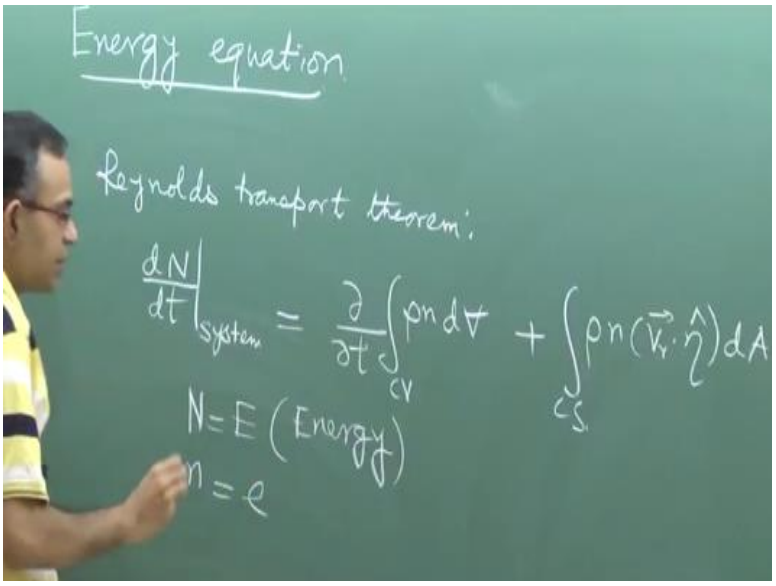
**Conduction and Convection Heat Transfer**
**Prof. S.K. Som**
**Prof. Suman Chakraborty**
**Department of Mechanical Engineering**
**Indian Institute of Technology – Kharagpur**

**Lecture - 27**
**Energy Equation - I**

So, so far we have discussed about the momentum equation and its applications. Today we will discuss about the energy equation.

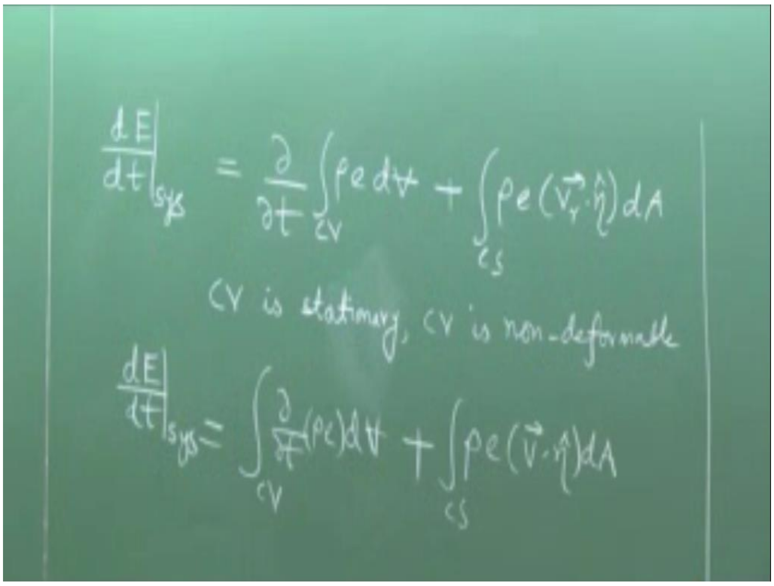
**(Refer Slide Time: 0:53)**




By Energy equation what we mean is that we want to conserve the total energy of the system. So, if we write the Reynolds Transport Theorem, if we write this, then for the total energy conservation N should be equal to E. That is the energy of the system. Remember this is the total energy as per the purview of the loss of thermodynamics. So, that means this is the sum of internal energy, kinetic energy and potential energy.

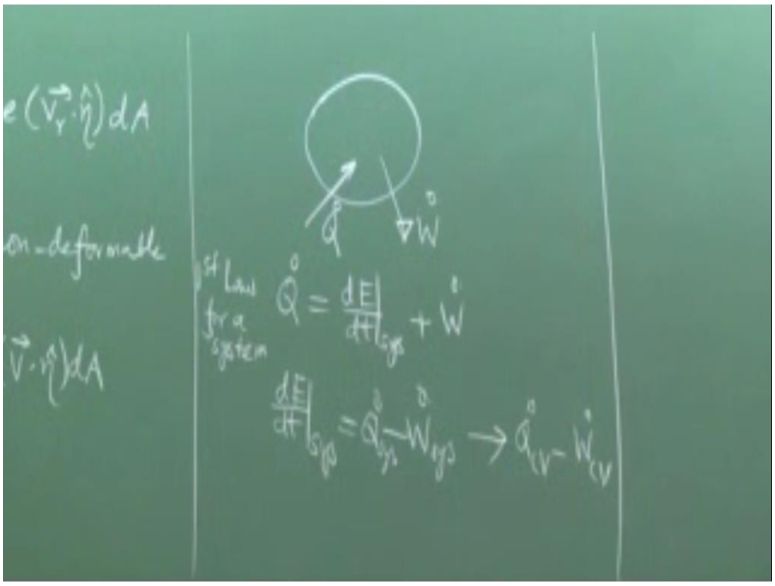
That is what we are talking about as the total energy. Now when N is equal to the total energy, n is the specific energy. That is the total energy parting with mass.

**(Refer Slide Time: 03:12)**




So, you can write. We will further assume that the control volume is steady and control volume is- sorry, stationary, not steady, and control volume is non-deformable. So, the stationary control volume implies that the relative velocity is same as the absolute velocity and non-deformable control volume means you can take this time derivate within the integral. So, that means, you can write

**(Refer Slide Time: 05:44)**

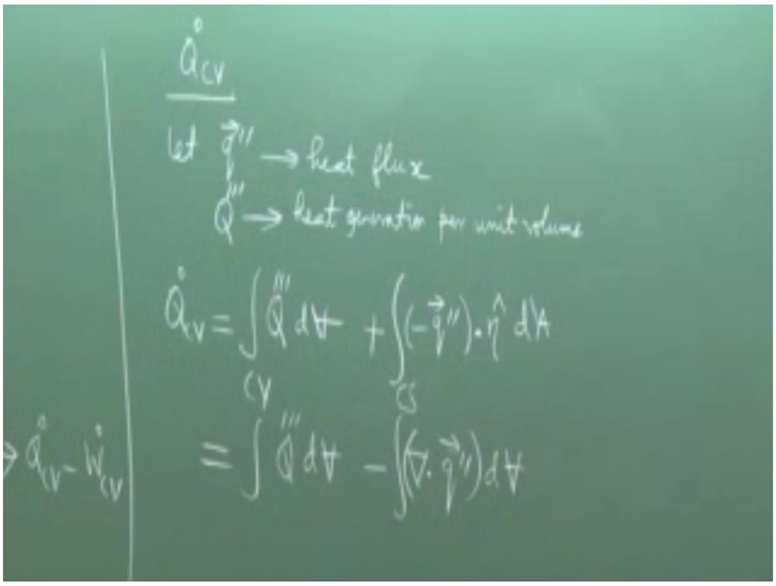



Now what we will do is. We will express this in terms of the first law of thermodynamics. So, think of a system on a controlled mass. The first law of thermodynamics in a rate expression can be written as, if you say that the heat transfer to the system is Q dot and the work done is W dot, then Q dot, this is the first law. First law for a system. Therefore, you can write, delta E delta t is equal to Q dot minus W dot.

And this is Q dot system minus W dot system, in the limit as delta t tends to zero, the system tends to the control volume. That is how we derive the Reynolds Transport theorem. So, therefore this will tend to Q dot cv minus W dot cv, where cv stands for the control volume. So, here in the left-hand side, we have expressed it in terms of the heat transfer and the work done.

Next our objective will be to write the expressions for heat transfer and work done. First let us write the expression for heat transfer.

**(Refer Slide Time: 08:12)**

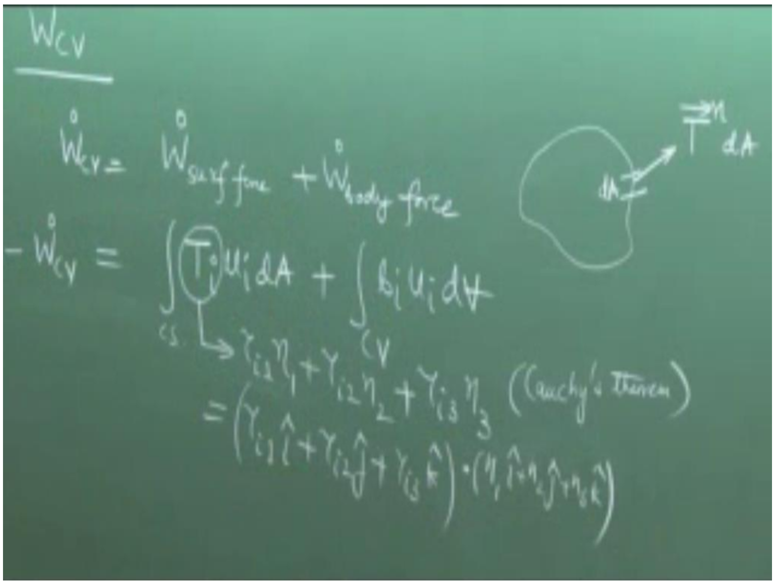



Just like in continuum mechanics you have two types of forces, surface force and body force, similarly heat transfer you have two types. One is surface heat transfer and another is volumetric heat transfer. Solet us say, let this be the heat flux, and - So, Q dot c. This is heat generation part unit volumes, so you multiply it with the elemental volume and integrate it over the control volume. So, this is for the volumetric part.

For the surface heat transfer you have to keep in mind that, if we have a heat flux like this, this is actually a negative heat transfer because heat transfer to the system or to the control volume is positive as per the first law of sign convention. I mean, we could also make a different sign convention, but by writing this we have made this sign convention only. So, this will be adjusted with a negative sign, we can write this as -

By using the divergence theorem, we are converting the area integral into the volume integral, so this is for the heat transfer. Next let us find out what is the work done.

**(Refer Slide Time: 11:43)**




Now in mechanics when we write work done, we write work done by the force as positive and in thermodynamics you have work done by a system against a resistance is actually a positive work. So, that means again we have a conflicting sign convention. To adjust to that, we will write, in terms of mechanics, whatever is the expression for work done when we substitute it in the loss of thermodynamics same expression, but with a negative sign.

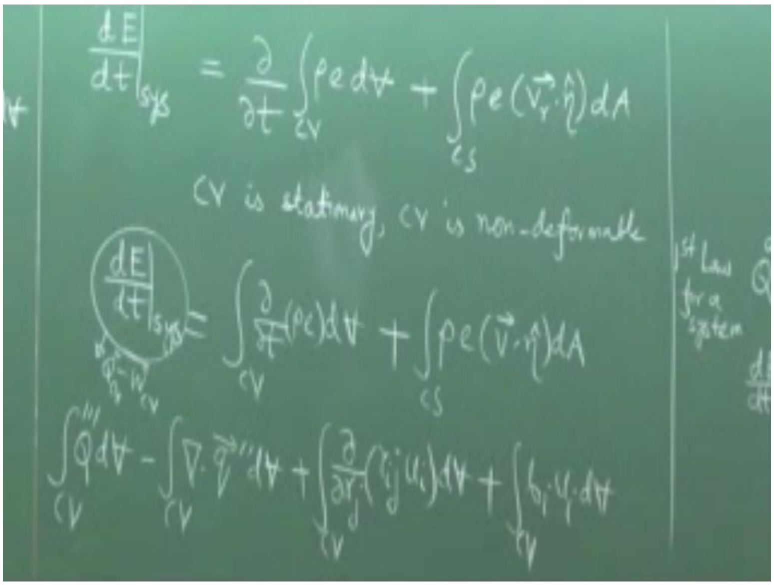
So, now work done is because of what, because of surface force and body force, right? Work done is because of presence of forces and there are two types of forces, surface force and body force. So, work done is due to what? Work done by work done due to surface force plus work done multiplied by body force. The surface force, work done due surface force. Now let us say that this is a controlled surface and let us say that you have an area dA.

What is the force per unit area on this? Recall, that is nothing but the traction vector. This is force per unit area. So, this time day is the total force and work done is the, what? The dot product of force and velocity, rate of work done, right? W dot. That is the dot product of force and velocity. So, this will be integral of this vector dot with velocity vector. That means Ti, Ui, dA.

Dot product of two vectors means basically multiplying their individual component and summing up. So, if there are two vectors are A and B that dot product is Ai, Vi, right? So, you have Ti plus rate of work done due to body force. Body force per unit volume when we derive Navier-Stokes equation we assume body force per unit volume is B, so bi, Ui, dV. This is b dot velocity.

So, basically, we will write this as minus of w dot cv to adjust for the sign or w dot cv equal to minus of these whatever. Now what is Ti? Ti could be written by using the Cauchy's theorem. So, Ti is this. Tau ij eta j, right? Cauchy's theorem. So, this you can write as this, just another way of writing this.

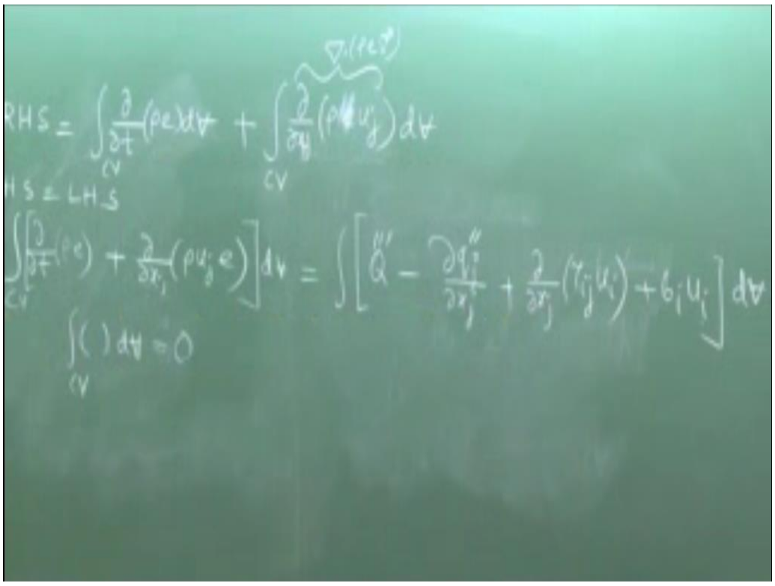
**(Refer Slide Time: 18:03)**




Now, Ti, Ui, dA, so let us give this a name tau i. So, how can we write this? Del dot tau i, u i. That means in index notation del del x j of… This is just another way of writing del dot tau i u i. So, now you can write this. So, we will now substitute that expression here. In the dE dt system it is Q dot minus W dot. So, we will substitute the Q dot and minus W dot. So, Q dot cv minus W dot cv. Q dot cv is integral of this.

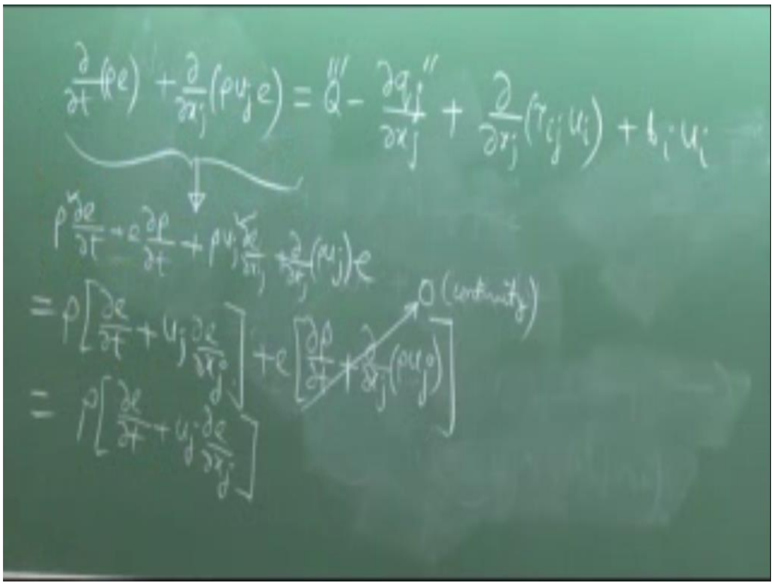
I am deliberately trying to mix up notations and symbols so that you get familiar with different types of symbols. Of course, this could be written as del del xj of qj, right? Same thing. This is heat transfer minus W dot cv. Now let us come to the right-hand side. So, left hand side all terms are expressed as volume integrals. Let us express right hand side all the terms also as volume integrals.

**(Refer Slide Time: 23:18)**




So, right hand side, what is this term? Del.rho e v or del del x j of rho e u j. So, this is del dot rho e v. So, now we can write, right hand side is equal to left hand side. Okay? So, all terms expressed as volume integrals. So, this is as good as writing integral of something into dV equal to zero, since the control volume is arbitrary, this is as good as something is equal to zero. So, that means you can write this.

**(Refer Slide Time: 26:23)**




Now, the left-hand side what form is this? Is it a conservative form or a non-conservative form? This is a conservative form because we started with a basic conservation equation. We can convert this into a non-conservative form by using the continuity equation. So, let us do that. So, what we have done is, we have used the product rule of derivate. Now we can write this is equal to rho by combining these terms and plus e.

So, this is zero y the continuity equation. So, this becomes rho, so if it is written in this way, then that is the non-conservative form of the equation. We have to keep in mind, again I am repeating. Although rho is coming out of the derivate that doesn’t mean that rho is a constant. Rho is coming out of the derivate because of simplification using the continuity equation. Now this equation, physically what does it represent?

It represents the conservation of total energy in a controlled volume perspective. Now, in heat transfer we are interested in conservation of thermal energy. So, if you have conservation of total energy how can you get conservation of thermal energy from there. So, in the total energy, you have mechanical energy plus thermal energy. So, you have to write an expression for concentration of mechanical energy, mechanical means kinetic and potential.

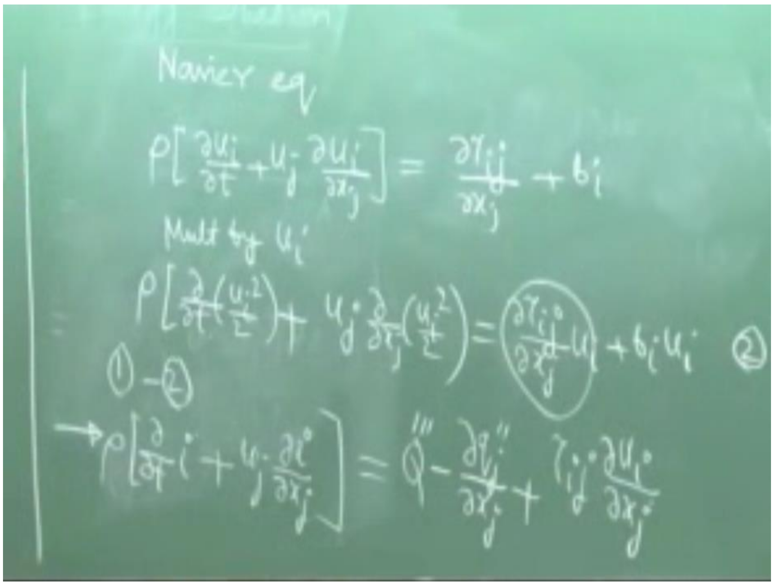
So, what essential is this, this is kinetic plus potential plus internal. So, we have to write something in terms of the internal energy only. So, we have to subtract the equation that involves kinetic and potential energy. That is the mechanical energy equation. Now, one has to be careful in one thing. In this equation when we write e actually we are including within it the internal energy, kinetic energy but not potential energy, why?

Because we are already having an expression for work done by body force. So, if we now again write potential energy here, then that will duplicate the same effect. So, either you do not write the work done by body force but put the corresponding potential energy here or you do not put the potential energy here but you put the work done by body force. So, here we have preferred to write the work done by body force.

Therefore, we will not include the potential energy here. Now, work done by potential energy if it is not included, if the energy, if the potential energy is not included that means work done by body force is already included then, this equation will give you the mechanical energy. So, this equation has given you the total energy where you have somewhere the potential energy effect and internal energy and kinetic energy.

We want to get another equation which will give us the mechanical energy, mechanical energy equivalent of the one. So, mechanical energy that means the kinetic and the potential, potential will be in form of work done by body force. So, which equation can give us.

**(Refer Slide Time: 33:45)**




So, Navier equation, what was it? It was an equation of momentum balance. So, in a conservative form, rho…This equation why we start with a Navier equation but not the Navier-Stokes equation is that, we want to generalize it for all types of fluids. So, once we commit ourselves to Navier-Stokes equation, we are restricted to Newtonian and Stokesian fluid.

But when we write the Navier equation or the Cauchy equation whatever we are not bothered about what is the special type of fluid that we are talking about for any fluid you can use this. So, now can you convert this momentum balance equation into an energy equation? Only a small trick you have to play to do that. See, you want to convert it into mechanical energy equation.

That is kinetic energy and potential. Potential will be work done because of the body force. So, how can you convert this to kinetic energy? You multiply this with Ui. So, then it becomes, Ui, del Ui, del T, that is del del T of Ui square by 2. Ui square by 2 is kinetic per unit mass. So, you multiply by Ui. So, rho -Let us say this is equation number one. This is equation number two.

Now you subtract equation number two from equation number one. What you get? You will get an equation that governs the change in internal energy only because this is kinetic, this contribution is due to potential and you will see that when you subtract this e includes what? Internal and kinetic. So, internal and kinetic minus kinetic will become only internal. So, if you subtract, equation two from equation one.

We are using the symbol i for internal energy, the reason is that we have already used u for velocity. So, just to not to confuse with the symbol, whatever symbol u that you have used for internal energy in thermodynamics that same is i. If you subtract from this term, this term, then what will be left? Use the product rule, one time will be cancelled. What will be left is tau i j, del u I, del x j.

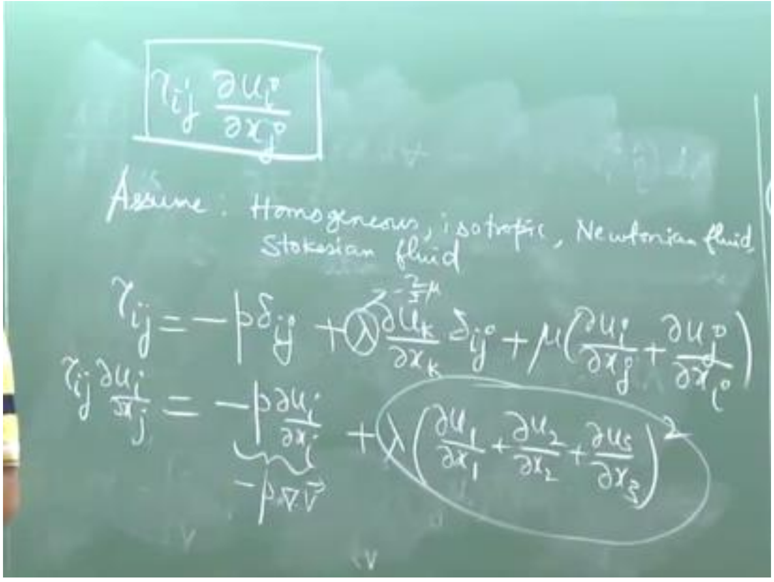
So, see, the change in internal energy, this is the total rate of change of internal energy, the rate of change of internal energy may be due to volumetric heat generation, surface heat transfer but also due to stresses in the flow field. So, this part, this contribution comes directly from fluid mechanics. So, tau ij del ui del xj physically what does it mean. So, this is like a heat source. That means this will result in heating.

You may ask that, why can’t it result in cooling. We will give an answer to that, that it will trivially give rise to heating and not cooling. So, it is something like this. Let us say that you are shearing the flow. If you are shearing the flow then two layers of the fluid are sliding against each other, so as if you are rubbing your palms. So, if you are rubbing your palms, this work is not utilized.

But this work is irreversibly converted into internal energy through viscous dissipation. So, this is called as viscous dissipation where there is an irreversible conversion of work to intermolecular form of energy. So, if you rub your palms you cannot get any work out of that. Simply it will generate heat. So, it is effectively like a heat source. But now the question always come that well. We are claiming that it is like a heat source, but where is the guarantee that it will always give rise to heating and not cooling.

So, we have to show that whatever is the effect associated with it, that effect associated with it because of viscous part of the stress, so remember that this tau ij also has hydrostatic part of the stress. So, we will isolate the hydrostatic part of the stress and consider only the deviatory part which is the viscous part and show that the viscous part contribution to this term is always positive. That means it will give rise to heating and not cooling.

**(Refer Slide Time: 41:39)**

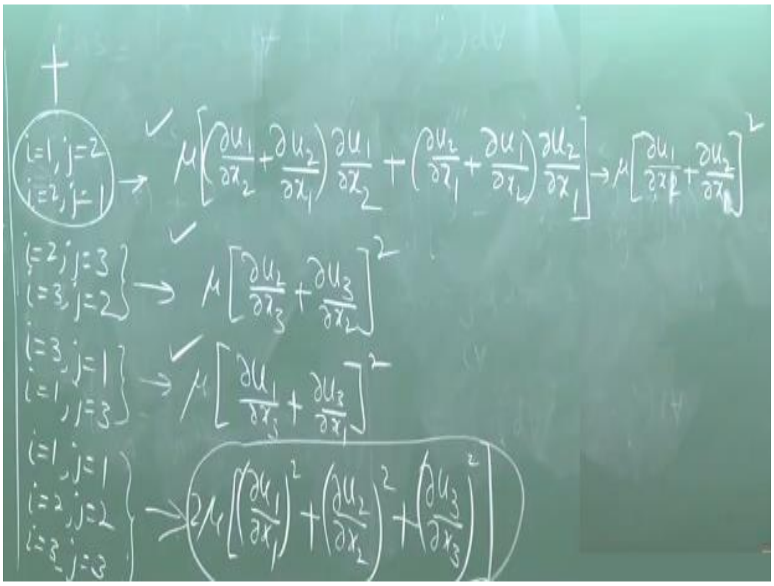



So, let us see the term tau ij. So, we assume homogeneous, isotropic and Newtonian fluid and also Stokesian fluid. So, tau ij is minus p delta ij, plus lambda, del Uk, del xk, delta ij plus mu, del ui, del xk plus del uj del xi. This we discussed in quite some length. So, tau ij del u by del xj. What will be the hydrostatic component? Remember delta ij is equal to one when j is equal to i. So, it will become minus p into del ui del xi. What is this?

This is minus p multiplied by del dot v. So, this is like the p dv work in thermodynamics because the divergence of the velocity vector gives the volumetric change. So, this is analogous to p dv. So, we will isolate that with the viscous component, remember this lambda is equal to minus two third of mu. Stoke side what this is. So, plus lambda, what is del Uk, del xk multiplied by delta ij, into del Ui, del xi?

See, del Uk, del xj is del U1, del x1, plus del U2 del x2 plus del U3 del x3 Delta ij into del Uy del xj is del u y, del xi. That is also del U1, del x1, plus del U2, del x2, plus del U3 del x3, so it becomes del u1, del x1 plus del 2 del x2, plus del u3, del x3 whole square This term, this into del ui del xj, plus the remaining terms.

**(Refer Slide Time: 46:03)**



So, here we will write several terms, first i equal one, j equal to two, i is equal to two, j equal to one. This is one set. Then i equal to 2 k equal to 3, i equal to 3, j equal to 2. This is one set. Then i equal to 3, j equal to 1, i equal to one, j equal to 3. This is another set. Then i is equal to one, j is equal to one, i is equal to 2, j is equal to 2, i is equal to 3, j is equal to 3. Total 9, right? So, first we will consider this set.

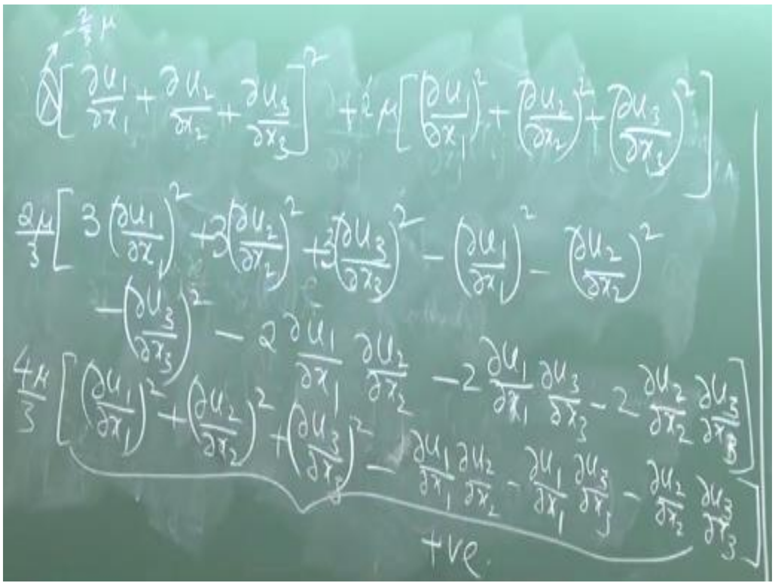
So, mu del u1 del x1 plus del u2 del x2 multiplied by del u1 del x1. Okay, let me write one term then it will be easier to write. Let me write the full one term. So, this is del u1 del x2 square plus del u2 del x1 square plus 2 multiplied by del u1 del x2 multiplied by del u2 del x1. So, it is like a plus b whole square formula, right? So, this is mu multiplied by…Okay. So, can you physically interpret this. This is like del u del y plus del v del x, rate of deformation.

So, square of rate of deformation times the viscosity is actually giving rise to a heating effect. Okay? So, how the kinematics of the fluid is getting converted into heat transfer effect, that is something which is not so intuitive but you can see from here. So, this is mu multiplied by… and this is … Now what will be this. i equal to one, j equal to one means? Right? So, now it is trivially clear that these 3 terms together are positive.

Of course, this term is positive, but these term together with these term whether it is positive negative. So, now it is trivially clear that these three terms together are positive. Of course, this term is positive, but these term together with these term whether it is positive or negative it has to be adjudged, why because lambda is negative. Lambda is not positive; lambda is minus two third of mu where mu is positive.

So, we will try to show that these term together with these term eventually becomes positive. So, let us try to do that exercise.

**(Refer Slide Time: 51:36)**




Lambda is equal to minus two third of mu. So, if you take 2 mu as common or 2 mu by 3 as common. So, these essentially becomes like, if you take 2 as common, so 3 minus 1 is 2. If you take 2 as common, then it is 4 mu by 3. So, this is a positive term. Like a square plus b square plus c square minus ab minus bc minus ca. That is half multiplied by a minus b whole square plus b minus c whole square plus c minus a whole square.

So, this is positive, right? So, our conclusion is that, the deviatory component of these will give rise to trivially heating and not cooling because this entire term, this plus whatever we have written, sum total of this is a positive term and that term is known as viscous dissipation. Now task is not yet complete because we have written the equation in terms of internal energy.

But in experiment we do not measure internal energy, we do not have any internal energy meter. We have say thermometer to measure temperature. So, we have to convert the internal energy based equation to a temperature based equation which will give us a governing equation for temperature which is the so-called energy equation in convection, that we will take up in the next lecture, thank you.