, so what is this, this is the average heat transfer coefficient times L. So, minus P into L into h average by m dot C p. Now what is the total heat transfer rate, Q dot. Let us say you transfer some heat from the wall to the fluid, so how can you measure what heat has been transferred from the wall to the fluid.

Let us say you are doing an experiment, so the situation is that you have some heat let us say you have a heating coil at the wall, you have a heater at the wall, you are transferring heat to the fluid. So how do you measure that what heat actually has been transferred to the fluid. So, you can measure the thermal energy at the inlet and you can measure the thermal energy at the outlet. So, the difference between these 2-thermal energy is the heat that is supplied from the wall.

So, you can say Q dot is nothing but m dot C p into T m at x equal to L minus T m at x equal to 0, right. This is just simple energy balance. So, this heat transfer this is nothing but the change in enthalpy. So, if you write the first law of thermodynamics for the control volume this is what you will get as a heat transfer. There is no (()) (22:18) done in this case. So, all the heat that is supplied is used to change this m dot C p into T that is m dot into h.

Of course, we neglect the changes in kinetic energy and potential energy. So, you can write these as m dot C p, see we can write this because this T wall is the constant so it is just adding and subtracting the same constant from the 2 terms, but why we have done is because this is the delta T at x equal to L and this is the delta T at x equal to 0. So, we can write m dot C p is equal to Q dot minus Q dot by delta T L minus delta T 0.

So, you can substitute that here and write Ln of delta T L by delta T 0, perimeter into length is what the total surface area, so this A is not cross-sectional area this is the surface area that is 2 pi r into L for a circular pipe of length L. So, minus area into h then in place of M this one you can write Q dot by delta T L minus delta T 0. So, Q dot is equal to A into h average into delta T L minus delta T 0 by Ln of delta T L by delta T 0.

So, you can see that normally what is Q dot A into h into delta T, the temperature difference between the 2 systems across which the heat transfer is taking place here one system is the wall another is the fluid. So, it would have been ideally A into h into T wall minus T m, but it is not A h into T wall minus T m. It is h into these, the reason is that T wall minus T m is not a constant. It is continuously varying with x.

So, if you make a plot of say T wall so T versus x. So, for constant T wall, so this is constant T wall. For constant T wall, what is T wall as the function of x, this is T wall constant. What about the T m, see look at this equation d T m d x, so d of T wall minus T m by T wall minus T m is this. So that means these T wall minus T m where is with e to the power minus x, right. Because log of these varies with minus x so this T wall minus T m varies with e to the power minus x.

So, that means T m has an exponential variation with x. So, for constant wall temperature you get assuming that the wall is again heated, this is T m as the function of x. This is an exponential curve. Why exponential curve you can look from the analytical expression. So, if you want to say that the rate of heat transfer is equal to A into h into delta T. Now, the delta T is different. Delta T at x equal to 0 is this, then it becomes this, it becomes this, it becomes this, like this.

So, delta T is continuously varying, so you require some equivalent average delta T. We have shown here by your calculation that average delta T is not this delta T plus this delta T by 2, not, it is these one delta T L by minus delta T 0 by Ln delta T L by delta T 0. This is called as LMTD or logarithmic mean temperature difference. so, why do we require a logarithmic mean temperature difference because the temperature difference itself is continuously varying.

So, what would be the logarithmic mean temperature difference, if the temperature difference is constant let us say that this T wall is the constant and T m is the constant, then what would be the logarithmic mean temperature difference or T wall like this T m like this. This difference is the constant. Then what would be the logarithmic mean temperature difference, no. The logarithmic mean temperature difference physically represents the equivalent temperature difference.

So, if this difference is a constant this constant value itself is the logarithmic mean temperature difference. See if or anything you can do it mathematically. How do you do it mathematically, see if a constant temperature difference is there. Then, it is delta T L equal to delta T 0. So, it is 0 by Ln 1, 0 by 0. You can use (()) (30:16) rule to find out what is the logarithmic mean temperature difference.

But my question is why should we do that from physical understanding we understand that the logarithmic mean temperature difference is the effective average temperature difference between the 2 fluids. If the 2 fluids temperature are continuously varying, you have to judiciously use that formula, but if the temperature difference itself is the constant then the logarithmic mean temperature difference will be that constant itself because it is the equivalent temperature difference.

So, we can say that to summarize so sometimes in short form this is written as delta T LM, logarithmic mean temperature difference, so this is just a short form of doing it, but it is the very important parameter and in one of the latter chapters on heat exchange here, what we learn which is a very important topic for practical engineering applications. This terminology will come over and again. So please make a note this very important terminology.

So, we have discussed about the situations of the constant wall heat flux and constant wall temperature somewhat qualitatively that is how the wall temperature and how the bulk mean temperature varies with x, but question is what is the rate of heat transfer that is what is the Nusselt number. So, we need to discuss about what is Nusselt number for thermally fully developed flow.

So, there may be situations when the flow is hydrodynamically fully developed, but it is not thermally fully developed like let me give you an example.

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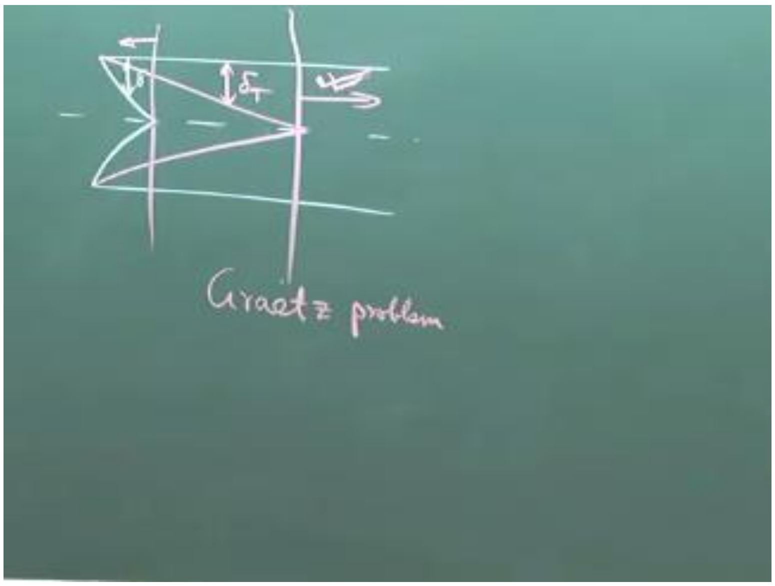

So, this is hydrodynamic boundary length and this is thermal boundary length. So, if you have a fluid say of value of high Prandtl number, then what will happen, this delta will be significantly more as compared to delta T, so the hydrodynamic boundary layer will grow very fast. So, the hydrodynamic boundary layers will merge here and the flow will become hydrodynamically fully developed.

So, from here to here, there is a region when the flow is hydrodynamically fully developed, but not thermally fully developed, right. So, that kind of problem is known as Graetz problem. This kind of problem is actually in the (()) (33:32) advanced level of convective heat transfer. So, we will not come into that here. There is another type of problem where the flow may be both hydrodynamically and thermally developing like this region.

But in this particular course, we will be concentrating on this region where the flow is both hydrodynamically and thermally fully developed and we will evaluate the Nussselt number for that cases.

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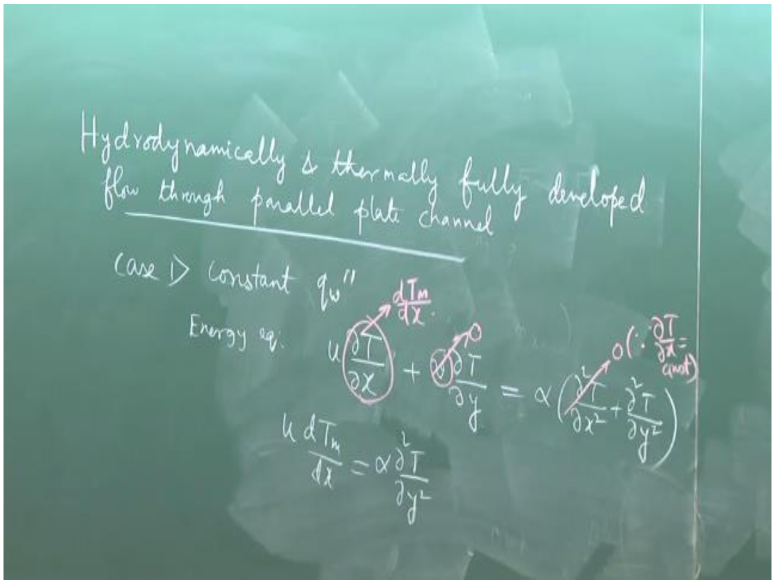

So, hydrodynamically and thermally fully developed flow through parallel plate channel. Case 1, constant wall heat flux. So, what answer can we expect for these types of problems. What are you assured of when you are solving a problem of thermally fully developed flow. We are interested about the Nusselt number. The Nusselt number will be a constant that is what is expected.

So, if we do not come up with the constant Nusselt number that means something is wrong in our analysis or approach. So, the Nusselt number will be a constant and our objective will to evaluate the constant value. So, we have to keep in mind that see what earlier cases we have studied for evaluation of Nusselt number flow over flat plate. The Nusselt number was of the form of Reynolds number to the power something say half into Prandtl number to the power something say half or one third.

This kind of expressions we have seen. So, Reynolds number to the power m into Prandtl number to the power n. In general, for force convection, Nusselt number is actually of the form

Reynolds number to the power m into Prandtl number to the power n, but for hydrodynamically and thermally fully developed flow for internal force convection that is the special case for Nusselt number becomes a constant.

But in general, if you have a situation, even the situation changes if the flow from laminar becomes turbulent, so then I mean you cannot workout those problems analytically you have to deal with sudden correlations based on experimental studies which engineers used for designing a system with the turbulent flow. I will come to 1 or 2 such expressions, but before that remember here we are assuming laminar flow, so constant wall heat flux.

Now let us write the governing equation, the energy equation. This is our governing equation. Now tell what will be the simplification. So, let us write it with different color may be, so in place of delta T delta x for constant wall heat flux, we can write d T m d x, which is the constant, whatever this term, v is equal to 0 for hydrodynamically fully developed flow, so this will be 0. What about this term in the right-hand side, delta T delta x, delta T delta x is what?

Delta T delta x is equal to constant for thermally fully developed flow with constant wall heat flux. Because delta T delta x is constant, delta 2 T delta x 2 is 0. So, you can write u d T m d x is equal to alpha delta 2 T delta y 2. Now, we will change these variables from T to theta, the nondimensional temperature which does not vary with x anymore for thermally fully developed flow.

Why we want to change the variable, because T is the function of both x and y, but for thermally fully developed flow theta is only a function of y, but not a function of x. So, theta is T minus T wall by T m minus T wall. So, you can write delta T delta y, remember T wall minus T m is the functions of x, so with respect to y derivative those are like constants.

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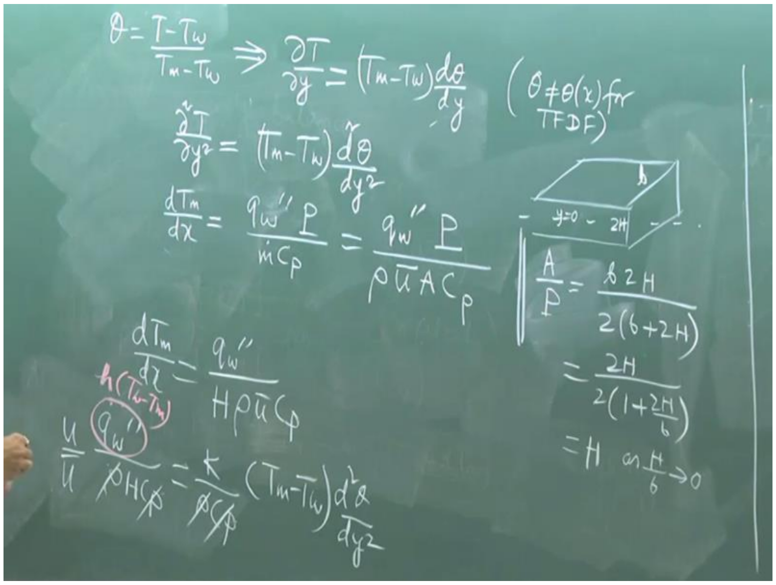

So, similarly the second derivative and what is d T m d x, d T m d x is what is that expression q double dash P by m dot C p, so q double dash in place of P, let us draw a parallel plate channel. Let us say that the width is b and height is 2 H with the center line is y equal to 0. So, what is the perimeter, so before that let us write another step. What is m dot, m dot is rho u average into A into C p by perimeter.

So basically, we have to calculate perimeter by area, so perimeter by area what is perimeter or let us calculate area by perimeter whatever. We will invert that area by perimeter what is that b into 2 H by 2 into b plus 2 H, right. Now, we have to consider a limit value calculating the area by perimeter what is that limit, limit is that h by b tends to 0 because by definition the parallel plate channel the width is infinitely large.

So, what we will do is we will divide both numerator and denominator by b, so this becomes 2 H by 2 into 1 plus 2 H by b, so this will become h as h by b tends to 0, right. So, when h by b tends to 0, this term is zero, this term is 1 and this 2 get cancelled, so it becomes H. So, you can write d T m d x is equal to q double prime into perimeter by area is 1 by h rho u average C p. So, u d T m d x is u by u average.

So, we are writing this equation, this differential equation u d T m d x is u by u average into q double prime by rho H C p is equal to alpha, alpha is K by rho C p into T m minus T wall into d

2 theta d y 2, ok. So, then rho C p gets cancelled. In place of Q wall what we can write Q wall is h into what, h into T wall minus T m.

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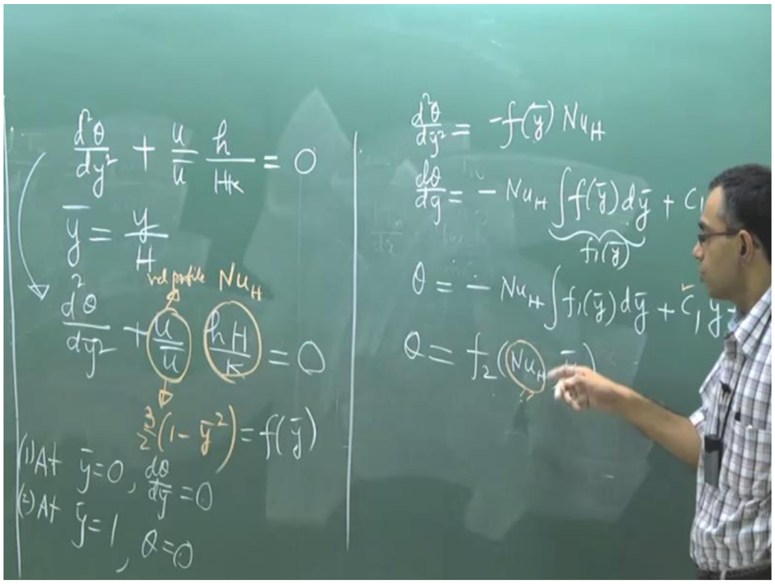

So, we can write d 2 theta d y 2 plus u by u average into h by H K equal to 0, right. Now you can non-dimensionalize like this, right. You can define a new parameter y bar as y by H, ok. So, if you define a new parameter y bar as y by H then this will be d 2 theta d y 2 plus u by u bar h H by K is equal to 0. So, there will be a 1 by H square here that H square coming here will make it h H by K.

So, in a non-dimensional form this becomes what. This is a Nusselt number based on H and what is this, this is the velocity profile. This is where fluid mechanics comes into the heat transfer calculation. So, the answer how theta will vary with y will depend on what will depend on the velocity profile. Now there are many types of velocity profiles which are possible like there may be a case when molten metal is flowing very slowly.

So, in that case the entire velocity profile may be almost uniform and that is called as the plug flow or slug flow. So, if have a plug flow or a slug flow that means uniform velocity profile then what is u by u average 1, right. So that is a very special case, very simple algebra to deal with. I am dot working it out here, but please take it as a homework and complete it by yourself that consider a plug flow with u by u average is equal to1, then complete the remaining derivation.

I will do the derivation for a more involved scenario when u by u average is not equal to 1, but it is a fully developed pressure driven flow that is Poiseuille flow. So, for a plane Poiseuille flow what is the velocity profile u by u average is equal to 3 by 2 into 1 minus y square by 8 square this was derived by Prof. Som when he was discussing about the exact solution of Navier-Stokes equation.

So, you substitute that here, so this is a like from the fully developed flow Navier-Stokes equation you can derive. There is no problem associated with this. So, now what are the boundary conditions, this is y bar, this is non-dimensional y. What are the boundary conditions at y bar equal to 0, what is the boundary condition? Y bar is equal to 0 is the center line, what is the boundary condition for theta at the center line.

Top wall and bottom wall are having symmetric boundary condition, so d theta d y equal to 0 at y equal to 0, it is the center line symmetry and at nondimensional y is equal to 1 that is the wall, what is theta. Remember the definition of theta, T minus T wall by T m minus T wall, so theta is 0. So, with this you have d 2 theta d y 2 is equal to some function of y. Let us say this is f y bar into Nusselt number with minus sign.

So, d theta d y is equal to minus Nusselt number, integral of f y d y bar plus some constant C 1, so when you integrate you will get theta is equal to, so this is let us say f 1 y, right. I have just written it generically. This integration is very easy because this is a simple polynomial, I do not waste time by doing the integration. I want to give you the frame work in which you can just put the numbers and values.

So, you can evaluate the constants C1 and C2 by using this 2 boundary conditions, but does this solve the problem. You have an expression for theta is equal to a function of some function f 2 Nusselt number and y. These are all nondimensional y, so make this as y bar, but this does not tell you exquisitely theta as a function of y because you do not still no what is Nusselt number, what is a value of the Nusselt number?

In fact, obtaining that is the objective and remember that although I have written it as a function of Nusselt number and y, actually Nusselt number is separated from the function of y. It is Nusselt number into some function of y. I mean it is not mixed with y. Nusselt number is outside this integration. So, it will come out to be Nusselt number into some function of y. Now you might argue that yes, we have used 2 boundary conditions for a second order differential equation.

So, what makes the situation that still the problem is not completely solved. One of the reasons is that actually this boundary condition theta equal to 0 at wall is not actually giving you any new information. Because theta by definition is T minus T wall by T m minus T wall, so at wall theta will always be 0 that is the definition of theta, but it does not depend on whether it is constant wall temperature or constant wall heat flux whatever.

It is just a simple English sentence T equal to T wall at wall, so nothing more than that. So, that is what that has been utilized here. So, we need to close this problem by putting some additional constraints what is that additional constraint that additional constraint is the definition of bulk mean temperature which we have not yet used. We have not yet used the definition of the bulk mean temperature.

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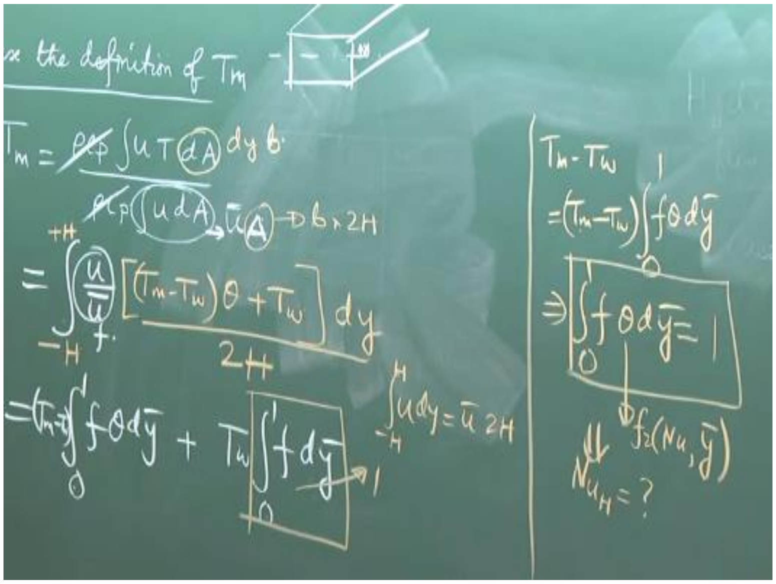

So, now use the definition of T m. The rho C p is constant for this problem. So, we cancel the rho C p part. So, we can write what is integral of u d A that is u average into A. So, we have integral of u by u average what is d A, d A is so if we have a parallel plate channel like this so what is d A, so at a distance y you take a small strip of a width d y. D A is d y into b, so d A is d y into b and A is b into 2 h. So, integral u by u average in place of p you can write T m minus T wall into theta plus T wall.

You can change the variable from y to y bar by putting this H within the derivative. So, it will become d y bar. This is the function f and integral from minus H to H is 2 into integral of 0 to H, because it is symmetric in the 2 sides. So, that 2 and this 2 will cancel. So, this will become integral of f theta into T m minus T wall d y plus T wall, right. We have absorbed the y by H within the derivative.

So, if that be the case then can you tell what is integral of f d y, nondimensional, see f is what f is the velocity profile. So, integral of f d y non-dimensionally it will become 1 from zero to 1, right. Because integral of u d y is u average into the height, so u by u average into integral of d y nondimensional is 1. So, this is from the condition that u integrally u d y is equal to u average into 2 H, so u by u average integral, this will become 1. So, this is equal to 1.

So, we can say that T m minus T wall is equal to T m minus T wall into integral of f theta d y that means integral of f theta d y equal to 1, this is the constraint. So, theta will be now you can write f 2 of Nusselt number and y bar. The functions f and f 2 are completely known, so this will tell you what is Nusselt number based on H. Now normally we can calculate the Nusselt number on the basis of H, but engineers refer to use a length scale which is called as hydraulic diameter.

So, what is hydraulic diameter, if have a circular pipe you have the actual diameter, but you do not have a circular pipe, there is nothing called a physical diameter, but something equivalent to the physical diameter is called as the hydraulic diameter. So, for as circular pipe what is the physical diameter.

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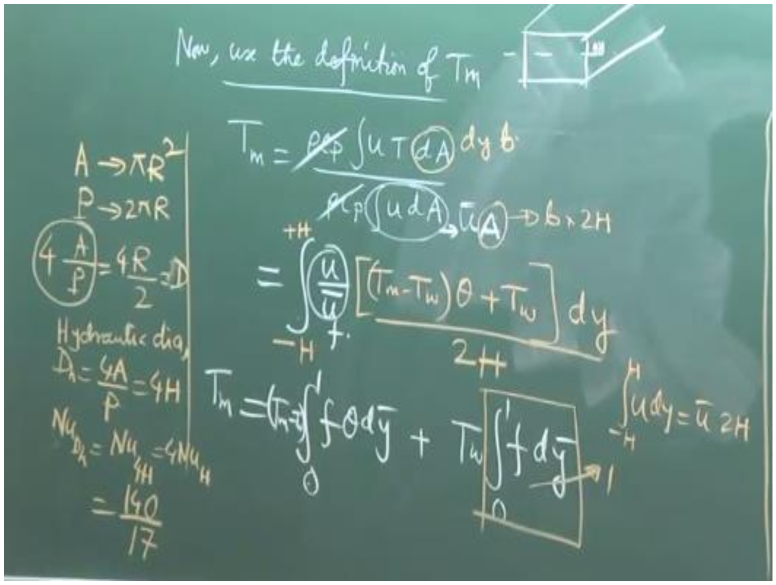

So, what is the area, area is pi r square. What is the perimeter 2 pi r, so what is area by perimeter? So how can you get diameter from here, multiply by 4, so these becomes diameter, ok. So, 4 into area by perimeter for a circular pipe is the actual diameter. For a noncircular geometry that is some effective diameter which is called hydraulic diameter. So, hydraulic diameter let us write it here is 4 into area by perimeter.

So, what will be the hydraulic diameter for a parallel plate channel the half height is H. Area by perimeter we have already calculated what was that H, so 4 area by perimeter is 4 H. So, the Nusselt number based on hydraulic diameter is the Nusselt number based on 4 H that is 4 times the Nusselt number based on H and if you evaluate it all these numbers I have given the outline, but if you now evaluate the value this value will come out to be 140 by 17.

So, this is your homework 140 by 17, okay. So, I have given the full frame work, in this frame work just you have to do the integrations, integral A f 1 f 2 all this and once you substitute, you will get the value of this, so we can see that the Nusselt number is the constant for thermally fully developed flow for this example, this is the value of the constant. We will stop here and we will continue in next lecture.