Conduction and Convection Heat Transfer
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Lecture – 28
Energy Equation - II and Thermal Boundary Layer – I

We continue with, what we were discussing in the previous lecture. So, in the previous lecture, we were attempting to derive the energy equation. So, what are the steps that we followed? We first derive, the total energy conservation then we subtracted that the mechanical energy conservation and then we got the thermal energy conservation equation.

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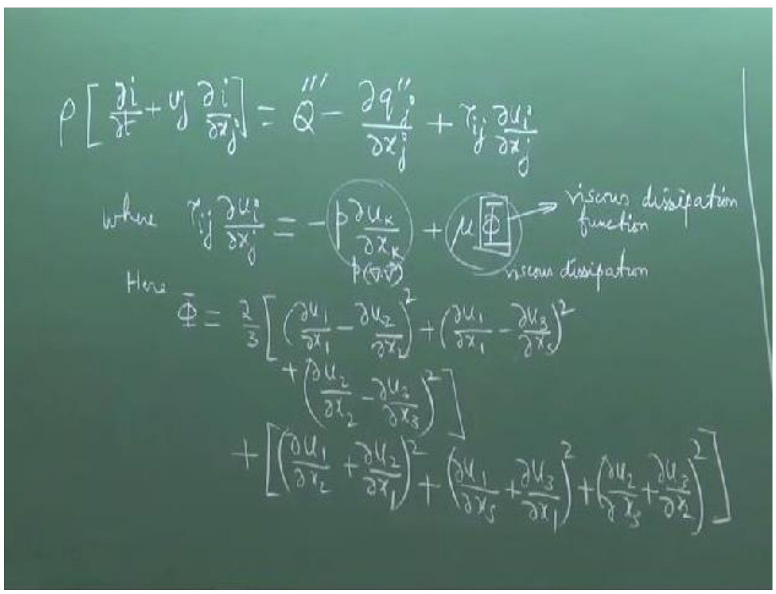

That equation is written here with i as the internal energy so various terms this is like the total rate of change of internal energy. This can be attributed to volumetric heating, surface heat flux and viscous dissipation. So, one part is viscous dissipation and this is pressure work or PDVP, P dot delta v, sorry p into delta dot v. So, p into Delta dot v that term is there which is just like the PDV work in thermodynamic. So, this is due to volumetric change whatever is the work done.

So that is p into delta dot p and this is called as viscous dissipation. This is due to irreversible conversion of the work to overcome the viscous interaction into intermolecular form of energy. And we have shown that this viscous dissipation for a Newtonian and Stokesian fluid is always a positive contribution that is it always gives rise to heating and not cooling. So, the expression that we drive towards the end of the previous lecture that summarizes to this expression.

You can see very well that this phi is called as viscous dissipation functions. So, this viscous dissipation function is a function of basically the sum squares of the velocity gradients or rates of deformation. So, if you know the different rates of deformation then it is some of the squares of rates of deformation plus some of squares of other terms involving the velocity gradients. So, velocity gradients are responsible for this term along with the viscosity of the fluid.

Now, we will complete the derivation of the energy of the energy equation by expressing the internal energy in terms of enthalpy and then enthalpy in terms of temperature. Because temperature is the measurable parameter so we should express the governing equation in terms of temperature. Now, to do that the left-hand side

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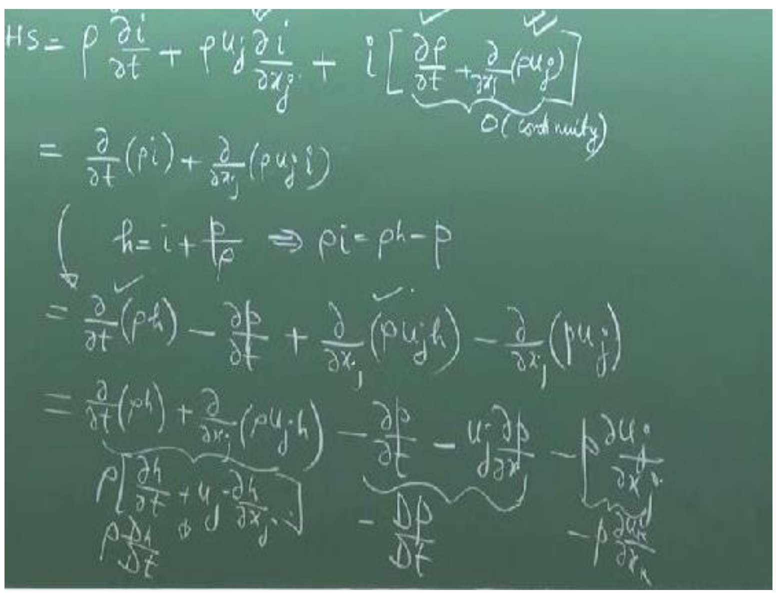

You can write –what we have done is see this trick, we will often play. We will convert as per our wish from none conservative form to a conservative form and then from a conservative form to a nonconservative form. So, in any of these cases we have to use the continuity equation. So, this is the continuity equation. This is actually zero by continuity equation.

Now, you can combine these terms and write and combine these terms. So, you can see that in one step we can convert the none conservative form to the conservative form. Then what we will do, we will write the internal energy in terms of enthalpy. So, h equal to i plus p/ rho that is the definition of enthalpy h equal to u plus pv that is written in different symbols. So, rho i equal to rho h minus p. So, we can club these two terms. This two terms again I will not repeat.

But what you can do is you can use the continuity equation to write it in none conservative form. These two terms correspond to the conservative form with enthalpy plus you have some correction terms for converting internal energy to enthalpy. So, these two terms together can be converted to a none conservative form how you do that? You simply use, the product rule for differentiating these two terms and use the continuity equation.

So, if you do that you will get rho. The remaining terms will be zero because of continuity equation. So, this in a short hand notation is rho, total derivative of enthalpy. You are familiar with the capital Dh/Dt that is the total rate of change is the change due to change with respect to time at a given location plus due to advection to a different location where you get a new velocity field and the new scalar field.

If this is rho Dh/Dt, what these two terms together are? Dp/Dt, right. And this terms you can write as minus p delta u k/ delta x k with an understanding that delta u j/ delta x j is equal to delta u k/ delta x k, whatever. Delta u j/ delta x j is what? Delta u 1/ delta x 1 plus delta u 2/ delta x 2 plus delta u 3/ delta x 3 so it does not matter whether you use j k l m whatever? So, I have written it in this way is because this term gets cancelled with the corresponding terms in the right-hand side.

So, when you convert the internal energy expression into enthalpy expression the first observation is that the PDV work gets cancelled from both sides. So, you are left with

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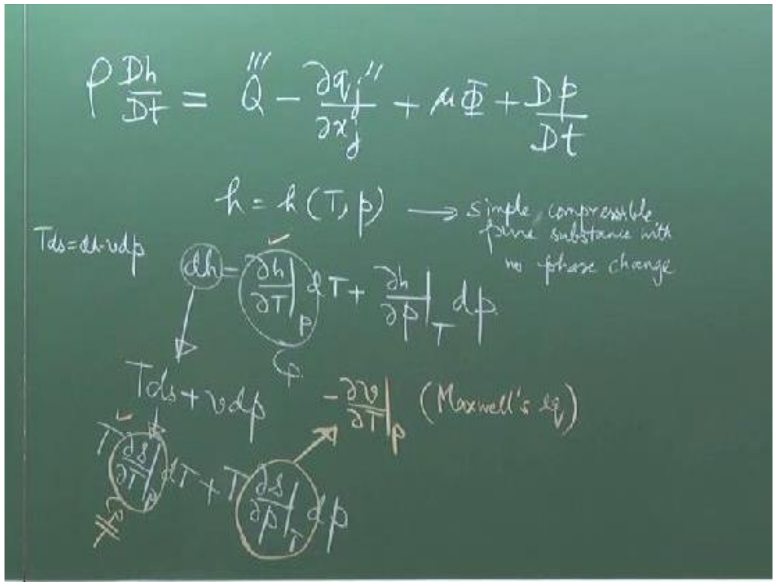

The minus DP/ Dt terms from left hand side we have brought to the right-hand side it has become plus Dp/Dt. Now, what we will do is we will write this enthalpy in terms of temperature. So, in general, what are the assumptions under which this is valid? We discussed it earlier, what are the assumptions under which this is valid? Simple, compressible pure substance with no change in phase. So, you can write dh. So, what is this? This is Cp.

By definition of Cp this is what is Cp. Now we will write this using one of the Tdh relationship, that Tdh is equal to dh minus udp. So, in place of dh we write Tds plus vdp. Next enthalpy is itself a function of temperature and pressure. So, you can write this as T. So, if you compare both sides then these terms and these terms they have the same coefficient. That means this will be what? Cp/ T.

And this term you can write express by changing from enthalpy to a measurable quantity by using one of the four Maxwell's equations. So, if you compare both sides now you can write

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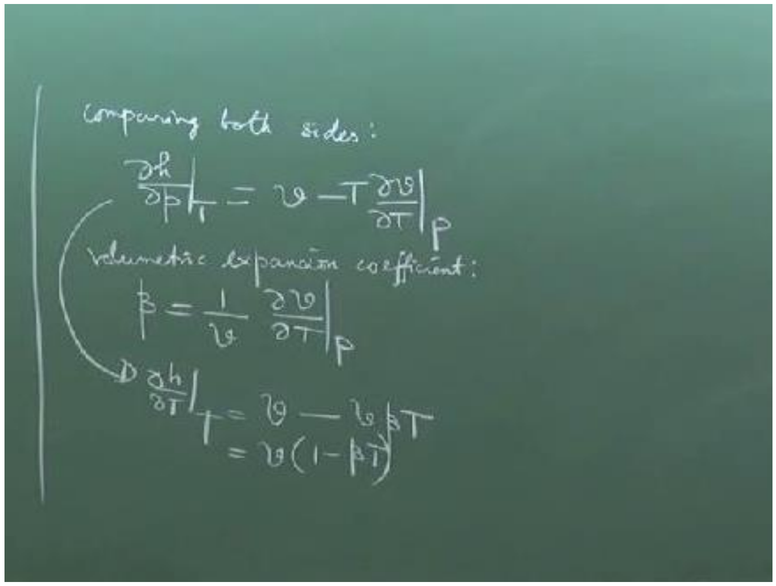

This is basically coefficient of Dp in both sides. Now, we know by the definition of volumetric expansion coefficient, beta is defined as this. So, volumetric expansion coefficient in qualitative in qualitative form is what? Change in volume per unit volume for each degree change in temperature and that has to be evaluated at some parametric conditions. So, the condition is here constant pressure.

So, you write dh equal to Cp dt in our expression what we require is capital Dh/ Dt of h. So, capital Dh/ Dt behaves mathematically in the same manner as small dh/dt. So, the same expression can be used so you can write

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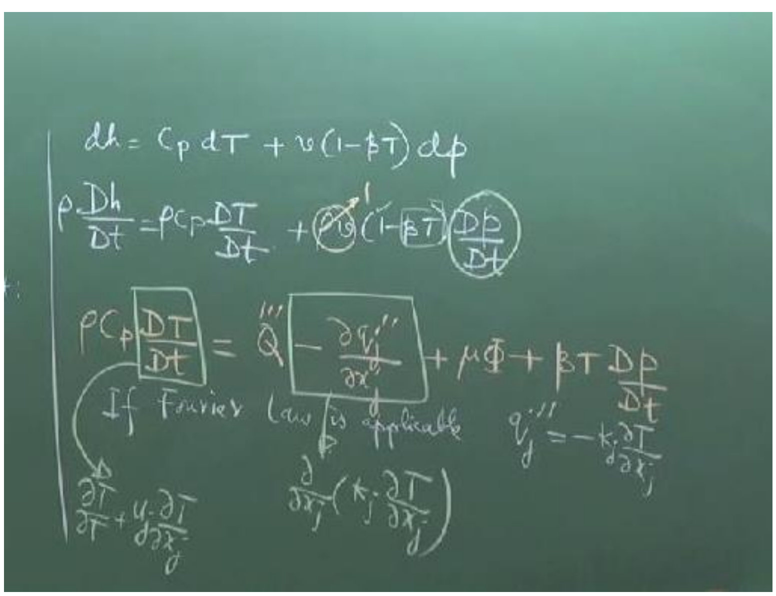

Dh/Dt equal to Cp DT/Dt plus. So, in the left-hand side what is required is rho, you multiply by rho all the terms. So, when you multiply all the terms rho into specific volume is equal one. Now you compare the left-hand side and right-hand side of this energy equation and we can observe one very interesting thing that this Dp/Dt and this Dp/Dt they get cancelled from both sides, right.

This is in the right-hand side and only this part not this part this part is still there only this part with Dp/Dt that gets cancelled. So, you are left with rho Cp. This equation still is not mathematically closed. Why it is not mathematically closed? Because it has an unknown temperature, it has also an unknown heat flux. So just like in Newton law of viscosity we expressed the stress in terms of the rate of deformation.

Here also we will express the heat flux in terms of temperature gradient. That is the constitutive behavior for heat transfer. Now, there are various mechanism by which the heat conduction can take place so there is no single constitutive behavior for the heat flux. So, for the heat flux there can be different constitutive behaviors but we will take as an example the constitutive behavior dictated by the Fourier law of heat conduction.

So, if Fourier law is applicable this is a very interesting thing and sometimes beginners have confusion on this. See we are driving an energy equation where fluid flow is also a part of the consideration. So, it is an equation, governing equation the equation that we have written there is an equation for convection. However, for heat flux we are still using the Fourier law of heat conduction. We are not using any other law that means that actually convection is not a fundamental mode of heat transfer.

The fundamental mode of heat transfer is still conduction but in convection what at best you can say what is happening is advection assisted conduction or advection assisted by conduction. whatever. So, the basic heat transfer mode still remain conduction because if you think of there is a solid boundary and from the solid boundary heat has to reach the fluid before it is advection that heat transfer from the solid boundary to the fluid is predominantly by, in fact is solely by conduction.

Because the fluid molecules adjacent to the solid boundary as stationary. So, conduction is the only mechanism by which heat get transferred from the solid boundary to the fluid and then from the fluid one layer to the other by combination of conduction and advection. So, conduction is present in all occasions and to calculate the heat flux we have to use law of conduction. So, then this term becomes.

This terms you can expand first in the non-conservative from even if you want to use the conservative from you can use the continuity equation to convert it to the conservative form just in the same we did for enthalpy and internal energy you can do it for temperature. So, this equation is the so-called energy equation that is the basic governing equation for convection. So, the left-hand side is the total change in the temperature again.

Rho into Cp comes out of the derivative because of mathematical simplification but not because of rho and Cp are constants. So, even if rho and Cp are not constants they will still come out of the derivative but they themselves may be variables. Right hand side this is the volumetric heat generation. This is the heat transfer due to the surface heat flux. This is the viscous dissipation and this is the pressure work.

So, in this term you can see that for flows with negligible compressibility effect this term is not important. Since in this particular course we will deal with incompressible flow, we will not consider that term to be any more important for the discussion and for the problems that we are solving subsequently. But wherever compressibility effects are important that last term is important. Example of application.

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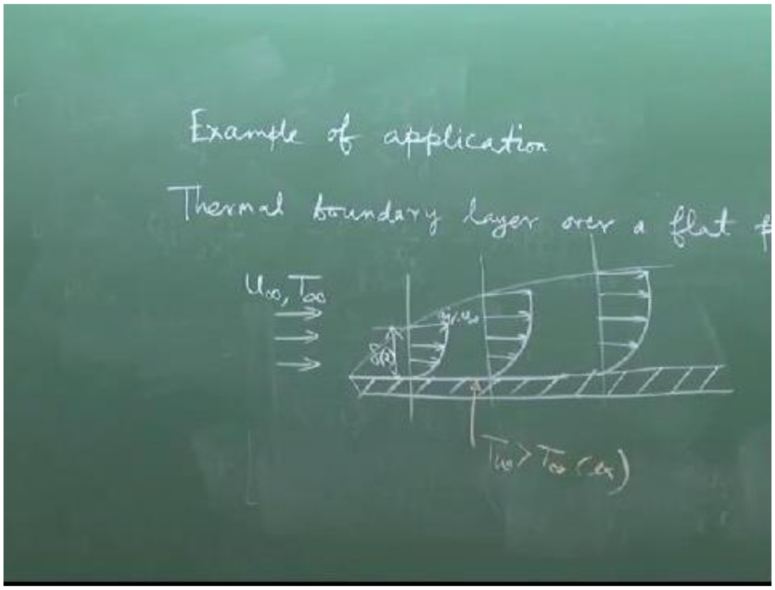

We consider the example of first Thermal boundary layers over a flat plate. So, first I will discuss about the problem qualitatively. Let us say that you have a fluid coming from far steam with a velocity u infinity and temperature T infinity. And let us say that the temperature of this wall is Tw which may be greater than T infinity or may be less than T infinity. Tw equal to T infinity is not a case of our interest because then there will be no heat transfer.

Because heat transfer is triggered by the temperature difference so we assume that T wall is not equal to T infinity as an example let us T wall greater then T infinity example. You can consider even the other example. Now as we have seen in fluid mechanics that there is a hydrodynamic boundary layer which grows because of viscous interactions. So, this is like 99 percent of u infinity and this thickness is delta which is a function of x.

This much we have understood while discussing about the hydrodynamic boundary layer. Now what about the heat transfer? Let us try to draw a separate sketch for discussing what happens for the heat transfer. Let us try to draw a separate sketch for discussing what happens for the heat transfer. So, let us draw the plate

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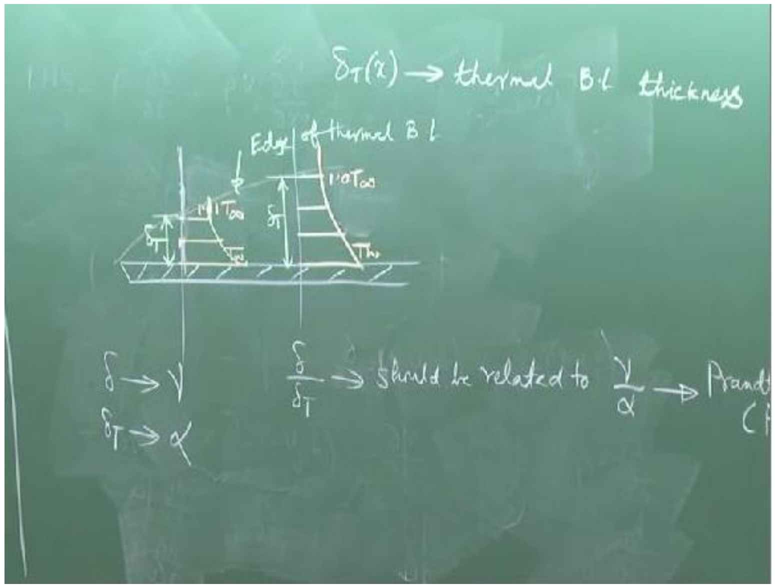

Consider a section here temperature is T wall, right. If you go further away from the wall little bit away the temperature is less than T wall so something like this. In this way, there will be a distance from the wall at which the temperature will almost come to T infinity, right. Is it 99 percent of T infinity? Will the temperature of the fluid become 99 percent of T infinity? Which here is more, temperature of the fluid or T infinity? I mean what is the lowest temperature of the fluid in other words?

The lowest temperature of the fluid is T infinity so temperature at any other location in the fluid has to be greater than T infinity for affective heat transfer from the wall to the fluid. So, the temperature where it assymtotically attains the value close to the T infinity is not 99% if the infinity but if you take as one percent gap one point zero one of T infinity, right. So, let us say this is one point zero one, T infinity. For all practical purposes, this is as good as T infinity.

Now, just like the velocity profile you can also plot the temperature profile. One important caution that in velocity profile we give vectors arrows, in temperature profile, please do not give vectors. Because I mean all of us understand that temperature is scalar and not a vector. So, do not give arrows just draw simple lines with arrow. So, this distance from the wall at which the temperature attains practically infinity this distance is called as thermal boundary layer thickness.

Just like the velocity where it attains practically infinity that distance from the wall is called a hydrodynamic boundary layer thickness. This is called as thermal boundary layer thickness. So now you have two boundary layers. In fluid mechanics, we have just one boundary layers so we talk about boundary layer. In heat transfer we have hydrodynamic boundary layer and thermal boundary layer. So, we have to distinguish these two by using these two different terminologies.

The boundary layer in fluid mechanics what we discussed from now onwards we will say hydrodynamic boundary layer and the heat transfer boundary layer is the thermal boundary layer. So, delta t which is the function of x is thermal boundary layer thickness. So, this thermal boundary layer as usual grows. So, the delta T, here is this one and this line is the age of the thermal boundary layer. Now I will ask you the very elementary and basic question that should first come to our mind.

What is the relationship between the thermal boundary layer thickness and the hydrodynamic boundary layer thickness that means can be say at least whether delta T is greater than delta, equal to delta, or less than delta at a given x. How can we say? What is the scientific bases from which we can talk about that? So, you understand that delta will depend on what? We have discussed it earlier.

Delta depends primarily for a given velocity field. Delta depends on which property of the fluid? Kinematic viscosity of the fluid. So, delta depends on nu. Similarly, delta T will depend on what? Alpha, the thermal diffusivity because more the thermal diffusivity greater will be the distance from the wall up to which the heating or cooling effect of the wall will be propagated. So just like Kinematic viscosity is a messenger of momentum disturbance in the fluid.

Thermal diffusivity is the messenger of the thermal disturbance within the fluid. So, if you have the thermal diffusivity that is k/rho C pk denotes the strength of conduction and rho Cp is the thermal inertia. So, it talks about the storage. So, conduction relative to the storage that ability is dictated by the thermal diffusivity so thermal diffusivity in many ways is analogous to the kinematic viscosity.

So, delta will scale with nu I mean it will be related to nu and delta T will be related to alpha. So, delta/ delta T should be related to nu/ alpha, these two have same dimensions meter square per second, as unit. So, this is a non-dimensional number called Prandtl number. So clearly depending on different values of Prandtl number it is possible that delta T may be greater than delta. Delta T may be equal to delta or delta T may be less than delta for Prandtl number equal to one delta T and delta are identical.

For Prandtl number less than one, delta less than delta T and for Prandtl number greater than one delta greater then delta T. So, with this little bit of qualitative understanding we will now derive the thermal boundary layer equations.

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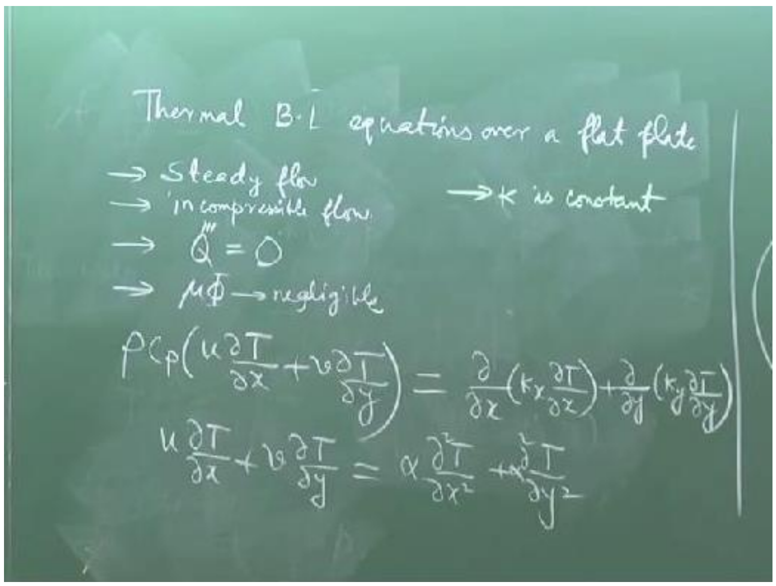

Thermal boundary layer equation over a flat plate. So, we will assume steady flow, incompressible flow. So, for incompressible flow the last term is not important in the energy equation, steady flow of course will mean that in the left-hand side the time derivative term will be zero. We are neglecting any volumetric heat generation and we are neglecting the viscous dissipation.

We will separately talk about certain problems let on where viscous dissipation is important normally for flow over a flat plate with open ambiance the viscous dissipation will not be important. Because viscous dissipation depends on square of the velocity of gradient so if the velocity gradient is very large then that will be important. So, in very small confinements if a fluid is constrained then viscous dissipation may be important.

So, we will talk about some such examples in this course but for flow over a flat plate we will assume that in general viscous dissipation may not be important. So, with this we will write this equation in the boundary layer co-ordinate. The boundary layer coordinates x, y coordinates we have discussed that what is a boundary layer coordinate. So, with the boundary layer coordinates so the left-hand side this term is zero because it is steady flow.

This terms becomes u delta T/ delta x plus v delta x plus v delta T/ delta y. Now, to proceed further we need to make a simplification and the simplification that we will make is that k is a constant. See we are not bothered about rho Cp are constants or not because anyway that is coming out of the derivatives but to bring k out of the derivatives we have to assume that k is a constant. So, we will make another assumption that k is constant.

That is thermal conductivity of fluid is constant. So, if you do that and then divide this k by rho Cp you will get the alpha in the right-hand side. So, your equation will become now this equation we can say that it is an energy equation for heat transfer for flow over a flat plate. But in terms of boundary layer consideration, the boundary layer consideration for hydrodynamic boundary layer what was the important consideration. Delta much, much less than x.

Here we will assume for thermal boundary layer theory that delta t much, much less than x.

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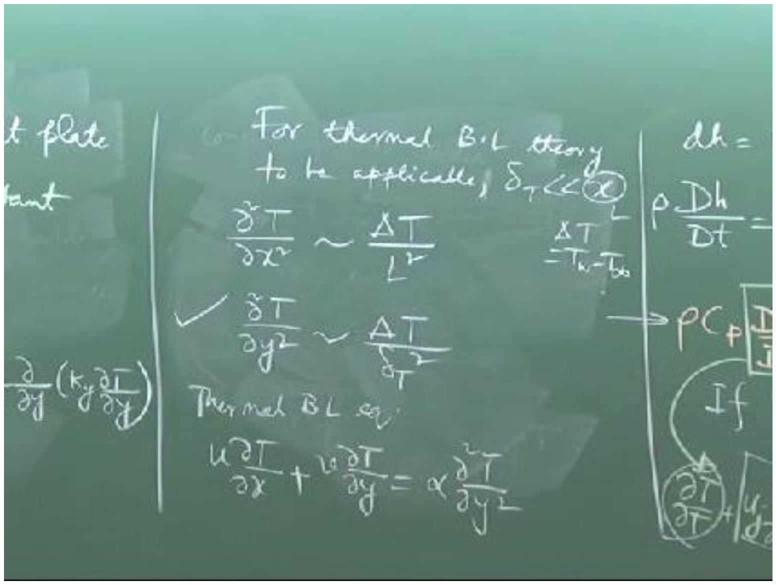

So, now what is the order of magnitude of this term? Yes. This is delta, delta x of delta T/ delta x that means some characteristic temperature difference by some square of characteristic length. What is the characteristic temperature difference? T1 minus T infinity we will call it in a short notation, delta capita T where delta T is T1 minus T infinity divided by L square. And so x characteristic is L for flow over a flat plate and this is what?

So out of these two-which one is more, clearly this is the dominating term. So, we will neglect this as compared to this so that gives rise to the thermal boundary layer equation. So, let us summarize the hydrodynamic and thermal boundary layer equations for flow over a flat plate before we solve these equations together.

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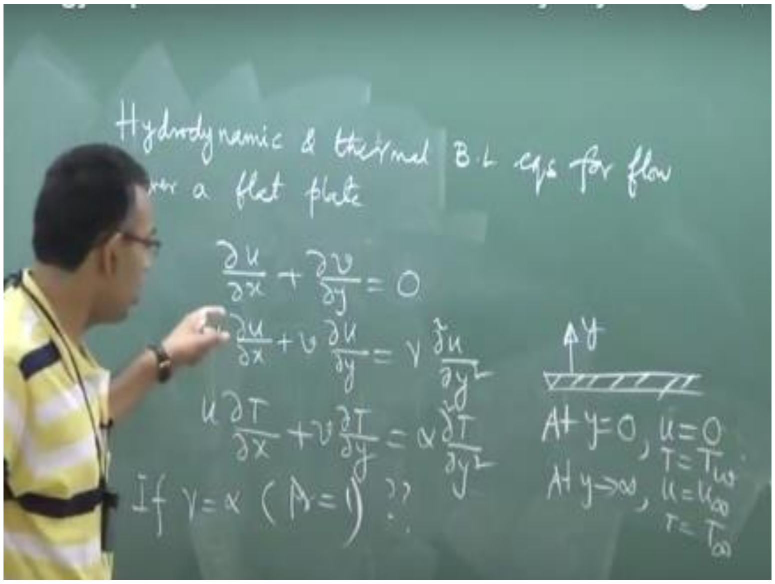

Remember, that our philosophy in forced convection is that the velocity field is already known we will use that velocity field to obtain the temperature field. So, the difference between the boundary layer momentum and energy equation is that this equation is what linear or none linear partial differential equation? It is none linear partial equation because of these terms u delta u/ delta x like that. But the energy equation is linear in T.

Because u is a separate function which you can get from the momentum equation solutions. So, once you get u then its linear in T. So, you can solve further temperature. Now it is very tempting to look into the similarity of these two equations because you see as if u is replaced by T and nu is replaced alpha. So, if you consider a situation if nu is equal to alpha that is Prandtl number is equal to one. then what happens?

If Prandtl number equal to one these two equations are the same basically same form. So, the question is will the solution be same so this is what Reynolds was thinking about. See Reynolds was a very cleaver scientist, mathematician whatever you call. Reynolds did a lot of work in fluid mechanics and his first thought was that how will I solve the energy equation. So, one possibility is that can I solve the energy equation without solving it?

It appears to be a time of paradox that how can you solve an equation without solving it. So, the possibility is that can I look into the analogy between these two equations and then using that analogy from the solution of these, we can directly tell what is they solution of this without solving this equation. And when Reynolds attempted that, that led to a very famous derivation in heat transfer which we will do now is known as Reynold's analogy.

Now, question is when you have the Reynolds analogy you also have to make sure that these equations are analogous not just in terms of equations but also boundary conditions. So, what are the boundary conditions? So, this is the flat plate. This is the y axis at y equal to zero what is the boundary condition? U equal to and t equal to T wall and the other boundary condition at y tends to infinity, u tends to u infinity and T tends to T infinity.

So, see all though for Prandtl number equal to one the equations are same but the boundary conditions they do not look the same. For example, it is a homogeneous boundary condition. It is a non- homogenous boundary condition and these values are different. So, can we convert these equations in such a way that not only the equations look similar the boundary conditions will also be the same.

The answer is very simple you renormalize the variables that is you define the new variable.

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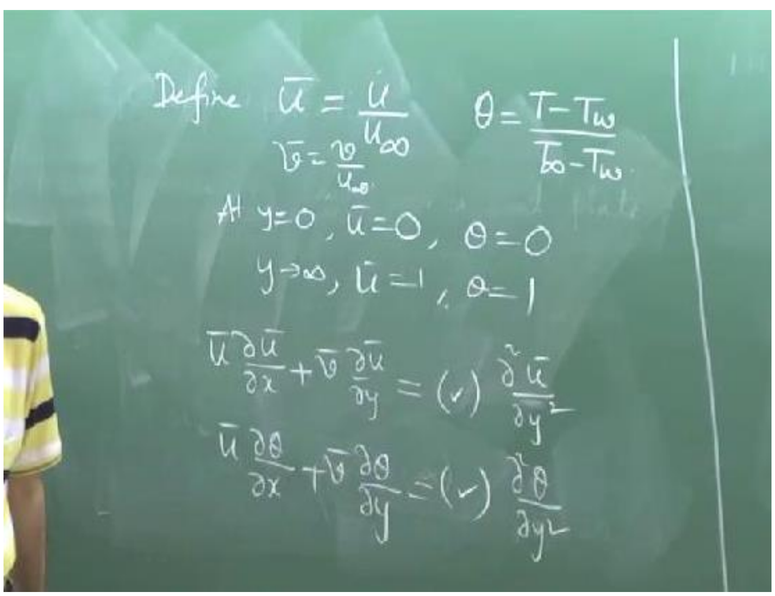

Always remember one thing I mean these are intuitive things but sometimes we do not give a thought to it. When you define none, dimensional temperature do not define d by d infinity like this because is the temperature difference that drives the heat transfer and not the absolute temperature itself. So, non-dimensional temperature are normally defined based on the ration of temperature differences and not temperatures.

So, non-dimensional velocity u/ u infinity but none dimensional temperature not T/ D infinity because it is the difference in temperature that drives the heat transfer. So now the boundary condition at y equal to zero, u bar equal to zero and theta equal to zero and y tends to infinity, u bar equal to one, and theta equal to one. And if cast that equation this is a very small exercise but I will leave it on you. You can just show that this equation will boil down to u bar delta u bar / delta x.

So, what you can do is you can change the variables from u so you also define v bar is equal to v/u infinity. So, you change the variables from u v to u bar, v bar and from d to theta by using this definition in that equation you will get equations again in the same form very little algebra. Nothing is there to show this even by observation you can say. So then now we are in a position that the governing equations and the boundary conditions are exactly the same.

What are the variables? Variables are u bar and theta. So, we can say that since governing differential equation and boundary conditions are same in form for u bar and theta we can conclude that u bar equal to theta. This is something which is not very intuitive because this either solution of a nonlinear equation this is a solution of a linear equation so some mathematical insight should get into that we will not be too much bothered about that but we will see what the consequence of this.

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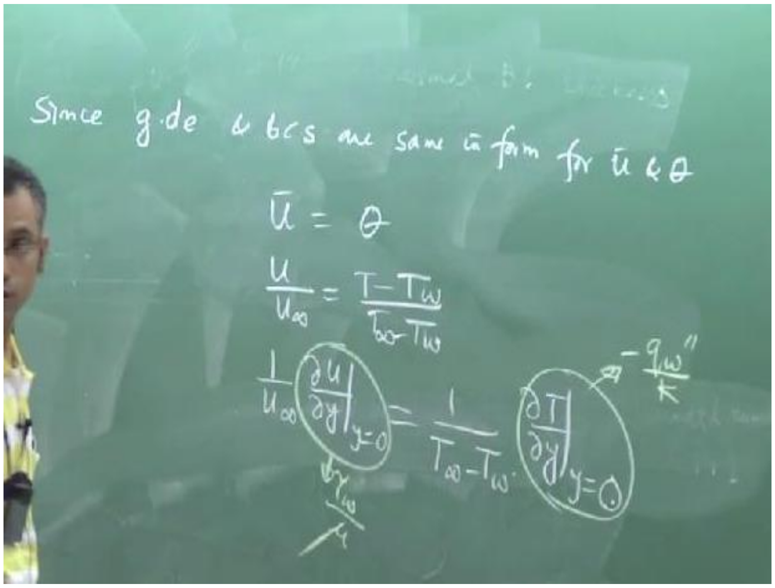

Is u/ u infinity equal to T minus T wall. Again, I am repeating that as an engineer we are not so much bothered about what is a temperature? What is the velocity? In fluid mechanics, what is the most important parameter the wall sheer stress that we are bothered about. And in heat transfer what is the most important parameter wall heat flux. So, wall sheer stress in fluid mechanics and wall heat flux in heat transfer.

And you can see that both will follow one from the other by differentiating this with respect to y at y equal to zero. So, we will differentiate both with respect to y at y equal to zero. So, what is this? This is tau w/ mu. And what is this? Minus wall hit plus by k. Q equal to minus k delta T/ delta y. So, 1/mu u infinity tau w.

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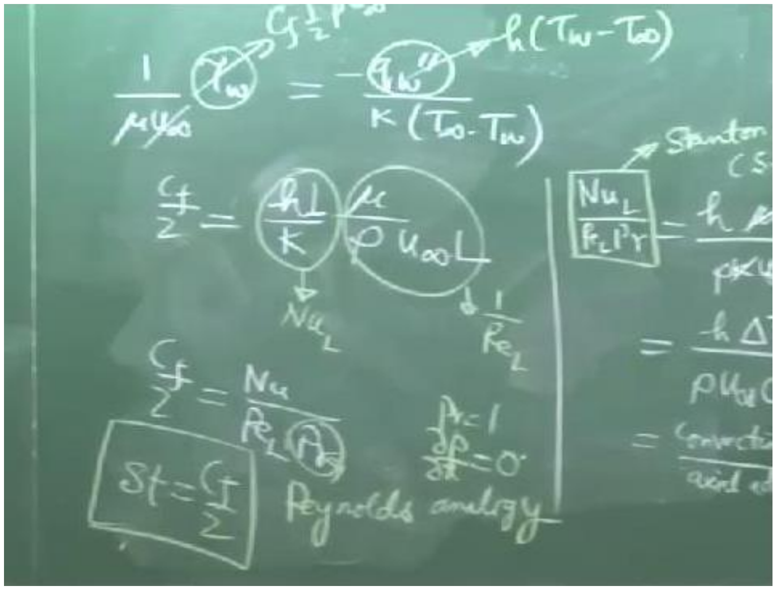

Now, tau w we can write in terms of skin friction coefficient, CF. So, tau w is equal to CF into half rho u infinity square. And wall heat flux? H into T w minus T infinity where h is the convective heat transfer coefficient. So, we can write one u infinity will cancel here Cf/2. So, we can multiply both numerator and denominator by L so what does it become? What is this? Nusselt number based on the length L and what is this 1/ Reynolds number.

For this particular problem, we can also write this as Nusselt number/ Reynolds number into Prandtl number. Why? Because we assume Prandtl number equal to one then only these two equations are the same nu are alpha are the same or Prandtl number equal to one. So, this is actually equal to one. Why we are doing this is because Nusselt number/ Reynolds number into Prandtl number has a very interesting physical interpretation. What is that?

Prandtl number is mu Cp/k. So, mu get canceled and k get canceled. What is this? You just multiply both numerator and denominator by delta T you will get a physical meaning. What is this? This is convection heat flux and this is axial advection. This is heat transfer due to fluid flow along x direction. So, this you can say that convection flux by axial advection plus. Because these are all none dimensional numbers this ration is also a non-dimensional ratio which is called Stanton number.

So, we can write that Stanton number equal to Cf/ 2 this is called as Reynolds analogy. The beauty of this analogy is that for fluid mechanics if you can find out what is Cf then you can say what is the corresponding heat transfer parameter without solving anything. But this being a very beautiful and simplistic expression there are major assumptions associated with that. So, what are the major assumptions associated with this Reynolds analogy?

The most important assumption is first of all there is no pressure gradient. That means it flows over a flat plate if there is a pressure gradient then you will have an extra pressure gradient term in the momentum equation then there will be no analogy of the momentum and the energy equation, right. So, there is no pressure gradient that means flow over a flat plate and then Prandtl number must be equal to one.

So Prandtl number equal to one, delta p/ delta x equal to zero, these two must be satisfied. So, this Reynold analogy is very nice but it can be applied only with Prandtl number equal to one in addition to the assumptions that we have considered. So, the situation is that when Prandtl number is not equal to one what happens? We will discuss about that in the next lecture. Thank you.