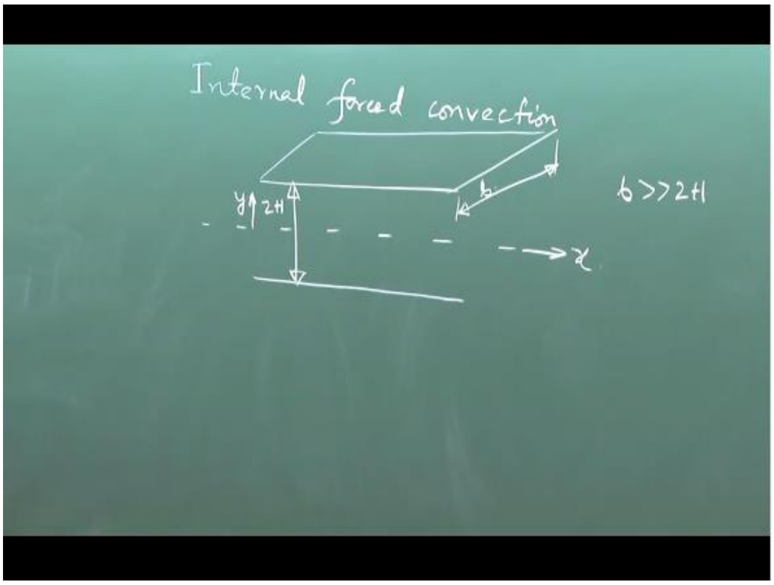
Conduction and Convection Heat Transfer

Prof. S.K. Som
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 31
Internal Forced Convection - I

In the previous lecture, we were discussing about flow and heat transfer over a flat plate. Now, today we will be discussing about internal flows. So, when we will be discussing about internal flows, we will briefly discuss about the important of fluid mechanics issues and then of course, we will give some more time on heat transfer issues with internal flows.

(Refer Slide Time: 00:58)



So first, we will try to understand what basically happens with internal flows. So, we are now studying internal forced convection. So, let us for simplicity consider something which we call as parallel plate channel. So, what is a parallel plate channel? Parallel plate channel is essentially 2 parallel plates where the width of the plate is like infinitely large as compared to its other dimensions.

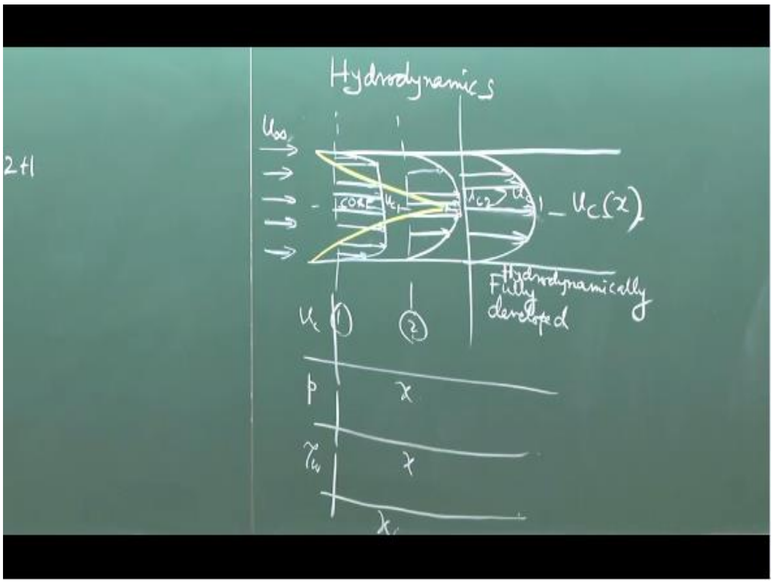
So, if you have a channel like this. So, this dimension let us call it b, let us call this as 2H, so b is much greater than 2H. So that it is basically a flow taking place in the x y plane. The z direction being infinitely large. It means that the gradient in the z direction is much negligible as compared to the gradients in the x and y direction. Now, this is a simplified paradigm**,** but it gives a very important physical insight.

Now, in reality even for rectangular channels, if the width of the rectangular channel is much larger than the height we can consider it like a parallel plate channel, although it is not physically 2 parallel plates, but in effect if the width is much larger than the height then it is like a parallel plate channel.

So, we will discuss about parallel plate channels and circular tubes or circular pipes, which are of importance in engineering and although the mathematics involved with the circular pipes will be different from that of a parallel plate channel simply because of the change of coordinate systems from Cartesian to the cylindrical polar coordinate system. But essentially, the concept, the physics is almost the same.

So, we will try to understand that carefully. So, first we will consider the hydrodynamics.

(Refer Slide Time: 03:33)



Let us say that fluid enters this channel with a velocity u infinity and temperature T infinity, but I am not writing the temperature issue here. Because here we are discussing only the hydrodynamics first. So, just as in case of flow over a single flat plate, for flow over, flow within a parallel plate channel. Essentially it is like flow confined between 2 individual plates.

So, the boundary layers will develop on individual flat plates. So, the boundary layers are developing like this and because of symmetry with respect to the centre line, the boundary layers will meet somewhere on the center line of the channel. Now, let us try to figure out the velocity distribution in here. Let us say 2 sections 1 and 2. So, you can see that within the boundary layer the velocity profile will be there then the velocity will be uniform.

So, the region where the velocity is uniform, this is called as the core region, outside the boundary layer - CORE region. Now, if you try to draw the velocity profile in section 2, what will happen? Now, let us say in the core region at section 1, the velocity is u c 1. Now, my question is that in the core region of section 2, if the velocity is u c 2. Is u c 2 equal to u c 1? Is it greater than u c 1 or is it less than u c 1?

So, let us assume that it’s a constant density flow. So, what remains constant between the sections 1 and 2 is the volume flow rate. The mass flow rate is constant, but if rho is constant that is equivalent to volume flow rate is constant. How do you get the volume flow rate? You get the volume flow rate by integrating the velocity profile. You get the volume flow rate by integrating the velocity profile.

So, integral of the velocity profile here must be same as the integral of the velocity profile here. Now here there is a greater region over which the fluid is slowed down. Here the fluid is slowed down only over this distance. Here the fluid is slowed down, over a greater distance. So, when the fluid is slowed down by a greater distance, what will happen? To compensate for this slowing down, the core velocity u c 2 must be greater than u c 1.

To make up for this slowing down over a larger distance right. So, one important thing we can follow is that one very key difference between internal and external flow is: In external flow, there is a u infinity which is a constant. Here, there is no constant u infinity. There is a centre line velocity or the core velocity, the core velocity is continuously changing with x. So, u c is a function of x. Alright, so it is not like u infinity which is a constant.

Now, let us try to schematically draw u c versus x, P versus x and tau wall versus x. So, u c versus x we have seen that the core velocity increases as you are moving along x but to what extent? How long? There is a situation when the boundary layers merge. After the boundary layers merge, if you want to get the velocity profile. The velocity profile becomes something like this.

This velocity profile is called as fully developed velocity profile because we are going to discuss about 2 different boundary layers: thermal and hydrodynamic. This is more specifically characterized as hydrodynamically fully developed flow. So, when the flow hydrodynamic becomes hydrodynamically fully developed, I mean it may appear that just when the boundary layers merge it becomes like that.

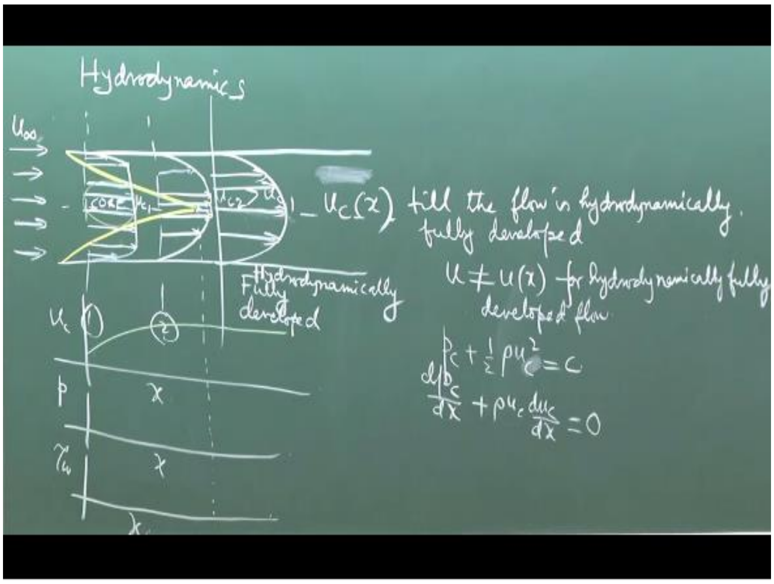
But in practice it takes a little bit of distance from there to become hydrodynamically fully developed, but at least for the under-graduate level without going into that complexity we will assume that after the boundary layers are merged the flow has become hydrodynamically fully developed. So, what is the big essence of hydrodynamically fully developed flow? So, here in the core region what happens?

The effect of the presence of the wall is not propagated. The effect of the presence of the wall is propagated only up to this distance. But, when you come to hydrodynamic fully developed flow, the entire fluid fills the effect of the wall, ok. And because the entire fluid now fills the effect of the wall. The velocity profile from now onwards does not change any more with x. So, hydrodynamically fully developed u c is a function of x till the flow is hydrodynamically fully developed

And u is not a function of x for hydrodynamically fully developed flow. So, with this concept now let us try to draw this graph of u centre line versus x. So, how will the graph look like? Let us assume that the flow has become hydrodynamically fully developed here. So, u centre line versus x. How will the graph look like? Or u core versus x? How will the graph look like? The core velocity increases till the flow becomes fully developed.

After it has become fully developed, it does not change any more right. What happens with pressure versus x or d p d x versus x, whatever? So, remember that in the core region, we can use the Bernoulli's equation. Because the core region does not feel the effect of the viscous force, right. So, this is for the core region.

(Refer Slide Time: 14:08)



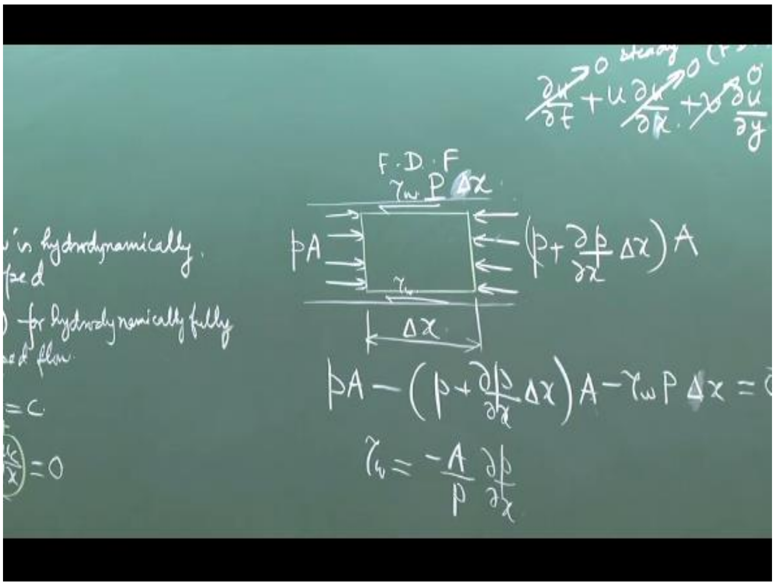
So, if you now differentiate with respect to x, d p c d x right. So, you can see what is this? What is d u c d x? positive or negative? Positive, u c is positive. That means d p c d x is negative. So, it shows that the pressure continuously decreases as the fluid enters the channel. Remember that this pressure may include gravity. So, this is here we have not considered the effect of gravity assuming that it is not important or we have put the gravity within the p.

So, the p is p plus rho g z that is called as piezometric pressure, ok. So, this p versus x is the piezometric pressure versus x, it may include the gravity part. That is why in the Bernoulli's equation separately the potential energy term, we have not reaching. Because it may be clubbed with the pressure term, so pressure versus x. Now, after the flow has become fully developed what is the situation? D u c d x will be 0. So, d p d x will be 0.

That means p will be - so p will be yes, why are you accepting what I am saying? This is the. see, I am saying that when the flow has become fully developed, this will be 0 and you are accepting it. When the flow has become fully developed the viscous effect has penetrated to the centre line. So, the Bernoulli's equation is no more valid, ok. So, you cannot use this equation after the flow has become fully developed.

This equation is only till the flow has become flow is becoming developed. So, when the flow has become fully developed what happens? So, let us try to make a talk about the fully developed flow.

(Refer Slide Time: 16:30)



The mathematics of the fully developed flow, the exact solution of the navier stokes equation has already been worked out by Professor Som. I am not getting into the exact solution of the navier stokes equation for fully developed flow, but I am trying to give you some physical and qualitative understanding which may be useful for our discussion. So, let us take a small volume of fluid.

Let us say if this is the channel so we have got convinced that we should not use this equation when the flow has become fully developed because viscous effects now penetrate to the core. So, you cannot use the Bernoulli's equation any more. Otherwise, it gives something upside. You see if this is 0 then d p c d x is 0. That means p c is the constant. If the pressure is the constant how it will drive a flow? Right.

So, if you take a fluid element like this. So, here if the pressure is P let us say the cross-sectional area is A, what is the pressure here? P plus let us say this length is delta x. P plus del p del x into delta x into A. This is the force due to pressure. What are the forces acting on this? This the fully developed flow. What are the forces which are acting on this? So, a fully developed flow means what? So, if you look into the acceleration of flow.

If it is a 2-dimensional flow so u del u del x let us say it's a steady flow. So, steady flow this term will be 0. So, this is the acceleration term right. Rho into this is the mass into acceleration per unit volume in the navier stokes equation. So, this is 0 for steady flow. What about del u del x? del u del x is equal to 0 for fully developed flow. When we say fully developed we are meaning here hydrodynamically fully developed.

When we come to thermal boundary layer, you have to be care full in distinguishing between hydrodynamically fully developed flow and thermally fully developed flow. They are 2 different concepts all together. Now what about this term? So, we can evaluate this term by looking into the continuity equation. So, this is 0 for fully developed flow. That means v is not a function of y. So, if v not a function of y, what can you say about v?

So, if you consider the y direction, consider this figure. What is the value of v at y equal to h? 0. Y equal to minus h also it is 0. That means because v is not a function of y, if you can find out the value of v at a particular value of y that should be true for all values of y right. That means v is 0 for all values of y. So, but if there were holes in the plate, there are technological situations in which you make holes in the plate. Then v is not 0 at the wall.

So, assuming that there is no penetration at the plates or no penetration at the solid boundaries, you will have v equal to 0 at wall. So v is equal to 0 at wall, this means v is equal to 0 for all y. So, if v is 0 then this term is 0. So, the entire, in the Navier-Stokes equation, the left-hand side becomes identically 0. So, the non-linearity of the Navier-Stokes equation is gone.

So, it becomes a linear equation which is also called as the strokes equation which you can solve by analytical tools. Now when the, so physically what is happening? Physically the flow is not accelerating. So, a fully developed flow physically is not accelerating. Until and unless there is a penetration at the solid boundary. If there is no penetration at the solid boundary that means a fully developed flow is not accelerating.

So, if the fully developed flow is not accelerating then what is happening? All forces are balanced that is why it is not accelerating. So, what are the forces acting on it? one is due to, so if you take an element like this a control volume like this one force is due to the pressure, other force is due to the wall shear stress. So, these 2 forces they are balance each other, so that there is no net acceleration.

That is the physics of a fully developed flow. So, if you have tau wall as the wall shear stress. So, you can write the force as tau wall into perimeter into delta x, right. So, for example if it is for the circular pipe what is the shear force? Wall shear stress into 2 pi r into l, right. So, the perimeter is 2 pi r. So, to make it general, so that you can apply it for any cross section we are writing tau wall into perimeter into length. Perimeter for a circular pipe is 2 pi r.

That is easy to visualize. So, now these forces are balanced. So, P into A minus P plus. So, you can write tau wall, right. Now, how do you calculate tau wall for a fully developed flow? For a fully developed flow, you get a velocity profile which is a function of y for a rectangular or a parallel plate channel and its function of, or for a circular pipe. So, depending on whether it’s a parallel plate channel or a circular pipe.

You have to calculate mu d u d y at the wall or mu d u d r at the wall. So, that will give you tau wall. So, question is, Is it a constant or not a constant? So, u is a function of y for a fully developed flow. So, d u d y is a function of y. When you put the value of y equal to y at the wall then that becomes a constant right. So, if the velocity is the function of y only then tau wall is the constant.

If velocity is the function of y only then tau wall is the constant. That means this is also a constant. That means for fully developed flow del p del x is the constant. That means del p del x becomes d p d x. So, p versus x is linear. So, p versus x when the flow is not fully developed is non-linear, but when the flow is fully developed it is linear, so this is linear. Tau wall versus x, the wall shear stress should be constant for a fully developed flow.

Because tau wall is mu d u d y at the wall, u is a function of y only. Sob d u d y at the wall is the constant. So, fully developed flow it’s a constant, this is constant. This is also constant. In the developing region, now there are two possibilities, one is it increases to this constant value or it decreases from a different value to this constant value. So, what it will be? So, tau wall scales with mu u c by delta right.

Mu d u d y is order of magnitude wise, mu into u centre line or u core by delta. Where delta is the local boundary layer thickness. This is delta. So, the interesting thing is u c increases with x right in the developing region, that we have seen. Delta also increases with x right. The boundary layer grows. So, what happens to this ratio? Right. You have a ratio where the numerator increases with x, the denominator also increases with x.

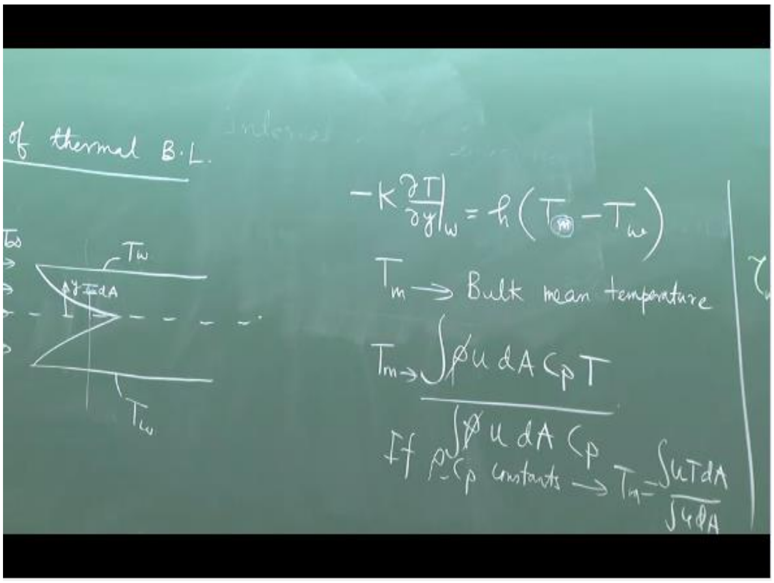
So, what happens to this ratio? See, that you can tell from a very intuitive thing, that when you start entering the channel what is delta? Delta is tending to 0, but u c is finite. So, tau wall will be very large as you enter the channel. After the both delta and u c are finite. That means it should asymptotically decrease from a very large value to this value as. So, this is large. Large tau wall then it comes to a constant.

See these 3 graphs, I will tell you that it is very important that as students of fluid mechanics, you should be able to sketch those qualitatively. See, you may be very strong in solving complex differential equations. That is very important but that is not all in learning fluid mechanics. So, this we totally develop these concepts by looking into the physics of the fully developed flow.

Without getting into the Navier-Stokes equations their exact solution and all these. At this kind of physical insight, you should try to develop in a… like understanding the fluid mechanics and heat transfers. So, when we are interested about the hydrodynamics in parallel we should also look into the issues of heat transfer. So, we will look in to that now. Just like the hydrodynamic boundary layer growing.

We will also have the thermal boundary layer growing. So, important take home message from this part of the analysis is that for the hydrodynamically fully developed flow the velocity profile is not a function of x, it is a function of y only. In the developing flow, the core velocity continuously varies with x, there is nothing called as u infinity.

(Refer Slide Time: 30:59)



Growth of thermal boundary layer. So, let us say that this fluid which was coming with a velocity u infinity was also coming with a temperature of T infinity and the wall temperature T wall. You may argue that the 2 walls may not be at the same temperature at the undergraduate level, we will not be entering into that complication. We will assume that it is symmetric with respect to the thermal problem also.

That means the 2 walls are at same temperature. But the temperature need not be constant, it may be a function of x. So just like the growth of hydrodynamic boundary layer, the thermal boundary layer also grows. Now, we have already learned that there is nothing called as u infinity in these type sub problems and analogously there is also nothing called as T infinity. T infinity is only outside the channel.

Once the flow enters the channel, the effective velocity of the fluid, the core temperature continuously changes and it is not T infinity. So, question is for internal flows when you write minus K del T del y at the wall, this is equal to h. So, we are writing a heat transfer equation say from fluid to the wall. So, we are writing the wall temperature. So, it is different from that of the fluid temperature.

So, we are writing the heat flux minus k del T del y at the wall is equal to h into this temperature difference. So, T, this is not T infinity. What is this? That we will discuss. What is this? So here just because the y axis is directed positively upwards, that is why this is this difference, otherwise if the y axis was from the plate down then it would have been T wall minus this. So, it is just because of the axis issue.

But the more important is, what is this T? Now this T is not T infinity because T infinity is no more the referenced temperature in the internal flow. Just like u infinity is no more the referenced velocity for internal flow. T infinity is no more the reference temperature for internal flow. So, if T infinity is no more the reference temperature for internal flow then what is the reference temperature?

That reference temperature is called as T m, which is bulk mean temperature. What is bulk mean temperature? Let us say that you have a fluid in a particular section. This section has non-uniform temperature. Now if you mix this entire fluid in a particular section very thoroughly so that the entire fluid comes to a uniform temperature in a particular section with the same thermal energy as that of the previous case. Then that equivalent uniform temperature is called as the bulk mean temperature.

So, the entire fluid in a particular section mix to come to a cross-sectionally uniform temperature. So, what is the utility of this concept? See, actually this problem is like a 2-dimensional problem. If it is a parallel plate the third dimension is not important. By introducing this bulk mean temperature, we are converting it into a cross-sectionally average problem or a 1 dimensional problem.

Because if we are using a bulk mean temperature that means we are no more interested with the variation in the y direction. We are using a temperature which is already cross-sectionally averaged out. So, that will now depend only on x. So, what is the definition of this? Now, what is the thermal energy of the fluid? Think of the first law of thermodynamics. If you have a flowing system, if you have a fluid that is flowing,

What is the thermal energy of it? In a first law energy balance equation, what is the thermal energy component that you write? M dot into h mass flow rate into specific enthalpy, right? So, for a, incompressible fluid that we are discussing here the h we can write as c p into T. So, now what is it? the T is not uniform over the entire cross sections, so what you do is that at location y, you take a small area d A.

So, what is the mass flow rate through this d A? rho u d A. This is m dot into c p into T. This integrated is the total thermal energy. So, that divided by m dot C p will give you some p average. This is integral of m dot C p T. This is integral of m dot C p. So, this will give you some sort of cross-sectionally average T. So, cross-sectionally average T is not integral of T d a by integral of d a. It is weighted with the velocity.

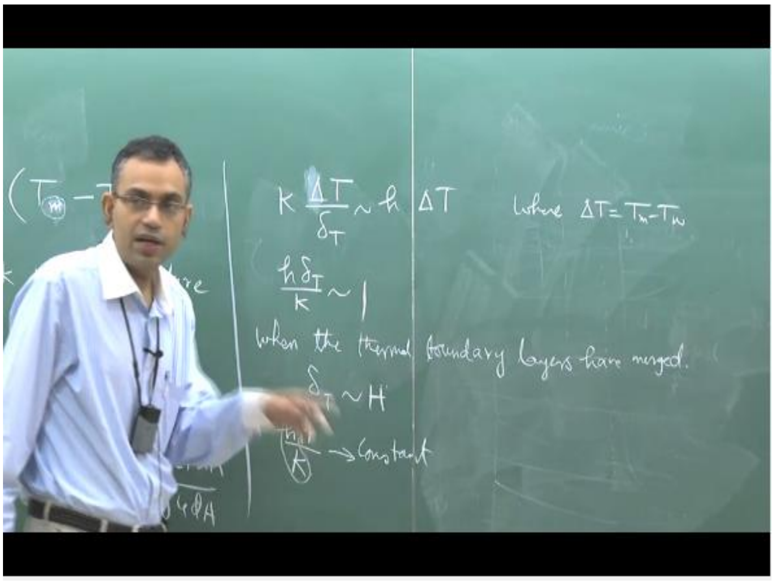
Because when you are calling it as a bulk mean temperature, it is also called as mixing cup temperature. The factor that is creating the mixing is the fluid flow. So, it has to be weighted by the velocity. So, when you write the thermal energy of a flowing system, it is not independent of velocity that is where convection is related to fluid mechanics. So, when you write m dot into h, in that m dot, the velocity is there.

So, you cannot write the average temperature without referring to the velocity. So, this is called as the bulk mean temperature, but if you have rho constant for this particular problem and C p also not changing. So, if rho C p are constants then T m is equal to integral of u T d A by integral of u d A. So, now so we can understand this T m is a function of what? X or y? it is a function of x because it is already integrated over y, so this is a function of x.

T wall is a function of x. T is a function of both x and y. T is a function of both x and y. but T m is a function of x, t wall is the function of x. So, h can be a function of whatever. So, del T del y at wall when you calculate so this T is a function of x y. So, del T del y at the wall is the function of what? x. So, this is the function of x that means h in general is the function of x, right.

Now, let us try to write an order of magnitude expression for this. So, let us say that delta T at a given x is the thermal boundary layer thickness.

(Refer Slide Time: 42:00)

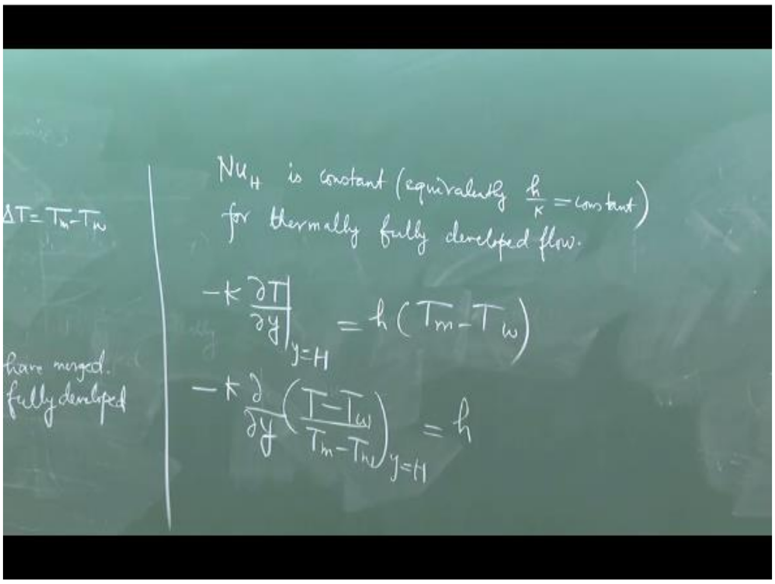


So, we can write K into h delta T by K is of the order of 1, right. It is of the order of 1, but it is not a constant. When it will become a constant? When delta T does not change anymore. That means when the 2 thermal boundary layers have merged. When the 2 thermal boundary layers are merged then delta T is half height of the channel right. Or for a circular pipe, it is the radius of the pipe.

So, when the thermal boundary layers are merged or met this is the half height of the channel H. So, then you can H by K, this is a constant. See why it is so? Because H by K is of the order of 1 by delta T and when the delta T becomes a constant? H by K becomes a constant and H by K becomes a constant means H by K into a constant length is also a constant. This is a non-dimensional way of representing H by K.

So, it is basically this H by K, that is a matter of interest, but non-dimensionally when expressed this is actually what? This is the Nusselt number. So, the Nusselt number becomes constant for a thermally fully developed flow. Thermally fully developed flow means the 2 thermal boundary layers are merged. So, this is called as thermally fully developed flow.

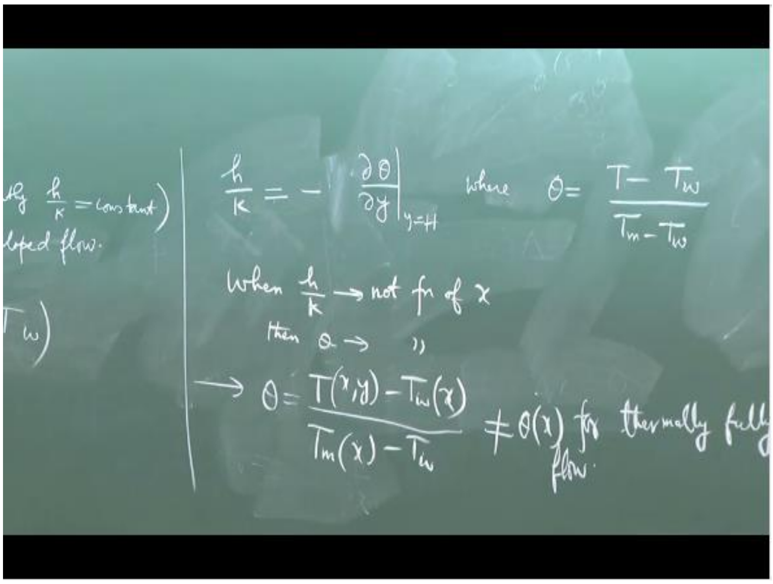
(Refer Slide Time: 45:11)



So Nusselt number based on H is constant equivalently H by K is constant for thermally fully developed flow. Now, we will try to manipulate with this equation minus k del T del y at y is equal to H is equal to H into T m minus T wall. So, in place of this we can write minus K del del y of T minus T wall. See, this is a y derivative because this is a y derivative any function of x is like a constant with respect to it.

So, we have taken this within the derivative. These are all functions of x and T wall is also a function of x.

(Refer Slide Time: 46:40)



So, we can write that h by k is equal to minus k del minus theta del y at y is equal to h. Where theta is T minus T wall by T m minus T wall, right. So, when H by K is not a function of x then theta is also not a function of x, right. Because this may be function of y, because at y equal to h, you will get a value which is not dependent on y in any way, but it is a function of x in general.

But for thermally fully developed flow this is not a function of x therefore this also should not be a function of x. If del theta del y is not a function of x that means theta is also not a function of x. So, in general to generalize it we should not think of special functions. But generalizing, that we can say that if del theta del y is not a function of x then in general, theta should also not be a function of x.

So, theta is not a function of x, this is another hallmark of thermally fully developed flow. So, this is 1 point Nusselt number is constant. The second point is theta is equal to T which is a function of x y minus T wall, which is a function of x by T m minus T wall. This is not equal to a function of x for thermally fully developed flow.

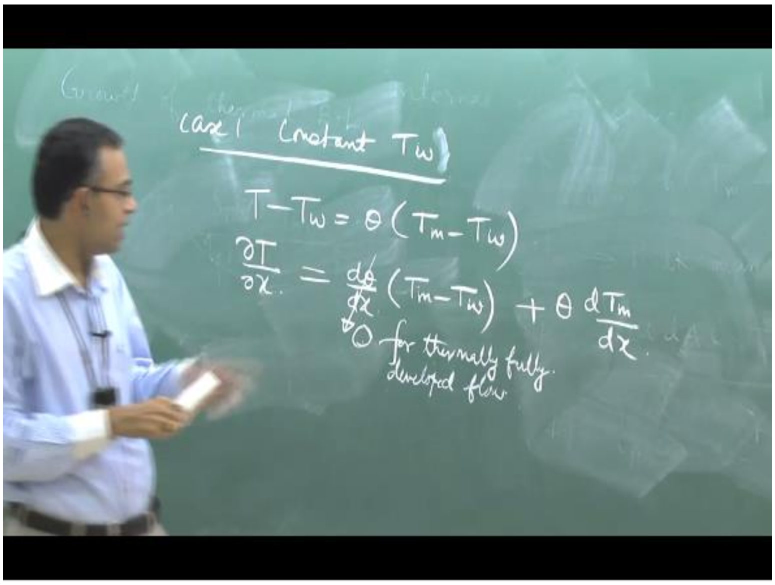
Now, you can clearly see that this does not mean that T is not a function of x. So, the reason is quiet clear. Let us say that you have 2 parallel plates like this. You have thermal boundary layers growing and now you go on say heating the plate. This is heat flux. So, if you go on heating the plate, how can it happen that the temperature has become a constant? So even if the thermal boundary layers have merged, temperature can still vary with x.

So, this is a very important difference between hydrodynamically and thermally fully developed flow. For hydrodynamically fully developed flow, u is not a function of x, but for thermally fully developed flow, T still remains a function of x, but this definition of non-dimensional temperature theta that is not a function of x. So, it is not a very intuitive thing because all these are: this has function of x y.

This is function of x, this is function of x, this is also a function of x, but this ratio is no more a f unction of x for thermally fully developed flow and why we have just proved it. Now, we will have some observations with regard to 2 different boundary conditions which we will discuss in this course: one is a constant wall temperature, another is a constant wall heat flux. So, constant T wall what is the consequence?

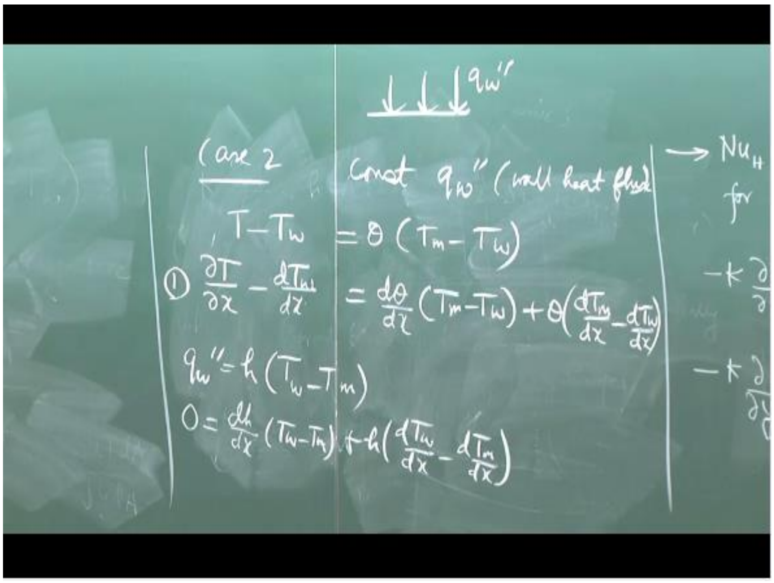
So, this is case one. Constant T wall, you have T minus T wall is equal to theta into T m minus T wall. Differentiate both sides with respect to x. So, this becomes del T del x right. What is the derivative with respect to x? 0 because T wall is constant is equal to d theta d x. So, this d theta d x is 0 for thermally fully developed flow.

(Refer Slide Time: 51:14)



So, you can write del T del x is equal to theta into d T m d x. When we will solve the energy equation, you will see that in the energy equation, you have term mu del T del x. So, in that del T del x, we will substitute theta into d T m d x. Then, the next important case which we will discuss in length over this course is the constant wall heat flux boundary condition.

(Refer Slide Time: 53:04)



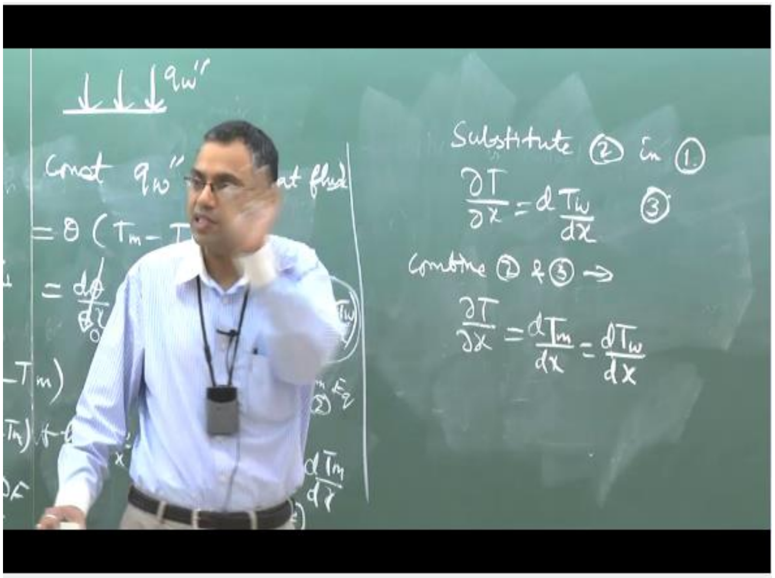
So, here you can write again T minus T wall is equal to theta into T m minus T wall. So, del T del x minus d T wall d x. Now, T wall it may be a function of x. If always remember that on a surface you cannot control the temperature and heat flux simultaneously. So, if you are controlling the heat flux, temperature will vary in its own way. It will be some function of x. Not that you can maintain it both isothermal and constant flux that is not physically possible.

So, you have del T del x minus d T wall d x is equal to. Now, what is the wall heat flux? Let us say, this is equation number 1. What is the wall heat flux? This is equal to h into. So, if you are directing the heat flux in this direction at the top wall then it is h into T wall minus T m, right. So, constant wall heat flux if you differentiate with respect to x, 0 is equal to d h d x into T wall minus T m plus h into d T wall d x minus d T m d x, right.

Now for thermally fully developed flow, one important thing. See these are very subtle concepts. For thermally fully developed flow, can we tell h is a constant? We can tell h is a constant provided case also a constant because the very basic premise of the thermally fully developed flow is H by K is a constant. But often when h is constant we say the flow is thermally fully developed and that is grossly valid, provided k is a constant.

So, if we assume that k is a constant then this is 0 for thermally fully developed flow if k is constant, right. So, if this 0 then that means you have d T wall d x is equal to d T m d x. This is equation number 2. Now, you substitute equation number 2 in equation number 1.

(Refer Slide Time: 56:30)



So, d T wall d x equal to d T m d x means this term will be 0 and this term is 0 for thermally fully developed flow. Because theta is not a function of x, right and this term becomes 0 from equation number 2. So, what is left? The left-hand side is 0 that means del T del x is equal to d T wall d x. This is equation 3. If you combine equation 2 and 3, you will get del T del x equal to d T m d x is equal to d T wall d x, right.

This is plain and simple comparing a (()) (57:42) combination of 2 and 3. Can we tell anything more about this? Just these 3 are equal. Can we go beyond that and tell something? These are not merely equal, but this shows that this is a constant, why? This is a function of x and y and this and this are functions of x. In general, if you have a general function of x and y is equal to a general function of x.

General function you have to keep that in mind, not specific function. So, this could be any arbitrary function of x y. A general function of x and y is the general function of x. This equality is possible only if h is a constant, ok. Otherwise, that equality may be valid only for special cases, but not for general functions. So that means not only these 3 are equal, but h is equal to a constant.

So, to summarize our discussion today for the constant wall temperature you can write del T del x is equal to theta into d T m d x for thermally fully developed flow with constant wall temperature. With constant wall heat flux, this is the condition. So, we have got expressions for del T del x in terms of the bulk mean temperature gradient. So that essentially has made one thing possible, mathematically what heat has given us?

That heat has given us an equivalent 1 dimensional formulation of the problem. Instead of del T del x, now we can use d T m d x or d T wall d x. Those are 1-dimensional parameters and similarly for constant wall temperature. So, we will take it up from here and we will continue in the next lecture. Thank you.